① Halla la tasa de variación media de $f(x) = \sqrt{x-1}$ en los intervalos $[1,2]$ y $[1,10]$

$$T.V.M.[a,b] = \frac{f(b) - f(a)}{b-a}$$

TASA DE VARIACIÓN MEDIA EN $[a,b]$

$$T.V.M\,[1,2] = \frac{f(2) - f(1)}{2-1} = \frac{\sqrt{1} - \sqrt{0}}{1} = 1$$

$$T.V.M\,[1,10] = \frac{f(10) - f(1)}{10-1} = \frac{\sqrt{9} - \sqrt{0}}{9} = \frac{3}{9} = \frac{1}{3}$$

② Dadas las funciones $f(x) = \frac{1-x^2}{3}$ y $g(x) = \frac{2}{x+1}$ determina la T.V.M. de ambas en los intervalos $[2,3]$ y $[-3,-2]$

$$T.V.M_f\,[2,3] = \frac{f(3) - f(2)}{3-2} = \frac{\frac{1-3^2}{3} - \frac{1-2^2}{3}}{1} = -\frac{5}{3}$$

$$T.V.M_f\,[-3,-2] = \frac{f(-2) - f(-3)}{-2-(-3)} = \frac{\frac{1-(-2)^2}{3} - \frac{1-(-3)^2}{3}}{1} = \frac{5}{3}$$

$$T.V.M_g\,[2,3] = \frac{g(3) - g(2)}{3-2} = \frac{\frac{2}{3+1} - \frac{2}{2+1}}{1} = -\frac{1}{6}$$

$$T.V.M_g\,[-3,-2] = \frac{g(-2) - g(-3)}{-2-(-3)} = \frac{\frac{2}{-2+1} - \frac{2}{-3+1}}{1} = -1$$

③ Dada $f(x) = x^2 + 2x$, obtener $f'(2)$ utilizando la definición de derivada de una función en un punto.

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Derivada de una función en el punto $x = a$

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$$f'(2) = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0} \frac{(2+h)^2+2(2+h)-8}{h} =$$

$$= \lim_{h\to 0} \frac{4+4h+h^2+4+2h-8}{h} = \lim_{h\to 0} \frac{h^2+6h}{h} = \lim_{h\to 0} \frac{h(h+6)}{h} =$$

$$= \lim_{h\to 0} (h+6) = 6$$

④ Utilizando la definición de derivada, calcula $f'(3)$ en las siguientes funciones:

a) $f(x) = \frac{3x-2}{7}$ b) $f(x) = x^2-4$

c) $f(x) = (x-5)^2$ d) $f(x) = \frac{2+x}{x}$

a) $$f'(3) = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h} = \lim_{h\to 0} \frac{\frac{3(3+h)-2}{7} - 1}{h} =$$

$$= \lim_{h\to 0} \frac{\frac{7+3h}{7} - 1}{h} = \lim_{h\to 0} \frac{7+3h-7}{7h} = \lim_{h\to 0} \frac{3h}{7h} = \frac{3}{7}$$

b) $$f'(3) = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h} = \lim_{h\to 0} \frac{(3+h)^2-4-5}{h} =$$

$$= \lim_{h\to 0} \frac{9+6h+h^2-9}{h} = \lim_{h\to 0} \frac{6h+h^2}{h} = \lim_{h\to 0} \frac{h(6+h)}{h} =$$

$$= \lim_{h\to 0} (6+h) = 6$$

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c) $f'(3) = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h} = \lim_{h\to 0} \frac{(3+h-5)^2-4}{h} =$

$= \lim_{h\to 0} \frac{(h-2)^2-4}{h} = \lim_{h\to 0} \frac{h^2-4h+4-4}{h} = \lim_{h\to 0} \frac{h^2-4h}{h} =$

$= \lim_{h\to 0} \frac{h(h-4)}{h} = \lim_{h\to 0} (h-4) = -4$

d) $f'(3) = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h} = \lim_{h\to 0} \frac{\frac{2+3+h}{3+h} - \frac{5}{3}}{h} =$

$= \lim_{h\to 0} \frac{\frac{5+h}{3+h} - \frac{5}{3}}{h} = \lim_{h\to 0} \frac{\frac{15+3h-15-5h}{3(3+h)}}{h} = \lim_{h\to 0} \frac{-2h}{3h(3+h)} =$

$= \lim_{h\to 0} \frac{-2}{9+3h} = -\frac{2}{9}$

⑤ Utilizando la definición derivada, calcula las derivadas de las siguientes funciones:

a) $f(x) = \frac{5x+1}{2}$ b) $f(x) = 3x^2-1$ c) $f(x) = \frac{1}{x-2}$

d) $f(x) = \frac{x-1}{x}$ e) $f(x) = x^2+2x+3$ f) $f(x) = \sqrt{x}$

g) $f(x) = \frac{3}{x}$ h) $f(x) = \operatorname{sen}(x)$ i) $f(x) = e^x$

j) $f(x) = \ln(x)$ k) $f(x) = \sqrt[3]{x}$ e) $f(x) = \operatorname{tg}(x)$

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$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$ FUNCIÓN DERIVADA

a) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{5(x+h)+1}{2} - \frac{5x+1}{2}}{h} =$

$= \lim_{h\to 0} \frac{\frac{5x+5h+1-5x-1}{2}}{h} = \lim_{h\to 0} \frac{5h}{2h} = \frac{5}{2}$

b) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{3(x+h)^2 - 1 - (3x^2-1)}{h} =$

$= \lim_{h\to 0} \frac{3(x^2+2xh+h^2) - 1 - 3x^2 + 1}{h} = \lim_{h\to 0} \frac{6xh+3h^2}{h} =$

$= \lim_{h\to 0} \frac{h(6x+3h)}{h} = 6x$

c) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{x+h-2} - \frac{1}{x-2}}{h} =$

$= \lim_{h\to 0} \frac{\frac{x-2-(x+h-2)}{(x+h-2)(x-2)}}{h} = \lim_{h\to 0} \frac{-h}{h(x+h-2)(x-2)} =$

$= \lim_{h\to 0} \frac{-1}{(x+h-2)(x-2)} = \frac{-1}{(x-2)^2}$

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d) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{x+h-1}{x+h} - \frac{x-1}{x}}{h} =$

$= \lim_{h\to 0} \frac{\frac{x^2+xh-x-x^2-xh+x+h}{x(x+h)}}{h} = \lim_{h\to 0} \frac{h}{xh(x+h)} =$

$= \lim_{h\to 0} \frac{1}{x(x+h)} = \frac{1}{x^2}$

e) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2+2(x+h)+3-(x^2+2x+3)}{h} =$

$= \lim_{h\to 0} \frac{x^2+2xh+h^2+2x+2h+3-x^2-2x-3}{h} = \lim_{h\to 0} \frac{2xh+h^2+2h}{h} =$

$= \lim_{h\to 0} \frac{h(2x+2+h)}{h} = \lim_{h\to 0} (2x+2+h) = 2x+2$

f) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} =$

$= \lim_{h\to 0} \frac{(\sqrt{x+h}-\sqrt{x})\cdot(\sqrt{x+h}+\sqrt{x})}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0} \frac{(\sqrt{x+h})^2-(\sqrt{x})^2}{h(\sqrt{x+h}+\sqrt{x})} =$

$= \lim_{h\to 0} \frac{h}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$

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g) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{3}{x+h} - \frac{3}{x}}{h} =$

$= \lim_{h\to 0} \frac{\frac{3x - 3x - 3h}{x(x+h)}}{h} = \lim_{h\to 0} \frac{-3h}{x h (x+h)} = \lim_{h\to 0} \frac{-3}{x(x+h)} =$

$= -\frac{3}{x^2}$

$\operatorname{sen}(\alpha+\beta) = \operatorname{sen}\alpha\cos\beta + \cos\alpha\operatorname{sen}\beta$

h) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\operatorname{sen}(x+h) - \operatorname{sen}(x)}{h} =$

$= \lim_{h\to 0} \frac{\operatorname{sen}x \cdot \cos h + \cos x \operatorname{sen} h - \operatorname{sen}x}{h} = \lim_{h\to 0} \frac{\cos x \operatorname{sen} h}{h} =$ ($\cos h \to 1$)

$= \lim_{h\to 0} \cos x \cdot \lim_{h\to 0} \frac{\operatorname{sen} h}{h} = \cos x$ ($\frac{\operatorname{sen} h}{h} \to 1$)

i) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h\to 0} \frac{e^x \cdot e^h - e^x}{h} =$

$= \lim_{h\to 0} \frac{e^x(e^h - 1)}{h} = \lim_{h\to 0} e^x \cdot \lim_{h\to 0} \frac{e^h - 1}{h} =$

$e^h = \lim_{h\to 0} \left[(1+h)^{1/h}\right]^h = e$

$= \lim_{h\to 0} e^x \cdot \lim_{h\to 0} \frac{(1+h)^{h/h} - 1}{h} = \lim_{h\to 0} e^x \cdot \lim_{h\to 0} \frac{1+h-1}{h} = e^x$

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j) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\ln(x+h)-\ln(x)}{h} =$

$= \lim_{h\to 0} \frac{\ln\left(\frac{x+h}{x}\right)}{h} = \lim_{h\to 0} \frac{1}{h}\cdot \ln\left(\frac{x+h}{x}\right) = \lim_{h\to 0} \ln\left(\frac{x+h}{x}\right)^{1/h}$

$= \ln\left(\lim_{h\to 0}\left(\frac{x+h}{x}\right)^{1/h}\right) = \ln e^{\lim_{h\to 0}\frac{1}{h}\left(\frac{x+h}{x}-1\right)} =$

$[1^\infty]$

$= \ln e^{\lim_{h\to 0}\frac{1}{h}\left(\frac{x+h-x}{x}\right)} = \ln e^{\lim_{h\to 0}\frac{1}{x}} = \ln e^{1/x} = \frac{1}{x}$

k) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt[3]{x+h}-\sqrt[3]{x}}{h} =$

$= \lim_{h\to 0} \frac{\left(\sqrt[3]{x+h}-\sqrt[3]{x}\right)\cdot\left(\sqrt[3]{(x+h)^2}+\sqrt[3]{(x+h)\cdot x}+\sqrt[3]{x^2}\right)}{h\left(\sqrt[3]{(x+h)^2}+\sqrt[3]{(x+h)x}+\sqrt[3]{x^2}\right)} =$

$= \lim_{h\to 0} \frac{\sqrt[3]{(x+h)^3}+\sqrt[3]{(x+h)^2\cdot x}+\sqrt[3]{(x+h)x^2}-\sqrt[3]{(x+h)^2 x}-\sqrt[3]{(x+h)x^2}-\sqrt[3]{x^3}}{h\left(\sqrt[3]{(x+h)^2}+\sqrt[3]{(x+h)x}+\sqrt[3]{x^2}\right)} =$

$= \lim_{h\to 0} \frac{x+h-x}{h\left(\sqrt[3]{(x+h)^2}+\sqrt[3]{(x+h)x}+\sqrt[3]{x^2}\right)} = \lim_{h\to 0} \frac{h}{h\left(\sqrt[3]{(x+h)^2}+\sqrt[3]{(x+h)x}+\sqrt[3]{x^2}\right)} =$

$= \lim_{h\to 0} \frac{1}{\sqrt[3]{(x+h)^2}+\sqrt[3]{(x+h)x}+\sqrt[3]{x^2}} = \frac{1}{3\sqrt[3]{x^2}}$

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e) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{tg(x+h)-tg(x)}{h} =$

$tg(\alpha+\beta) = \frac{tg\alpha + tg\beta}{1 - tg\alpha \cdot tg\beta}$

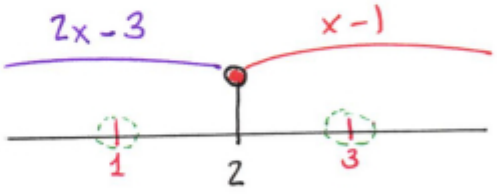
$= \lim_{h\to 0} \frac{\frac{tg(x)+tg(h)}{1-tg(x)\,tg(h)} - tg(x)}{h} = \lim_{h\to 0} \frac{\frac{tg(x)+h}{1-h\cdot tg\,x} - tg\,x}{h} =$

Si $h\to 0 \Rightarrow tg(h) \approx h$

$= \lim_{h\to 0} \frac{tg(x) + h - tg\,x + h\,tg^2 x}{h(1-h\cdot tg(x))} = \lim_{h\to 0} \frac{h(1+tg^2(x))}{h(1-h\cdot tg(x))} = 1 + tg^2(x)$

⑥ Dada $f(x) = \begin{cases} 2x-3 & \text{si } x<2 \\ x-1 & \text{si } x\geq 2 \end{cases}$ halla $f'(1)$ y $f'(3)$ utilizando la definición de derivada

$f(x) = \begin{cases} 2x-3 & \text{si } x<2 \\ x-1 & \text{si } x\geq 2 \end{cases}$



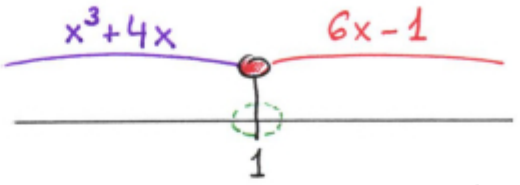
$f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{2(1+h)-3+1}{h} = \lim_{h\to 0} \frac{2h}{h} = 2$

$f'(3) = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h} = \lim_{h\to 0} \frac{3+h-1-2}{h} = \lim_{h\to 0} \frac{h}{h} = 1$

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(7) Dada la función $f(x)=\begin{cases} x^3+4x & \text{si } x<1 \\ 6x-1 & \text{si } x\geq 1 \end{cases}$, ¿existe $f'(1)$?

$$f(x)=\begin{cases} x^3+4x & \text{si } x<1 \\ 6x-1 & \text{si } x\geq 1 \end{cases}$$



$$f'(1)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h} \Rightarrow$$

$$\Rightarrow \begin{cases} \lim\limits_{h\to 0^+}\dfrac{f(1+h)-f(1)}{h}=\lim\limits_{h\to 0^+}\dfrac{6(1+h)-1-5}{h}=\lim\limits_{h\to 0^+}\dfrac{6\cancel{h}}{\cancel{h}}=6 \\ \lim\limits_{h\to 0^-}\dfrac{f(1+h)-f(1)}{h}=\lim\limits_{h\to 0^-}\dfrac{(1+h)^3+4(1+h)-5}{h}= \\ =\lim\limits_{h\to 0^-}\dfrac{\cancel{1}+3h+3h^2+h^3+\cancel{4}+4h-\cancel{5}}{h}=\lim\limits_{h\to 0^-}\dfrac{\cancel{h}(7+3h+h^2)}{\cancel{h}}=7 \end{cases}$$

Como los laterales son distintos $\Rightarrow \nexists \lim\limits_{h\to 0}\dfrac{f(1+h)-f(1)}{h} \Rightarrow \nexists f'(1)$

Nota Esos límites laterales que hemos evaluado se llaman DERIVADAS LATERALES

$$\left.\begin{aligned} f'(a^-)&=\lim_{h\to 0^-}\frac{f(a+h)-f(a)}{h} \\ f'(a^+)&=\lim_{h\to 0^+}\frac{f(a+h)-f(a)}{h} \end{aligned}\right\} \quad \exists f'(a) \Leftrightarrow f'(a^-)=f'(a^+)$$

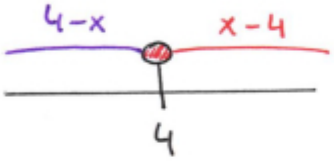
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⑧ Expresa la función $f(x) = |x-4|$ y luego calcula $f'(3)$, $f'(5)$ y $f'(4)$ utilizando la definición de derivada

$$f(x) = |x-4|$$

$x - 4 = 0 \Rightarrow x = 4$



$$\Longrightarrow f(x) = \begin{cases} 4-x & \text{si } x < 4 \\ x-4 & \text{si } x \geq 4 \end{cases}$$

$$f'(3) = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h} = \lim_{h\to 0} \frac{4-(3+h)-1}{h} = \lim_{h\to 0} \frac{-h}{h} = -1$$

$$f'(5) = \lim_{h\to 0} \frac{f(5+h)-f(5)}{h} = \lim_{h\to 0} \frac{(5+h)-4-1}{h} = \lim_{h\to 0} \frac{h}{h} = +1$$

$$f'(4) = \lim_{h\to 0} \frac{f(4+h)-f(4)}{h} \Rightarrow$$

$$\Rightarrow \begin{cases} \lim\limits_{h\to 0^-} \dfrac{f(4+h)-f(4)}{h} = \lim\limits_{h\to 0^-} \dfrac{4-(4+h)-0}{h} = \lim\limits_{h\to 0^-} \dfrac{-h}{h} = -1 \\ \lim\limits_{h\to 0^+} \dfrac{f(4+h)-f(4)}{h} = \lim\limits_{h\to 0^+} \dfrac{(4+h)-4-0}{h} = \lim\limits_{h\to 0^+} \dfrac{h}{h} = +1 \end{cases}$$

Como los laterales son distintos $\Rightarrow \nexists \lim\limits_{h\to 0} \dfrac{f(4+h)-f(4)}{h} \Rightarrow \nexists f'(4)$

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(9) Calcula la derivada de las siguientes funciones:

a) $f(x) = 6$ b) $f(x) = x^4$ c) $f(x) = \sqrt[5]{x}$

d) $f(x) = \sqrt[4]{x^3}$ e) $f(x) = \frac{1}{x^5}$ f) $f(x) = \sqrt[7]{x^4}$

g) $f(x) = 6x^3$ h) $f(x) = \sqrt[3]{3x^4}$ i) $f(x) = \frac{3x}{x^2 \cdot \sqrt[3]{x}}$

j) $f(x) = 5x^2 - 4x^3$ k) $f(x) = x^2 + \sqrt{x}$ l) $f(x) = (x^3 + x^2) \cdot (1 - x^3)$

m) $f(x) = \frac{5x}{x^2 - 1}$ n) $f(x) = \frac{x^2 + 2}{x+1}$ o) $f(x) = \frac{x^3 - 5x^2}{4}$

p) $f(x) = \frac{x}{e}$ q) $f(x) = \frac{2}{x}$ r) $f(x) = \frac{x^4 - 5x^3 + x^2}{2}$

s) $f(x) = (x^2+5)^3$ t) $f(x) = \frac{(x+8)^3}{(x-2)^2}$ u) $f(x) = \frac{x^2+3}{(1-x)^2}$

v) $f(x) = \frac{x+2}{(x-3)^2}$ w) $f(x) = \sqrt{\frac{x+2}{x-2}}$ x) $f(x) = \frac{6}{(x^2-3)^2}$

y) $f(x) = \left(1 + \frac{3}{x}\right)^3$ z) $f(x) = \sqrt{\frac{24 - \sqrt{3}}{\pi^2 + 5}}$

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a) $f(x)=6 \longrightarrow f'(x)=0$

b) $f(x)=x^4 \longrightarrow f'(x)=4x^3$

c) $f(x)=\sqrt[5]{x} \longrightarrow f'(x)=\dfrac{1}{5\sqrt[5]{x^4}}$

d) $f(x)=\sqrt[4]{x^3}=x^{3/4} \longrightarrow f'(x)=\dfrac{3}{4}x^{-1/4}=\dfrac{3}{4x^{1/4}}=\dfrac{3}{4\sqrt[4]{x}}$

e) $f(x)=\dfrac{1}{x^5}=x^{-5} \longrightarrow f'(x)=-5x^{-6}=\dfrac{-5}{x^6}$

f) $f(x)=\sqrt[7]{x^4}=x^{4/7} \longrightarrow f'(x)=\dfrac{4}{7}x^{-3/7}=\dfrac{4}{7x^{3/7}}=\dfrac{4}{7\sqrt[7]{x^3}}$

g) $f(x)=6x^3 \longrightarrow f'(x)=18x^2$

h) $f(x)=\sqrt[3]{3x^4}=\sqrt[3]{3}\cdot\sqrt[3]{x^4}=\sqrt[3]{3}\cdot x^{4/3}$

$f'(x)=\sqrt[3]{3}\cdot\dfrac{4}{3}x^{1/3}=\dfrac{4\sqrt[3]{3}\cdot\sqrt[3]{x}}{3}=\dfrac{4\sqrt[3]{3x}}{3}$

i) $f(x)=\dfrac{3x}{x^2\cdot\sqrt[3]{x}}=\dfrac{3}{x\sqrt[3]{x}}=\dfrac{3}{\sqrt[3]{x^4}}=3\cdot x^{-4/3}$

$f'(x)=3\cdot\left(\dfrac{-4}{3}\right)\cdot x^{-7/3}=\dfrac{-4}{x^{7/3}}=\dfrac{-4}{\sqrt[3]{x^7}}=\dfrac{-4}{x^2\sqrt[3]{x}}$

j) $f(x)=5x^2-4x^3 \longrightarrow f'(x)=10x-12x^2$

k) $f(x)=x^2+\sqrt{x} \longrightarrow f'(x)=2x+\dfrac{1}{2\sqrt{x}}$

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l) $f(x) = (x^3+x^2)\cdot(1-x^3) = x^3 - x^6 + x^2 - x^5$

$f'(x) = 3x^2 - 6x^5 + 2x - 5x^4$

m) $f(x) = \dfrac{5x}{x^2-1}$

$f'(x) = \dfrac{5(x^2-1) - 5x\cdot 2x}{(x^2-1)^2} = \dfrac{5x^2 - 5 - 10x^2}{(x^2-1)^2} = \dfrac{-5x^2-5}{(x^2-1)^2}$

n) $f(x) = \dfrac{x^2+2}{x+1}$

$f'(x) = \dfrac{2x(x+1) - (x^2+2)\cdot 1}{(x+1)^2} = \dfrac{2x^2+2x-x^2-2}{(x+1)^2} = \dfrac{x^2+2x-2}{(x+1)^2}$

o) $f(x) = \dfrac{x^3-5x^2}{4} \longrightarrow f'(x) = \dfrac{3x^2-10x}{4}$

p) $f(x) = \dfrac{x}{2} \longrightarrow f'(x) = \dfrac{1}{2}$

q) $f(x) = \dfrac{2}{x} \longrightarrow f'(x) = \dfrac{0\cdot x - 2\cdot 1}{x^2} = -\dfrac{2}{x^2}$

r) $f(x) = \dfrac{x^4-5x^3+x^2}{2} \longrightarrow f'(x) = \dfrac{4x^3-15x^2+2x}{2}$

s) $f(x) = (x^2+5)^3 \longrightarrow f'(x) = 3(x^2+5)^2\cdot 2x = 6x\cdot(x^2+5)^2$

t) $f(x) = \dfrac{(x+8)^3}{(x-2)^2}$

$f'(x) = \dfrac{3(x+8)^2\cdot(x-2)^{\cancel{2}} - (x+8)^3\cdot 2\cancel{(x-2)}}{(x-2)^{\cancel{4}3}} = \dfrac{3(x+8)^2(x-2) - 2(x+8)^3}{(x-2)^3} =$

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$$= \frac{(x+8)^2 \cdot [3(x-2)-2(x+8)]}{(x-2)^3} = \frac{(x+8)^2 \cdot (x-22)}{(x-2)^3} = \frac{x^3-6x^2-288x-1408}{(x-2)^3}$$

u) $f(x) = \dfrac{x^2+3}{(1-x)^2}$

$$f'(x) = \frac{2x(1-x)^2 - (x^2+3)\cdot 2(1-x)\cdot(-1)}{(1-x)^4} = \frac{2x(1-x)+2(x^2+3)}{(1-x)^3} =$$

$$= \frac{2x-2x^2+2x^2+6}{(1-x)^3} = \frac{2x+6}{(1-x)^3}$$

v) $f(x) = \dfrac{x+2}{(x-3)^2}$

$$f'(x) = \frac{1\cdot(x-3)^2 - (x+2)\cdot 2(x-3)\cdot 1}{(x-3)^4} = \frac{x-3-2(x+2)}{(x-3)^3} = \frac{-x-7}{(x-3)^3}$$

w) $f(x) = \sqrt{\dfrac{x+2}{x-2}}$

$$f'(x) = \frac{1}{2\cdot\sqrt{\frac{x+2}{x-2}}} \cdot \frac{1\cdot(x-2)-(x+2)\cdot 1}{(x-2)^2} = \frac{1}{2\sqrt{\frac{x+2}{x-2}}} \cdot \frac{-4}{(x-2)^2} =$$

$$= \frac{-2}{(x-2)^2\cdot\sqrt{\frac{x+2}{x-2}}} = \frac{-2}{(x-2)\cdot\sqrt{\frac{(x-2)^2(x+2)}{(x-2)}}} = \frac{-2}{(x-2)\sqrt{x^2-4}}$$

x) $f(x) = \dfrac{6}{(x^2-3)^2} \longrightarrow f'(x) = \dfrac{0\cdot(x^2-3)^2 - 6\cdot 2(x^2-3)\cdot 2x}{(x^2-3)^4} = \dfrac{-24x}{(x^2-3)^3}$

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y) $f(x) = \left(1 + \frac{3}{x}\right)^3$

$$f'(x) = 3 \cdot \left(1 + \frac{3}{x}\right)^2 \cdot \left(\frac{-3}{x^2}\right) = \frac{-9}{x^2}\left(1 + \frac{3}{x}\right)^2 = \frac{-9x^2 - 54x - 81}{x^4}$$

z) $f(x) = \sqrt{\frac{24 - \sqrt{3}}{\pi^2 + 5}} \longrightarrow f'(x) = 0$ ($f(x)$ es constante!!)

(10) Calcula la derivada de las siguientes funciones:

a) $f(x) = 2^x$ b) $f(x) = e^x$ c) $f(x) = \log_3(x)$

d) $f(x) = \ln(x)$ e) $f(x) = \sqrt[3]{x} + \sqrt[4]{x}$ f) $f(x) = -\frac{1}{x} + x$

g) $f(x) = 3^x - \text{arctg}(x)$ h) $f(x) = 2\text{sen}(x) - 3\cos(x)$

i) $f(x) = \text{sen}x \cdot \cos x$ j) $f(x) = x \cdot e^x$ k) $f(x) = x^3 \cdot \ln x$

l) $f(x) = (x^2 + 2x) \cdot \text{sen}x$ m) $f(x) = (e^x - x) \cdot \log_3(x)$ n) $f(x) = x \cdot \text{sen}(x)$

o) $f(x) = 4x \cdot \text{sen}\,x + x^3 \cos x$ p) $f(x) = e^x \cdot \text{tg}\,x$ q) $f(x) = \frac{\text{sen}(x)}{x}$

r) $f(x) = \frac{\sqrt{x}}{\cos x}$ s) $f(x) = \frac{\text{sen}x + 2x}{x - 3}$ t) $f(x) = \sqrt{x^2 + x}$

u) $f(x) = (\text{arcsen}(x))^3$ v) $f(x) = \ln(x^2 + 3)$ w) $f(x) = e^{x^2 + 7x - 4}$

x) $f(x) = \text{tg}\left(\frac{x}{x-5}\right)$ y) $f(x) = \text{arctg}\left(\frac{x}{x+1}\right)$ z) $f(x) = \ln\left(\frac{1+x}{1-x}\right)$

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a) $f(x) = 2^x \longrightarrow f'(x) = 2^x \cdot \ln(2)$

b) $f(x) = e^x \longrightarrow f'(x) = e^x$

c) $f(x) = \log_3(x) \longrightarrow f'(x) = \dfrac{1}{x \cdot \ln(3)}$

d) $f(x) = \ln(x) \longrightarrow f'(x) = \dfrac{1}{x}$

e) $f(x) = \sqrt[3]{x} + \sqrt[4]{x} \longrightarrow f'(x) = \dfrac{1}{3\sqrt[3]{x^2}} + \dfrac{1}{4\sqrt[4]{x^3}}$

f) $f(x) = -\dfrac{1}{x} + x \longrightarrow f'(x) = \dfrac{1}{x^2} + 1 = \dfrac{1+x^2}{x^2}$

g) $f(x) = 3^x - \operatorname{arctg}(x) \longrightarrow f'(x) = 3^x \ln 3 - \dfrac{1}{1+x^2}$

h) $f(x) = 2\operatorname{sen}(x) - 3\cos x \longrightarrow f'(x) = 2\cos(x) + 3\operatorname{sen}(x)$

i) $f(x) = \operatorname{sen} x \cdot \cos x \longrightarrow f'(x) = \cos^2 x - \operatorname{sen}^2 x = \cos(2x)$

j) $f(x) = x \cdot e^x \longrightarrow f'(x) = 1 \cdot e^x + x \cdot e^x = e^x(1+x)$

k) $f(x) = x^3 \cdot \ln x \longrightarrow f'(x) = 3x^2 \cdot \ln x + x^{\cancel{3}\,2} \cdot \dfrac{1}{\cancel{x}} = x^2(3\ln x + 1)$

l) $f(x) = (x^2+2x)\operatorname{sen} x \longrightarrow f'(x) = (2x+2)\operatorname{sen} x + (x^2+2x)\cos x$

m) $f(x) = (e^x - x) \cdot \log_3(x) \longrightarrow f'(x) = (e^x - 1)\log_3(x) + (e^x - x) \cdot \dfrac{1}{x \ln(3)}$

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n) $f(x) = x \cdot \text{sen}(x) \longrightarrow f'(x) = \text{sen}\,x + x \cdot \cos x$

o) $f(x) = 4x\,\text{sen}\,x + x^3 \cos x$

$f'(x) = 4\,\text{sen}\,x + 4x\cos x + 3x^2\cos x - x^3\,\text{sen}\,x =$

$= \text{sen}\,x \cdot (4 - x^3) + \cos x \cdot (4x + 3x^2)$

p) $f(x) = e^x \cdot \text{tg}\,x \longrightarrow f'(x) = e^x \cdot \text{tg}\,x + e^x(1 + \text{tg}^2 x) = e^x(1 + \text{tg}\,x + \text{tg}^2 x)$

q) $f(x) = \dfrac{\text{sen}(x)}{x} \longrightarrow f'(x) = \dfrac{x\cos x - \text{sen}\,x}{x^2}$

r) $f(x) = \dfrac{\sqrt{x}}{\cos x} \longrightarrow f'(x) = \dfrac{\frac{1}{2\sqrt{x}} \cdot \cos x + x \cdot \text{sen}(x)}{\cos^2 x}$

s) $f(x) = \dfrac{\text{sen}\,x + 2x}{x - 3}$

$f'(x) = \dfrac{(\cos x + 2)(x-3) - (\text{sen}\,x + 2x)\cdot 1}{(x-3)^2} = \dfrac{(x-3)\cos x + 2x - 6 - \text{sen}\,x - 2x}{(x-3)^2} =$

$= \dfrac{(x-3)\cos x - \text{sen}\,x - 6}{(x-3)^2}$

t) $f(x) = \sqrt{x^2 + x} \longrightarrow f'(x) = \dfrac{1}{2\sqrt{x^2+x}} \cdot (2x+1) = \dfrac{2x+1}{2\sqrt{x^2+x}}$

u) $f(x) = (\text{arcsen}(x))^3 \longrightarrow f'(x) = 3(\text{arcsen}(x))^2 \cdot \dfrac{1}{\sqrt{1-x^2}}$

v) $f(x) = \ln(x^2+3) \longrightarrow f'(x) = \dfrac{1}{x^2+3} \cdot 2x = \dfrac{2x}{x^2+3}$

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w) $f(x) = e^{x^2+7x-4} \longrightarrow f'(x) = e^{x^2+7x-4} \cdot (2x+7)$

x) $f(x) = tg\left(\frac{x}{x-5}\right)$

$$f'(x) = \frac{1}{\cos^2\left(\frac{x}{x-5}\right)} \cdot \frac{1(x-5)-x}{(x-5)^2} = \frac{-5}{(x-5)^2 \cdot \cos^2\left(\frac{x}{x-5}\right)}$$

y) $f(x) = \text{arctg}\left(\frac{x}{x+1}\right)$

$$f'(x) = \frac{1}{1+\left(\frac{x}{x+1}\right)^2} \cdot \frac{1\cdot(x+1)-x\cdot 1}{(x+1)^2} = \frac{1}{\frac{(x+1)^2+x^2}{(x+1)^2}} \cdot \frac{1}{(x+1)^2} =$$

$$= \frac{1}{2x^2+2x+1}$$

z) $f(x) = \ln\left(\frac{1+x}{1-x}\right)$

$$f'(x) = \frac{1}{\frac{1+x}{1-x}} \cdot \frac{1\cdot(1-x)+(1+x)}{(1-x)^2} = \frac{2}{(1+x)(1-x)} = \frac{2}{1-x^2}$$

(11) Calcula las derivadas de las siguientes funciones:

a) $f(x) = \frac{x^2-3}{x^2+3}$ b) $f(x) = \frac{x+1}{(2-x)^2}$ c) $f(x) = \frac{3x^2}{x+\sqrt{x}}$

d) $f(x) = (0'5 - \frac{x}{10})^4$ e) $f(x) = \frac{x^3}{(x+1)^2}$ f) $f(x) = \left(\frac{x^2+1}{x}\right)^3$

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g) $f(x) = \dfrac{x}{(2x+1)^3}$ h) $f(x) = e^{4x}\cdot(x-1)$ i) $f(x) = \dfrac{(1-x)^2}{e^x}$

j) $f(x) = \sqrt{2^x}$ k) $f(x) = \ln(2x-1)$ l) $f(x) = \ln(x^2-1)$

m) $f(x) = \ln(\sqrt{1-x})$ n) $f(x) = \dfrac{\ln x}{e^x}$ o) $f(x) = e^{x^2+1}$

p) $f(x) = \operatorname{sen} x\cdot\cos^2 x$ q) $f(x) = \dfrac{\operatorname{sen}^2 x}{1+\cos^2 x}$ r) $f(x) = \operatorname{sen}^2(x^2)$

s) $f(x) = \cos^3(2x+1)$ t) $f(x) = \log_2\left(\dfrac{1}{x}\right)$ u) $f(x) = \sqrt[3]{\operatorname{sen}(x^2)}$

v) $f(x) = \sqrt{\dfrac{1+2x}{1-2x}}$ w) $f(x) = \sqrt{x+\sqrt{x}}$ x) $f(x) = \ln\sqrt{\dfrac{x}{x+1}}$

y) $f(x) = \ln(\operatorname{sen}(\sqrt{e^x}))$ z) $f(x) = \ln\sqrt{\dfrac{x-1}{x+1}}$

a) $f(x) = \dfrac{x^2-3}{x^2+3}$

$$f'(x) = \frac{2x(x^2+3)-(x^2-3)\cdot 2x}{(x^2+3)^2} = \frac{2x(x^2+3-x^2+3)}{(x^2+3)^2} = \frac{12x}{(x^2+3)^2}$$

b) $f(x) = \dfrac{x+1}{(2-x)^2}$

$$f'(x) = \frac{1\cdot(2-x)^2-(x+1)\cdot 2(2-x)\cdot(-1)}{(2-x)^4} = \frac{2-x+2x+2}{(2-x)^3} = \frac{x+4}{(2-x)^3}$$

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c) $f(x) = \dfrac{3x^2}{x+\sqrt{x}}$

$$f'(x) = \frac{6x(x+\sqrt{x}) - 3x^2\left(1 + \frac{1}{2\sqrt{x}}\right)}{(x+\sqrt{x})^2} = \frac{6x^2+6x\sqrt{x} - 3x^2 - \frac{3x^2}{2\sqrt{x}}}{(x+\sqrt{x})^2} =$$

$$= \frac{3x^2+6x\sqrt{x} - \frac{3x^2}{2\sqrt{x}}}{(x+\sqrt{x})^2} = \frac{6x^2\sqrt{x} + 12x^2 - 3x^2}{2\sqrt{x}(x+\sqrt{x})^2} = \frac{6x^2\sqrt{x} + 9x^2}{2\sqrt{x}(x+\sqrt{x})^2} =$$

$$= \frac{6x^2\sqrt{x} + 9x^2}{2\sqrt{x}(x^2+2x\sqrt{x}+x)} = \frac{x(6x\sqrt{x} + 9x)}{x\cdot 2\sqrt{x}\cdot(x+2\sqrt{x}+1)} = \frac{\sqrt{x}(6x+9\sqrt{x})}{2\sqrt{x}(\sqrt{x}+1)^2} =$$

$$= \frac{6x+9\sqrt{x}}{2(\sqrt{x}+1)^2}$$

d) $f(x) = \left(0'5 - \dfrac{x}{10}\right)^4$

$$f'(x) = 4\left(0'5 - \frac{x}{10}\right)^3 \cdot \left(\frac{-1}{10}\right) = -\frac{2}{5}\cdot\left(0'5 - \frac{x}{10}\right)^3 = \frac{-2(5-x)^3}{5000} =$$

$$= \frac{-(5-x)^3}{2500}$$

e) $f(x) = \dfrac{x^3}{(x+1)^2}$

$$f'(x) = \frac{3x^2\cdot(x+1)^2 - x^3\cdot 2(x+1)}{(x+1)^4} = \frac{3x^3+3x^2-2x^3}{(x+1)^3} = \frac{x^3+3x^2}{(x+1)^3}$$

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f) $f(x) = \left(\frac{x^2+1}{x}\right)^3$

$$f'(x) = 3\left(\frac{x^2+1}{x}\right)^2 \cdot \frac{2x\cdot x - (x^2+1)}{x^2} = \frac{3(x^2+1)^2}{x^2}\cdot\frac{x^2-1}{x^2} = \frac{3(x^2+1)(x^4-1)}{x^4}$$

g) $f(x) = \frac{x}{(2x+1)^3}$

$$f'(x) = \frac{1\cdot(2x+1)^3 - x\cdot 3(2x+1)^2\cdot 2}{(2x+1)^6} = \frac{2x+1-6x}{(2x+1)^4} = \frac{-4x+1}{(2x+1)^4}$$

h) $f(x) = e^{4x}\cdot(x-1)$

$$f'(x) = e^{4x}\cdot 4\cdot(x-1) + e^{4x} = e^{4x}(4x-4+1) = e^{4x}(4x-3)$$

i) $f(x) = \frac{(1-x)^2}{e^x}$

$$f'(x) = \frac{2(1-x)\cdot(-1)\cdot e^x - (1-x)^2\cdot e^x}{(e^x)^2} = \frac{-2+2x-(1-2x+x^2)}{e^x} = \frac{-x^2+4x-3}{e^x}$$

j) $f(x) = \sqrt{2^x} = 2^{x/2}$

$$f'(x) = 2^{x/2}\cdot\frac{1}{2}\cdot\ln(2) = \frac{1}{2}\cdot\ln 2\cdot\sqrt{2^x}$$

k) $f(x) = \ln(2x-1) \longrightarrow f'(x) = \frac{1}{2x-1}\cdot 2 = \frac{2}{2x-1}$

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e) $f(x) = \ln(x^2-1) \longrightarrow f'(x) = \dfrac{1}{x^2-1} \cdot 2x = \dfrac{2x}{x^2-1}$

m) $f(x) = \ln\left(\sqrt{1-x}\right)$

$$f'(x) = \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{1-x}} \cdot (-1) = \frac{-1}{2(1-x)} = \frac{1}{2(x-1)}$$

n) $f(x) = \dfrac{\ln x}{e^x}$

$$f'(x) = \frac{1/x \cdot e^x - \ln x \cdot e^x}{(e^x)^2} = \frac{\frac{1}{x} - \ln x}{e^x} = \frac{1 - x\cdot\ln x}{x \cdot e^x}$$

o) $f(x) = e^{x^2+1} \longrightarrow f'(x) = e^{x^2+1} \cdot 2x$

p) $f(x) = \operatorname{sen} x \cdot \cos^2 x$

$$f'(x) = \cos x \cdot \cos^2 x + \operatorname{sen} x \cdot 2\cos x \cdot (-\operatorname{sen} x) = \cos^3 x - 2\operatorname{sen}^2 x \cdot \cos x$$

q) $f(x) = \dfrac{\operatorname{sen}^2 x}{1+\cos^2 x}$

$$f'(x) = \frac{2\operatorname{sen} x \cdot \cos x \cdot (1+\cos^2 x) - \operatorname{sen}^2 x \cdot 2\cos x \cdot (-\operatorname{sen} x)}{(1+\cos^2 x)^2} =$$

$$= \frac{2\operatorname{sen} x \cos x + 2\operatorname{sen} x \cdot \cos^3 x + 2\operatorname{sen}^3 x \cdot \cos x}{(1+\cos^2 x)^2} = \frac{2\operatorname{sen} x \cos x \left(1 + \cos^2 x + \operatorname{sen}^2 x\right)}{(1+\cos^2 x)^2} =$$

$$= \frac{(2\operatorname{sen} x \cos x) \cdot 2}{(1+\cos^2 x)^2} = \frac{2\operatorname{sen}(2x)}{(1+\cos^2 x)^2}$$

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r) $f(x) = \text{sen}^2(x^2)$

$$f'(x) = 2\,\text{sen}(x^2)\cdot\cos(x^2)\cdot 2x = 2x\,\text{sen}(2x^2)$$

s) $f(x) = \cos^3(2x+1)$

$$f'(x) = 3\cos^2(2x+1)\cdot(-\text{sen}(2x+1))\cdot 2 = -6\cos^2(2x+1)\cdot\text{sen}(2x+1)$$

t) $f(x) = \log_2\left(\frac{1}{x}\right)$

$$f'(x) = \frac{1}{\frac{1}{x}\cdot\ln 2}\cdot\frac{-1}{x^2} = \frac{-1}{x\cdot\ln 2}$$

u) $f(x) = \sqrt[3]{\text{sen}(x^2)}$

$$f'(x) = \frac{1}{3\sqrt[3]{\text{sen}^2(x^2)}}\cdot\cos(x^2)\cdot 2x = \frac{2x\cos(x^2)}{3\sqrt[3]{\text{sen}^2(x^2)}}$$

v) $f(x) = \sqrt{\frac{1+2x}{1-2x}}$

$$f'(x) = \frac{1}{2\sqrt{\frac{1+2x}{1-2x}}}\cdot\frac{2(1-2x)-(1+2x)\cdot(-2)}{(1-2x)^2} = \frac{2(1-2x+1+2x)}{2(1-2x)^2\cdot\sqrt{\frac{1+2x}{1-2x}}} =$$

$$= \frac{2}{(1-2x)\cdot\sqrt{\frac{(1-2x)^2\cdot(1+2x)}{(1-2x)}}} = \frac{2}{(1-2x)\sqrt{1-4x^2}}$$

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w) $f(x) = \sqrt{x+\sqrt{x}}$

$$f'(x) = \frac{1}{2\sqrt{x+\sqrt{x}}} \cdot \left(1 + \frac{1}{2\sqrt{x}}\right) = \frac{1 + \frac{1}{2\sqrt{x}}}{2\sqrt{x+\sqrt{x}}}$$

x) $f(x) = \ln \sqrt{\frac{x}{x+1}}$

$$f'(x) = \frac{1}{\sqrt{\frac{x}{x+1}}} \cdot \frac{1}{2\sqrt{\frac{x}{x+1}}} \cdot \frac{1(x+1) - x \cdot 1}{(x+1)^2} = \frac{1}{2 \cdot \frac{x}{x+1}} \cdot \frac{1}{(x+1)^2} =$$

$$= \frac{1}{2x(x+1)}$$

y) $f(x) = \ln(\operatorname{sen} \sqrt{e^x})$

$$f'(x) = \frac{1}{\operatorname{sen}(\sqrt{e^x})} \cdot \cos(\sqrt{e^x}) \cdot \frac{1}{2\sqrt{e^x}} \cdot e^x = \frac{\sqrt{e^x}}{2 \cdot \operatorname{tg}(\sqrt{e^x})}$$

z) $f(x) = \ln \sqrt{\frac{x-1}{x+1}}$

$$f'(x) = \frac{1}{\sqrt{\frac{x-1}{x+1}}} \cdot \frac{1}{2\sqrt{\frac{x-1}{x+1}}} \cdot \frac{1(x+1) - (x-1) \cdot 1}{(x+1)^2} = \frac{1}{\frac{2(x-1)}{x+1}} \cdot \frac{2}{(x+1)^2} =$$

$$= \frac{1}{(x-1)(x+1)} = \frac{1}{x^2-1}$$

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(12) Calcula las derivadas de las siguientes funciones:

$a_1)\ f(x) = e^{4x}$ $a_2)\ f(x) = \log_5(7x+2)$ $a_3)\ f(x) = 5\cdot e^{x^2+3x}$

$b_1)\ f(x) = (x^3+1)(x^3-1)$ $b_2)\ f(x) = x^3\cdot e^x$ $b_3)\ f(x) = x^2\cdot \ln(x^2)$

$c_1)\ f(x) = e^x \cdot \operatorname{tg} x$ $c_2)\ f(x) = \operatorname{sen} x \cdot \cos(2x)$ $c_3)\ f(x) = \dfrac{1}{x-1}$

$d_1)\ f(x) = \dfrac{-x}{x+1}$ $d_2)\ f(x) = \dfrac{3}{x^2-1}$ $d_3)\ f(x) = \dfrac{x^2-3}{x^2+3}$

$e_1)\ f(x) = \dfrac{\ln(x^2+5)}{x}$ $e_2)\ f(x) = 5x^3 - 3x^2 + 4$ $e_3)\ f(x) = (x^2+1)^3$

$f_1)\ f(x) = \sqrt[3]{x^2+3}$ $f_2)\ f(x) = x^2\cdot \ln(x)$ $f_3)\ f(x) = \dfrac{x^2}{\ln x}$

$g_1)\ f(x) = \operatorname{sen}(3x)$ $g_2)\ f(x) = (\operatorname{sen} x)^5$ $g_3)\ f(x) = e^x \cdot \operatorname{sen} x$

$h_1)\ f(x) = \cos(3x+1)$ $h_2)\ f(x) = e^{7x^2}$ $h_3)\ f(x) = e^{\operatorname{sen}(2x)}$

$i_1)\ f(x) = e^x + \ln x$ $i_2)\ f(x) = \dfrac{\log x^2}{x}$ $i_3)\ f(x) = \operatorname{sen}(3x+1)\cdot\cos(3x+1)$

$j_1)\ f(x) = \operatorname{sen}(x^2-5x+7)$ $j_2)\ f(x) = x\cdot e^{2x+1}$ $j_3)\ f(x) = \dfrac{2}{(x+3)^3}$

$k_1)\ f(x) = \dfrac{x^3}{e} + \dfrac{x^2+7x}{5}$ $k_2)\ f(x) = \dfrac{1}{\sqrt{x-4}}$ $k_3)\ f(x) = \dfrac{e^x + e^{-x}}{2}$

$l_1)\ f(x) = \dfrac{x^3}{(x-1)^2}$ $l_2)\ f(x) = \sqrt{\dfrac{x^3}{x^2-4}}$ $l_3)\ f(x) = \operatorname{tg}^3(x^2)$

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$m_1)\ f(x) = \sqrt{\operatorname{sen}(x)}$ $\quad m_2)\ f(x) = 7^{x+1}\cdot e^{-x}$ $\quad m_3)\ f(x) = \ln x + e^{\sqrt{x}}$

$n_1)\ f(x) = e^{2x}\cdot \operatorname{tg} x$ $\quad n_2)\ f(x) = \log\left(\frac{x^2}{3-x}\right)$ $\quad n_3)\ f(x) = \sqrt{\ln x}$

$o_1)\ f(x) = (x^2-7x)^3$ $\quad o_2)\ f(x) = \left(\frac{x}{1+x^2}\right)^2$ $\quad o_3)\ f(x) = \operatorname{arctg}\left(\frac{x}{1+x}\right)$

$p_1)\ f(x) = \frac{x\cdot e^{-x}}{3^x}$ $\quad p_2)\ f(x) = \operatorname{arcsen}(x^2)$ $\quad p_3)\ f(x) = \ln(\operatorname{tg}(2x))$

$q_1)\ f(x) = \ln\sqrt{\frac{x}{x-1}}$ $\quad q_2)\ f(x) = \ln\left(\frac{\sqrt{x}}{1+\operatorname{sen}^2 x}\right)$ $\quad q_3)\ f(x) = \operatorname{arctg}\left(\sqrt{\frac{x}{x+2}}\right)$

$r_1)\ f(x) = \frac{\cos x + \cos(x+1)}{\operatorname{sen} x - \operatorname{sen}(x+1)}$ $\quad r_2)\ f(x) = \ln\sqrt{\frac{1-\operatorname{sen} x}{1+\operatorname{sen} x}}$ $\quad r_3)\ f(x) = \ln e^{\operatorname{sen} x}$

$s_1)\ f(x) = \operatorname{arctg}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$ $\quad s_2)\ f(x) = \ln\left(\frac{x}{x+1}\right)^x$ $\quad s_3)\ f(x) = \sqrt{e^x + e^{-x}}$

$t_1)\ f(x) = \ln\left(\sqrt{\frac{x^3\cdot\cos x}{\operatorname{sen} x}}\right)$ $\quad t_2)\ f(x) = \frac{e^{x+3}-1}{x^2-9}$ $\quad t_3)\ f(x) = \operatorname{arcsen}(\sqrt{1-\operatorname{sen}^2 x})$

$u_1)\ f(x) = \operatorname{arcsen}\left(\frac{1}{1+x^2}\right)$ $\quad u_2)\ f(x) = \frac{\ln\sqrt{x+1}}{x}$ $\quad u_3)\ f(x) = e^{\ln(\operatorname{sen}^2 x)}$

$v_1)\ f(x) = \operatorname{tg}(1+\operatorname{sen} x)$ $\quad v_2)\ f(x) = \operatorname{tg}^3(e^{x^2})$ $\quad v_3)\ f(x) = 27 + (x-1)^4$

$w_1)\ f(x) = \operatorname{sen}^2 x + \cos^2 x$ $\quad w_2)\ f(x) = \frac{e^x+1}{e^x-1}$ $\quad w_3)\ f(x) = \frac{2+\cos x}{2-\cos x}$

$x_1)\ f(x) = \operatorname{arcsen}\left(\frac{x}{\sqrt{1+x^2}}\right)$ $\quad x_2)\ f(x) = \operatorname{arccos}\left(\frac{1}{\sqrt{1+x^2}}\right)$ $\quad x_3)\ f(x) = e^{5x}$

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$Y_1)\ f(x) = \sqrt{\sec(x)}$ $\quad Y_2)\ f(x) = \operatorname{arccotg}\left(\frac{1}{x}\right)$ $\quad Y_3)\ f(x) = \operatorname{arcsec}(x)$

$Z_1)\ f(x) = \dfrac{\cos(2x)+\operatorname{sen}(2x)}{\cos(2x)-\operatorname{sen}(2x)}$ $\quad Z_2)\ f(x) = \ln\sqrt{\dfrac{x^2\operatorname{sen}x}{\cos x}}$ $\quad Z_3)\ f(x) = \ln(1+x)^x$

$a_1)\ f(x) = e^{4x} \longrightarrow f'(x) = e^{4x}\cdot 4$

$a_2)\ f(x) = \log_5(7x+2) \longrightarrow f'(x) = \dfrac{1}{(7x+2)\cdot \ln(5)}\cdot 7 = \dfrac{7}{(7x+2)\ln(5)}$

$a_3)\ f(x) = 5\cdot e^{x^2+3x} \longrightarrow f'(x) = 5\cdot e^{x^2+3x}\cdot(2x+3)$

$b_1)\ f(x) = (x^3+1)\cdot(x^3-1) = x^6-1 \longrightarrow f'(x) = 6x^5$

$b_2)\ f(x) = x^3\cdot e^x \longrightarrow f'(x) = 3x^2\cdot e^x + x^3\cdot e^x = e^x(3x^2+x^3)$

$b_3)\ f(x) = x^2\cdot \ln(x^2) \longrightarrow f'(x) = 2x\cdot\ln(x^2) + \cancel{x^2}\cdot\dfrac{1}{\cancel{x^2}}\cdot 2x = 2x(\ln(x^2)+1)$

$c_1)\ f(x) = e^x\cdot tgx \longrightarrow f'(x) = e^x\cdot tgx + e^x(1+tg^2x) = e^x(1+tgx+tg^2x)$

$c_2)\ f(x) = \operatorname{sen}x\cdot\cos(2x)$

$f'(x) = \cos x\cdot\cos(2x) + \operatorname{sen}(x)\cdot(-\operatorname{sen}(2x))\cdot 2 = \cos x\cdot\cos(2x) - 2\operatorname{sen}(x)\cdot\operatorname{sen}(2x)$

$c_3)\ f(x) = \dfrac{1}{x-1}$

$f'(x) = \dfrac{0\cdot(x-1)-1\cdot 1}{(x-1)^2} = -\dfrac{1}{(x-1)^2}$

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$d_1)\ f(x) = \dfrac{-x}{x+1}$

$$f'(x) = \frac{-1(x+1)+x}{(x+1)^2} = \frac{-1}{(x+1)^2}$$

$d_2)\ f(x) = \dfrac{3}{x^2-1} \longrightarrow f'(x) = \dfrac{-3\cdot 2x}{(x^2-1)^2} = \dfrac{-6x}{(x^2-1)^2}$

$d_3)\ f(x) = \dfrac{x^2-3}{x^2+3} \longrightarrow f'(x) = \dfrac{2x(x^2+3)-(x^2-3)\cdot 2x}{(x^2+3)^2} = \dfrac{2x\cdot 6}{(x^2+3)^2} = \dfrac{12x}{(x^2+3)^2}$

$e_1)\ f(x) = \dfrac{\ln(x^2+5)}{x}$

$$f'(x) = \frac{\frac{1}{x^2+5}\cdot 2x\cdot x - \ln(x^2+5)\cdot 1}{x^2} = \frac{2x^2-(x^2+5)\cdot\ln(x^2+5)}{x^2(x^2+5)}$$

$e_2)\ f(x) = 5x^3-3x^2+4 \longrightarrow f'(x) = 15x^2-6x$

$e_3)\ f(x) = (x^2+1)^3 \longrightarrow f'(x) = 3(x^2+1)^2\cdot 2x = 6x(x^2+1)^2$

$f_1)\ f(x) = \sqrt[3]{x^2+3} \longrightarrow f'(x) = \dfrac{1}{3\sqrt[3]{(x^2+3)^2}}\cdot 2x = \dfrac{2x}{3\sqrt[3]{(x^2+3)^2}}$

$f_2)\ f(x) = x^2\cdot\ln x \longrightarrow f'(x) = 2x\cdot\ln x + x^2\cdot\dfrac{1}{x} = x(2\ln x+1)$

$f_3)\ f(x) = \dfrac{x^2}{\ln x}$

$$f'(x) = \frac{2x\cdot\ln x - x^2\cdot 1/x}{(\ln x)^2} = \frac{2x\ln x - x}{(\ln x)^2}$$

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$g_1)\ f(x) = \text{sen}(3x) \longrightarrow f'(x) = \cos(3x)\cdot 3 = 3\cos(3x)$

$g_2)\ f(x) = (\text{sen}(x))^5 \longrightarrow f'(x) = 5(\text{sen}\,x)^4 \cdot \cos x$

$g_3)\ f(x) = e^x \cdot \text{sen}\,x \longrightarrow f'(x) = e^x \cdot \text{sen}\,x + e^x \cdot \cos x$

$h_1)\ f(x) = \cos(3x+1) \longrightarrow f'(x) = -\text{sen}(3x+1)\cdot 3 = -3\text{sen}(3x+1)$

$h_2)\ f(x) = e^{7x^2} \longrightarrow f'(x) = e^{7x^2}\cdot 14x$

$h_3)\ f(x) = e^{\text{sen}(2x)} \longrightarrow f'(x) = e^{\text{sen}(2x)}\cdot \cos(2x)\cdot 2$

$i_1)\ f(x) = e^x + \ln x \longrightarrow f'(x) = e^x + \frac{1}{x}$

$i_2)\ f(x) = \frac{\log x^2}{x} \longrightarrow f'(x) = \frac{\frac{1}{x^2\cdot \ln 10}\cdot 2x\cdot x - \log x^2}{x^2} = \frac{\frac{2}{\ln 10} - \log x^2}{x^2}$

$i_3)\ f(x) = \text{sen}(3x+1)\cdot\cos(3x+1)$

$f'(x) = \cos(3x+1)\cdot 3\cdot\cos(3x+1) + \text{sen}(3x+1)\cdot(-\text{sen}(3x+1))\cdot 3 =$

$= 3\left(\cos^2(3x+1) - \text{sen}^2(3x+1)\right) = 3\cos(6x+2)$

$j_1)\ f(x) = \text{sen}(x^2-5x+7)$

$f'(x) = \cos(x^2-5x+7)\cdot(2x-5) = (2x-5)\cos(x^2-5x+7)$

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$j_2)\ f(x) = x\cdot e^{2x+1} \longrightarrow f'(x) = 1\cdot e^{2x+1} + x\cdot e^{2x+1}\cdot 2 = e^{2x+1}\cdot(1+2x)$

$j_3)\ f(x) = \dfrac{2}{(x+3)^3} \longrightarrow f'(x) = \dfrac{-2\cdot 3(x+3)^2\cdot 1}{(x+3)^6} = \dfrac{-6}{(x+3)^4}$

$k_1)\ f(x) = \dfrac{x^3}{2} + \dfrac{x^2+7x}{5} \longrightarrow f'(x) = \dfrac{3x^2}{2} + \dfrac{2x+7}{5}$

$k_2)\ f(x) = \dfrac{1}{\sqrt{x-4}} \longrightarrow f'(x) = \dfrac{-1\cdot\frac{1}{2\sqrt{x-4}}}{\left(\sqrt{(x-4)}\right)^2} = \dfrac{-1}{2\sqrt{(x-4)^3}}$

$k_3)\ f(x) = \dfrac{e^x+e^{-x}}{2} \longrightarrow f'(x) = \dfrac{e^x+e^{-x}\cdot(-1)}{2} = \dfrac{e^x-e^{-x}}{2}$

$l_1)\ f(x) = \dfrac{x^3}{(x-1)^2}$

$$f'(x) = \frac{3x^2(x-1)^2 - x^3\cdot 2(x-1)}{(x-1)^4} = \frac{3x^3-3x^2-2x^3}{(x-1)^3} = \frac{x^3-3x^2}{(x-1)^3}$$

$l_2)\ f(x) = \sqrt{\dfrac{x^3}{x^2-4}}$

$$f'(x) = \frac{1}{2\sqrt{\frac{x^3}{x^2-4}}}\cdot\frac{3x^2(x^2-4)-x^3\cdot 2x}{(x^2-4)^2} = \frac{3x^4-12x^2-2x^4}{2(x^2-4)^2\cdot\sqrt{\frac{x^3}{x^2-4}}} =$$

$$= \frac{x^4-12x^2}{2(x^2-4)^2\cdot x\cdot\sqrt{\frac{x}{x^2-4}}} = \frac{x(x^3-12x)}{2x(x^2-4)^2\cdot\sqrt{\frac{x}{x^2-4}}} = \frac{x^3-12x}{2(x^2-4)^2\cdot\sqrt{\frac{x}{x^2-4}}}$$

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$l_3)\ f(x) = tg^3(x^2)$

$$f'(x) = 3\cdot tg^2(x^2)\cdot\left(1+tg^2(x^2)\right)\cdot 2x = 6x\, tg^2(x^2)\cdot\left(1+tg^2(x^2)\right)$$

$m_1)\ f(x) = \sqrt{sen(x)} \longrightarrow f'(x) = \dfrac{1\cdot\cos x}{2\sqrt{sen\,x}}$

$m_2)\ f(x) = 7^{x+1}\cdot e^{-x} \longrightarrow f'(x) = 7^{x+1}\cdot ln\,7\cdot e^{-x} + 7^{x+1}\cdot e^{-x}\cdot(-1) =$

$$= 7^{x+1}\cdot e^{-x}\left(ln\,7 - 1\right)$$

$m_3)\ f(x) = ln\,x + e^{\sqrt{x}} \longrightarrow f'(x) = \dfrac{1}{x} + e^{\sqrt{x}}\cdot\dfrac{1}{2\sqrt{x}}$

$n_1)\ f(x) = e^{2x}\cdot tg\,x \longrightarrow f'(x) = e^{2x}\cdot 2\cdot tg\,x + e^{2x}\cdot\left(1+tg^2x\right) =$

$$= e^{2x}\left(2\,tg\,x + 1 + tg^2x\right)$$

$n_2)\ f(x) = log\left(\dfrac{x^2}{3-x}\right)$

$$f'(x) = \frac{1}{\left(\frac{x^2}{3-x}\right)\cdot ln\,10}\cdot\frac{2x(3-x)+x^2}{(3-x)^2} = \frac{6x-x^2}{x^2\cdot(3-x)\cdot ln\,10} =$$

$$= \frac{x(6-x)}{x^2\cdot(3-x)\cdot ln\,10} = \frac{6-x}{x(3-x)\,ln\,10}$$

$n_3)\ f(x) = \sqrt{ln\,x}$

$$f'(x) = \frac{1}{2\sqrt{ln\,x}}\cdot\frac{1}{x} = \frac{1}{2x\sqrt{ln\,x}}$$

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$O_1)\ f(x) = (x^2-7x)^3 \longrightarrow f'(x) = 3(x^2-7x)^2 \cdot (2x-7)$

$O_2)\ f(x) = \left(\frac{x}{1+x^2}\right)^2$

$$f'(x) = 2\cdot\left(\frac{x}{1+x^2}\right)\cdot \frac{1\,(1+x^2)-x\cdot 2x}{(1+x^2)^2} = \frac{2x}{1+x^2}\cdot\frac{1-x^2}{(1+x^2)^2} =$$

$$= \frac{2x-2x^3}{(1+x^2)^3}$$

$O_3)\ f(x) = \text{arctg}\left(\frac{x}{1+x}\right)$

$$f'(x) = \frac{1}{1+\left(\frac{x}{1+x}\right)^2}\cdot\frac{1\,(1+x)-x}{(1+x)^2} = \frac{1}{\frac{(1+x)^2+x^2}{(1+x)^2}}\cdot\frac{1}{(1+x)^2} =$$

$$= \frac{1}{2x^2+2x+1}$$

$P_1)\ f(x) = \frac{x\cdot e^{-x}}{3^x}$

$$f'(x) = \frac{\left(1\cdot e^{-x}+x\cdot e^{-x}\cdot(-1)\right)\cdot 3^x - x\cdot e^{-x}\cdot 3^x\cdot \ln 3}{(3^x)^2} =$$

$$= \frac{e^{-x}\left(1-x\cdot e^{-x}-x\cdot e^{-x}\cdot \ln 3\right)}{3^x}$$

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P2) $f(x) = \text{arcsen}(x^2)$

$$f'(x) = \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x = \frac{2x}{\sqrt{1-x^4}}$$

P3) $f(x) = \ln(\text{tg}(2x))$

$$f'(x) = \frac{1}{\text{tg}(2x)} \cdot \frac{1}{\cos^2(2x)} \cdot 2 = \frac{2}{\text{sen}(2x)\cdot\cos(2x)}$$

9.1) $f(x) = \ln\sqrt{\frac{x}{x-1}}$

$$f'(x) = \frac{1}{\sqrt{\frac{x}{x-1}}} \cdot \frac{1}{2\sqrt{\frac{x}{x-1}}} \cdot \frac{1(x-1)-x}{(x-1)^2} = \frac{-1}{2\cdot\left(\frac{x}{x-1}\right)\cdot(x-1)^2} = \frac{-1}{2x^2-2x}$$

9.2) $f(x) = \ln\left(\frac{\sqrt{x}}{1+\text{sen}^2x}\right)$

$\text{sen}(2x) = 2\text{sen}x\cdot\cos x$

$$f'(x) = \frac{1}{\frac{\sqrt{x}}{(1+\text{sen}^2x)}} \cdot \frac{\frac{1}{2\sqrt{x}}\cdot(1+\text{sen}^2x) - \sqrt{x}\cdot 2\text{sen}x\cdot\cos x}{(1+\text{sen}^2x)^2} =$$

$$= \frac{1+\text{sen}^2x - 2x\cdot\text{sen}(2x)}{2x(1+\text{sen}^2x)}$$

9.3) $f(x) = \text{arctg}\left(\sqrt{\frac{x}{x+2}}\right)$

$$f'(x) = \frac{1}{1+\left(\sqrt{\frac{x}{x+2}}\right)^2} \cdot \frac{1}{2\sqrt{\frac{x}{x+2}}} \cdot \frac{1\cdot(x+2)-x}{(x+2)^2} =$$

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$$= \frac{1}{\frac{x+2+x}{x+2}} \cdot \frac{1}{2\sqrt{\frac{x}{x+2}}} \cdot \frac{2}{(x+2)^2} = \frac{1}{(2x+2)\cdot(x+2)\cdot\sqrt{\frac{x}{x+2}}} =$$

$$= \frac{1}{(2x+2)\sqrt{\frac{(x+2)^2\cdot x}{(x+2)}}} = \frac{1}{(2x+2)\cdot\sqrt{x^2+2x}}$$

$r_1)$ $f(x) = \dfrac{\cos x + \cos(x+1)}{\operatorname{sen} x - \operatorname{sen}(x+1)}$

$$f'(x) = \frac{(-\operatorname{sen} x - \operatorname{sen}(x+1))\cdot(\operatorname{sen} x - \operatorname{sen}(x+1)) - (\cos x + \cos(x+1))\cdot(\cos x - \cos(x+1))}{(\operatorname{sen} x - \operatorname{sen}(x+1))^2}$$

$$= \frac{-\operatorname{sen}^2(x) + \operatorname{sen}(x)\cdot\operatorname{sen}(x+1) - \operatorname{sen}(x+1)\operatorname{sen} x + \operatorname{sen}^2(x+1) - (\cos^2(x) - \cos^2(x+1))}{(\operatorname{sen} x - \operatorname{sen}(x+1))^2} =$$

$$= \frac{-(\operatorname{sen}^2 x + \cos^2 x) + (\operatorname{sen}^2(x+1) + \cos^2(x+1))}{(\operatorname{sen} x - \operatorname{sen}(x+1))^2} = 0$$

$r_2)$ $f(x) = \ln\sqrt{\dfrac{1-\operatorname{sen} x}{1+\operatorname{sen} x}}$

$$f'(x) = \frac{1}{\sqrt{\frac{1-\operatorname{sen} x}{1+\operatorname{sen} x}}} \cdot \frac{1}{2\sqrt{\frac{1-\operatorname{sen} x}{1+\operatorname{sen} x}}} \cdot \frac{-\cos x\cdot(1+\operatorname{sen} x) - (1-\operatorname{sen} x)\cdot\cos x}{(1+\operatorname{sen} x)^2} =$$

$$= \frac{1}{2\cdot\left(\frac{1-\operatorname{sen} x}{1+\operatorname{sen} x}\right)} \cdot \frac{\cos x\,(-1-\operatorname{sen} x - 1+\operatorname{sen} x)}{(1+\operatorname{sen} x)^2} = \frac{-2\cdot\cos x}{2\underbrace{(1-\operatorname{sen} x)(1+\operatorname{sen} x)}_{1-\operatorname{sen}^2 x \,=\, \cos^2 x}} = \frac{-1}{\cos x}$$

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$r_3)\ f(x) = \ln\left(e^{\operatorname{sen} x}\right) = \operatorname{sen} x \longrightarrow f'(x) = \cos x$

$s_1)\ f(x) = \operatorname{arctg}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$

$$f'(x) = \frac{1}{1+\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)^2} \cdot \frac{1}{2\sqrt{\frac{1-\cos x}{1+\cos x}}} \cdot \frac{\operatorname{sen} x(1+\cos x) + (1-\cos x)\cdot \operatorname{sen} x}{(1+\cos x)^2} =$$

$$= \frac{1}{\frac{1+\cos x + 1 - \cos x}{1+\cos x}} \cdot \frac{1}{2\sqrt{\frac{1-\cos x}{1+\cos x}}} \cdot \frac{2\operatorname{sen} x}{(1+\cos x)^2} =$$

$$= \frac{\operatorname{sen} x}{2(1+\cos x)\sqrt{\frac{1-\cos x}{1+\cos x}}} = \frac{\operatorname{sen} x}{2\cdot\sqrt{\frac{(1+\cos x)^2\cdot(1-\cos x)}{(1+\cos x)}}} = \frac{\operatorname{sen} x}{2\sqrt{1-\cos^2 x}} =$$

$$= \frac{\operatorname{sen} x}{2\cdot \operatorname{sen} x} = \frac{1}{2}$$

$s_2)\ f(x) = \ln\left(\frac{x}{x+1}\right)^x = x\cdot \ln\left(\frac{x}{x+1}\right)$

$$f'(x) = 1\cdot \ln\left(\frac{x}{x+1}\right) + x\cdot \frac{1}{\frac{x}{x+1}} \cdot \frac{1\cdot(x+1) - x}{(x+1)^2} = \ln\left(\frac{x}{x+1}\right) + \frac{1}{x+1}$$

$s_3)\ f(x) = \sqrt{e^x + e^{-x}}$

$$f'(x) = \frac{1}{2\sqrt{e^x+e^{-x}}}\cdot\left(e^x + e^{-x}\cdot(-1)\right) = \frac{e^x - e^{-x}}{2\sqrt{e^x+e^{-x}}}$$

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$t_1)\ f(x) = \ln \sqrt{\frac{x^3 \cdot \cos x}{\operatorname{sen} x}}$

$$f'(x) = \frac{1}{\sqrt{\frac{x^3\cdot\cos x}{\operatorname{sen} x}}} \cdot \frac{1}{2\sqrt{\frac{x^3\cdot\cos x}{\operatorname{sen} x}}} \cdot \frac{\left(3x^2\cdot\cos x + x^3\cdot(-\operatorname{sen} x)\right)\operatorname{sen} x - x^3\cdot\cos^2 x}{(\operatorname{sen} x)^2} =$$

$$= \frac{3x^2\cos x \operatorname{sen} x - x^3\cdot(\overbrace{\operatorname{sen}^2 x + \cos^2 x}^{1})}{2\cdot\frac{x^3\cdot\cos x}{\operatorname{sen} x}\cdot(\operatorname{sen} x)^2} = \frac{x^2(3\cos x\operatorname{sen} x - x)}{2x^3\cos x\operatorname{sen} x} =$$

$$= \frac{3\cos x\operatorname{sen} x - x}{2x\cos x\operatorname{sen} x} = \frac{3}{2x} - \frac{1}{\operatorname{sen}(2x)}$$

$t_2)\ f(x) = \dfrac{e^{x+3} - 1}{x^2 - 9}$

$$f'(x) = \frac{e^{x+3}(x^2-9) - (e^{x+3}-1)\cdot 2x}{(x^2-9)^2} = \frac{e^{x+3}(x^2-2x-9) + 2x}{(x^2-9)^2}$$

$t_3)\ f(x) = \operatorname{arcsen}\left(\sqrt{1-\operatorname{sen}^2 x}\right)$

$$f'(x) = \frac{1}{\sqrt{1-\left(\sqrt{1-\operatorname{sen}^2 x}\right)^2}} \cdot \frac{1}{2\sqrt{1-\operatorname{sen}^2 x}} \cdot (-2\operatorname{sen} x\cdot\cos x) =$$

$$= \frac{1}{\sqrt{\underbrace{1-\cos^2 x}_{\operatorname{sen}^2 x}}} \cdot \frac{1}{2\sqrt{\cos^2 x}} \cdot -2\operatorname{sen} x\cos x = \frac{-2\operatorname{sen} x\cos x}{2\operatorname{sen} x\cos x} = -1$$

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$\mu_1)\ f(x) = \operatorname{arcsen}\left(\frac{1}{1+x^2}\right)$

$$f'(x) = \frac{1}{\sqrt{1-\left(\frac{1}{1+x^2}\right)^2}} \cdot \frac{-1\cdot 2x}{(1+x^2)^2} = \frac{1}{\sqrt{\frac{(1+x^2)^2-1}{(1+x^2)^2}}} \cdot \frac{-2x}{(1+x^2)^2} =$$

$$= \frac{-2x}{(1+x^2)^2 \cdot \frac{1}{(1+x^2)} \cdot \sqrt{1+2x^2+x^4-1}} = \frac{-2x}{(1+x^2)\sqrt{x^2(2+x^2)}} =$$

$$= \frac{-2x}{(1+x^2)\cdot x \cdot \sqrt{2+x^2}} = \frac{-2}{(1+x^2)\sqrt{2+x^2}}$$

$\mu_2)\ f(x) = \frac{\ln\sqrt{x+1}}{x} = \frac{\ln(x+1)^{1/2}}{x} = \frac{\ln(x+1)}{2x}$

$$f'(x) = \frac{\frac{1}{x+1}\cdot 2x - 2\ln(x+1)}{4x^2} = \frac{2\left(\frac{x}{x+1} - \ln(x+1)\right)}{2\cdot 4x^2} = \frac{x-(x+1)\ln(x+1)}{2x^2(x+1)}$$

$\mu_3)\ f(x) = e^{\ln(\operatorname{sen}^2 x)} = \operatorname{sen}^2 x \longrightarrow f'(x) = 2\operatorname{sen}x\cos x = \operatorname{sen}(2x)$

$v_1)\ f(x) = \operatorname{tg}(1+\operatorname{sen}x) \longrightarrow f'(x) = \left(1+\operatorname{tg}^2(1+\operatorname{sen}x)\right)\cdot\cos x$

$v_2)\ f(x) = \operatorname{tg}^3(e^{x^2}) \longrightarrow f'(x) = 3\cdot\operatorname{tg}^2(e^{x^2})\left(1+\operatorname{tg}^2(e^{x^2})\right)\cdot e^{x^2}\cdot 2x =$

$$= 6x\cdot e^{x^2}\cdot\operatorname{tg}^2(e^{x^2})\left(1+\operatorname{tg}^2(e^{x^2})\right)$$

$v_3)\ f(x) = 27+(x-1)^4 \longrightarrow f'(x) = 4(x-1)^3$

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$W_1)\ f(x) = \operatorname{sen}^2 x + \cos^2 x = 1 \longrightarrow f'(x) = 0$

$W_2)\ f(x) = \dfrac{e^x + 1}{e^x - 1}$

$$f'(x) = \frac{e^x(e^x-1) - (e^x+1)\cdot e^x}{(e^x-1)^2} = \frac{e^x(e^x - 1 - e^x - 1)}{(e^x-1)^2} = \frac{-2e^x}{(e^x-1)^2}$$

$W_3)\ f(x) = \dfrac{2+\cos x}{2-\cos x}$

$$f'(x) = \frac{-\operatorname{sen} x(2-\cos x) - (2+\cos x)\cdot \operatorname{sen} x}{(2-\cos x)^2} = \frac{\operatorname{sen} x(-2+\cos x - 2 - \cos x)}{(2-\cos x)^2} =$$

$$= \frac{-4\operatorname{sen} x}{(2-\cos x)^2}$$

$X_1)\ f(x) = \operatorname{arcsen}\left(\dfrac{x}{\sqrt{1+x^2}}\right)$

$$f'(x) = \frac{1}{\sqrt{1-\left(\frac{x}{\sqrt{1+x^2}}\right)^2}} \cdot \frac{1\cdot\sqrt{1+x^2} - x\cdot\frac{1}{2\sqrt{1+x^2}}\cdot 2x}{1+x^2} =$$

$$= \frac{1}{\sqrt{\frac{1+x^2-x^2}{1+x^2}}} \cdot \frac{1+x^2-x^2}{(1+x^2)\cdot\sqrt{1+x^2}} = \frac{1}{\frac{1}{\sqrt{1+x^2}}} \cdot \frac{1}{(1+x^2)\cdot\sqrt{1+x^2}} = \frac{1}{1+x^2}$$

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$x_2)\ f(x) = \arccos\left(\frac{1}{\sqrt{1+x^2}}\right)$

$$f'(x) = \frac{-1}{\sqrt{1-\left(\frac{1}{\sqrt{1+x^2}}\right)^2}} \cdot \frac{-1 \cdot \frac{1}{2\sqrt{1+x^2}} \cdot 2x}{1+x^2} = \frac{-1}{\sqrt{\frac{1+x^2-1}{1+x^2}}} \cdot \frac{-x}{(1+x^2)\cdot\sqrt{1+x^2}} =$$

$$= \frac{x}{\frac{x}{\sqrt{1+x^2}} \cdot (1+x^2)\cdot\sqrt{1+x^2}} = \frac{1}{1+x^2}$$

$x_3)\ f(x) = e^{5x} \longrightarrow f'(x) = e^{5x}\cdot 5 = 5\cdot e^{5x}$

$y_1)\ f(x) = \sqrt{\sec(x)}$

$$f'(x) = \frac{1}{2\sqrt{\sec(x)}} \cdot \sec(x)\cdot tg(x) = \frac{\sec(x)\cdot tg(x)}{2\sqrt{\sec(x)}}$$

$y_2)\ f(x) = \operatorname{arccotg}\left(\frac{1}{x}\right)$

$$f'(x) = \frac{-1}{1+\left(\frac{1}{x}\right)^2} \cdot -\frac{1}{x^2} = \frac{1}{\frac{x^2+1}{x^2}} \cdot \frac{1}{x^2} = \frac{1}{x^2+1}$$

$y_3)\ f(x) = \operatorname{arcsec}(x)$

$$f'(x) = \frac{1}{x\sqrt{x^2-1}}$$

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$z_1)\ f(x) = \dfrac{\cos(2x)+\operatorname{sen}(2x)}{\cos(2x)-\operatorname{sen}(2x)}$

$$f'(x) = \frac{(-\operatorname{sen}(2x)\cdot 2+\cos(2x)\cdot 2)\cdot(\cos(2x)-\operatorname{sen}(2x))-(\cos(2x)+\operatorname{sen}(2x))(-\operatorname{sen}(2x)\cdot 2-\cos(2x)\cdot 2)}{(\cos(2x)-\operatorname{sen}(2x))^2}$$

$$= \frac{4\operatorname{sen}^2(2x)+4\cos^2(2x)}{\cos^2(2x)-2\operatorname{sen}(2x)\cdot\cos(2x)+\operatorname{sen}^2(4x)} = \frac{4}{1-\operatorname{sen}(4x)}$$

$z_2)\ f(x) = \ln\sqrt{\dfrac{x^2\cdot\operatorname{sen}x}{\cos x}} = \ln\sqrt{x^2\cdot\operatorname{tg}x} = \ln(x^2\cdot\operatorname{tg}x)^{1/2} = \dfrac{1}{2}\ln(x^2\cdot\operatorname{tg}x)$

$$f'(x) = \frac{1}{2}\cdot\frac{1}{x^2\cdot\operatorname{tg}x}\cdot\left(2x\operatorname{tg}x+x^2(1+\operatorname{tg}^2x)\right) = \frac{x}{2x^2\operatorname{tg}x}\left[2\operatorname{tg}(x)+x(1+\operatorname{tg}^2x)\right]$$

$$= \frac{1}{2x\operatorname{tg}x}\cdot\left[2\operatorname{tg}(x)+x(1+\operatorname{tg}^2x)\right] = \frac{1}{x}+\frac{1+\operatorname{tg}^2x}{2\operatorname{tg}x} =$$

$$= \frac{1}{x}+\frac{\frac{1}{\cos^2x}}{2\frac{\operatorname{sen}x}{\cos x}} = \frac{1}{x}+\frac{1}{2\operatorname{sen}x\cos x} = \frac{1}{x}+\frac{1}{\operatorname{sen}(2x)}$$

$z_3)\ f(x) = \ln(1+x)^x = x\cdot\ln(1+x)$

$$f'(x) = 1\cdot\ln(1+x)+x\cdot\frac{1}{1+x} = \ln(1+x)+\frac{x}{1+x}$$

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(13) Calcula la primera y la segunda derivada de las funciones:

a) $f(x) = 5x^3 + 2x^2 - 7$ b) $f(x) = e^{x^2}$ c) $f(x) = \ln(x^2-1)$

a) $f(x) = 5x^3 + 2x^2 - 7$

$f'(x) = 15x^2 + 4x$

$f''(x) = 30x + 4$

b) $f(x) = e^{x^2}$

$f'(x) = e^{x^2} \cdot 2x$

$f''(x) = e^{x^2} \cdot 2x \cdot 2x + e^{x^2} \cdot 2 = e^{x^2}(4x^2+2)$

c) $f(x) = \ln(x^2-1)$

$f'(x) = \frac{1}{x^2-1} \cdot 2x = \frac{2x}{x^2-1}$

$f''(x) = \frac{2(x^2-1) - 2x \cdot 2x}{(x^2-1)^2} = \frac{2x^2 - 2 - 4x^2}{(x^2-1)^2} = \frac{-2x^2-2}{(x^2-1)^2}$

(14) Dada la función $f(x) = 2(e^x+2) - x - \frac{x^2}{2}$, comprueba que se cumple la ecuación $f''(x) = f'(x) + x$

$f(x) = 2(e^x+2) - x - \frac{x^2}{2}$

$f'(x) = 2e^x - 1 - x$

$f''(x) = 2e^x - 1$

$f''(x) = f'(x) + x$

$2e^x - 1 = 2e^x - 1 - x + x = 2e^x - 1$

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(15) Resuelve la ecuación $f'(x)=0$, siendo:

a) $f(x)=\dfrac{x^2+1}{x^2-1}$ b) $f(x)=\dfrac{x^3}{x^2-1}$ c) $f(x)=\dfrac{x^2+1}{x}$

a) $f(x)=\dfrac{x^2+1}{x^2-1}$

$$f'(x)=\frac{2x(x^2-1)-(x^2+1)\cdot 2x}{(x^2-1)^2}=\frac{2x(x^2-1-x^2-1)}{(x^2-1)^2}=\frac{-4x}{(x^2-1)^2}$$

$$f'(x)=0 \longrightarrow \frac{-4x}{(x^2-1)^2}=0 \Rightarrow -4x=0 \Rightarrow \boxed{x=0}$$

b) $f(x)=\dfrac{x^3}{x^2-1}$

$$f'(x)=\frac{3x^2(x^2-1)-x^3\cdot 2x}{(x^2-1)^2}=\frac{3x^4-3x^2-2x^4}{(x^2-1)^2}=\frac{x^4-3x^2}{(x^2-1)^2}$$

$$f'(x)=0 \longrightarrow \frac{x^4-3x^2}{(x^2-1)^2}=0 \Rightarrow x^2(x^2-3)=0$$

$\rightarrow \boxed{x=0}$

$\rightarrow x^2-3=0 \rightarrow \boxed{x=\sqrt{3}}$, $\boxed{x=-\sqrt{3}}$

c) $f(x)=\dfrac{x^2+1}{x}$

$$f'(x)=\frac{2x\cdot x-(x^2+1)}{x^2}=\frac{x^2-1}{x^2}$$

$$f'(x)=0 \longrightarrow \frac{x^2-1}{x^2}=0 \longrightarrow x^2-1=0 \Rightarrow x^2=1$$

$\rightarrow \boxed{x=1}$

$\rightarrow \boxed{x=-1}$

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(16) Calcula la derivada n-ésima de cada una de las siguientes funciones:

a) $f(x) = e^{-x}$ b) $f(x) = 2^x$ c) $f(x) = x \cdot e^x$

a) $f(x) = e^{-x}$

$f'(x) = e^{-x} \cdot (-1) = -e^{-x}$

$f''(x) = -e^{-x} \cdot (-1) = e^{-x}$

$f'''(x) = e^{-x} \cdot (-1) = -e^{-x}$

$f^{IV}(x) = -e^{-x} \cdot (-1) = e^{-x}$

$\Rightarrow f^{n)}(x) = (-1)^n \cdot e^{-x}$

b) $f(x) = 2^x$

$f'(x) = 2^x \cdot \ln 2$

$f''(x) = 2^x \ln 2 \cdot \ln 2 = 2^x \cdot (\ln 2)^2$

$f'''(x) = 2^x \cdot (\ln 2)^2 \cdot \ln 2 = 2^x \cdot (\ln 2)^3$

$\Rightarrow f^{n)}(x) = (\ln 2)^n \cdot 2^x$

c) $f(x) = x \cdot e^x$

$f'(x) = 1 \cdot e^x + x \cdot e^x = e^x (x+1)$

$f''(x) = e^x (x+1) + e^x \cdot 1 = e^x (x+2)$

$f'''(x) = e^x (x+2) + e^x \cdot 1 = e^x (x+3)$

$\Rightarrow f^{n)}(x) = e^x (x+n)$

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(17) Encuentra la derivada de las siguientes funciones expresadas en forma implícita:

a) $x^2+y^2=4$ b) $x^2+2y^2=9$ c) $\frac{x^2}{16}+\frac{y^2}{9}=1$

d) $x^2+y^2+2x-4y-25=0$ e) $\frac{x^2}{25}-\frac{y^2}{16}=1$ f) $y+\cos(xy)=0$

g) $y=\operatorname{tg}(x\cdot y)$ h) $xy+\operatorname{sen} y=5$ i) $y^6+y-2x^2=0$

j) $y=xy+\operatorname{sen}(xy)$ k) $y=e^y+x$ l) $y=x^3-2xy+y^2$

m) $x=\operatorname{arctg}(x-y)$ n) $x^3+y^3=xy$ o) $\operatorname{sen} y=x\cdot y$

p) $y^2=\frac{x-y}{x+y}$ q) $\frac{y}{x}=1+\ln y$ r) $x^2+y^2-2xy=0$

s) $x=\cos(x\cdot y)$ t) $x^3+3y^2-2ay=0$ u) $e^{2y}-\ln x^3=3$

v) $x^3+y^3+xy=0$ w) $y^3-2xy+7=3x+1$ x) $(2y^2+3)^3=5x^3-3x$

y) $\sqrt{x}+\sqrt{y}=5$ z) $y^6+y^4-2x^2=0$

a) $x^2+y^2=4$

$2x+2y\cdot y'=0 \Rightarrow 2y\cdot y'=-2x \Rightarrow \boxed{y'=-\frac{x}{y}}$

b) $x^2+2y^2=9$

$2x+4y\cdot y'=0 \Rightarrow 4y\cdot y'=-2x \Rightarrow \boxed{y'=\frac{-x}{2y}}$

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c) $\frac{x^2}{16} + \frac{y^2}{9} = 1$

$\frac{2x}{16} + \frac{2y \cdot y'}{9} = 0 \Rightarrow \frac{2y\,y'}{9} = \frac{-2x}{16} \Rightarrow \boxed{y' = \frac{-9x}{16y}}$

d) $x^2+y^2+2x-4y-25=0$

$2x + 2y \cdot y' + 2 - 4y' = 0 \Rightarrow 2yy' - 4y' = -2x - 2$

$2y'(y-2) = -2(x+1) \Rightarrow \boxed{y' = -\frac{x+1}{y-2}}$

e) $\frac{x^2}{25} - \frac{y^2}{16} = 1$

$\frac{2x}{25} - \frac{2yy'}{16} = 0 \Rightarrow \frac{2y\,y'}{16} = \frac{2x}{25} \Rightarrow \boxed{y' = \frac{16x}{25y}}$

f) $y + \cos(x \cdot y) = 0$

$y' - \operatorname{sen}(x \cdot y) \cdot [y + x \cdot y'] = 0 \Rightarrow y' - y\operatorname{sen}(xy) - x \cdot y' \operatorname{sen}(xy) = 0$

$\Rightarrow y' - xy'\operatorname{sen}(xy) = y\operatorname{sen}(xy) \Rightarrow y'(1 - x\operatorname{sen}(xy)) = y\operatorname{sen}(xy)$

$\Rightarrow \boxed{y' = \frac{y\operatorname{sen}(xy)}{1 - x\operatorname{sen}(xy)}}$

g) $y = \operatorname{tg}(xy)$

$y' = \frac{1}{\cos^2(xy)}(y + x \cdot y') \Rightarrow y'\cos^2(xy) = y + xy' \Rightarrow$

$\Rightarrow y' \cdot \cos^2(xy) - x \cdot y' = y \Rightarrow \boxed{y' = \frac{y}{\cos^2(xy) - x}}$

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h) $xy + \text{sen}(y) = 5$

$y + x \cdot y' + \cos(y) \cdot y' = 0 \Rightarrow xy' + y'\cos(y) = -y \Rightarrow$

$\Rightarrow y'(x + \cos(y)) = -y \Rightarrow \boxed{y' = \dfrac{-y}{x + \cos(y)}}$

i) $y^6 + y - 2x^2 = 0$

$6y^5 \cdot y' + y' - 4x = 0 \Rightarrow 6y^5 \cdot y' + y' = 4x \Rightarrow$

$\Rightarrow y' \cdot (6y^5 + 1) = 4x \Rightarrow \boxed{y' = \dfrac{4x}{6y^5 + 1}}$

j) $y = xy + \text{sen}(xy)$

$y' = y + x \cdot y' + \cos(x \cdot y) \cdot (y + xy') \Rightarrow$

$\Rightarrow y' - x \cdot y' - xy'\cos(xy) = y + y\cos(xy) \Rightarrow$

$\Rightarrow y'(1 - x - x\cos(xy)) = y + y\cos(xy) \Rightarrow \boxed{y' = \dfrac{y + y\cos(xy)}{1 - x - x\cos(xy)}}$

k) $y = e^y + x$

$y' = e^y \cdot y' + 1 \Rightarrow y' - e^y \cdot y' = 1$

$\Rightarrow y'(1 - e^y) = 1 \Rightarrow \boxed{y' = \dfrac{1}{1 - e^y}}$

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l) $y = x^3 - 2xy + y^2$

$y' = 3x^2 - (2y + 2x \cdot y') + 2y \cdot y' \Rightarrow y' + 2xy' - 2yy' = 3x^2 - 2y$

$\Rightarrow y'(1 + 2x - 2y) = 3x^2 - 2y \Rightarrow \boxed{y' = \frac{3x^2 - 2y}{1 + 2x - 2y}}$

m) $x = \operatorname{arctg}(x - y)$

$1 = \frac{1}{1 + (x-y)^2} \cdot (1 - y') \Rightarrow 1 + (x-y)^2 = 1 - y' \Rightarrow \boxed{y' = -(x-y)^2}$

n) $x^3 + y^3 = xy$

$3x^2 + 3y^2 \cdot y' = y + x \cdot y' \Rightarrow 3y^2 \cdot y' - x \cdot y' = y - 3x^2 \Rightarrow$

$\Rightarrow y'(3y^2 - x) = y - 3x^2 \Rightarrow \boxed{y' = \frac{y - 3x^2}{3y^2 - x}}$

o) $\operatorname{sen} y = x \cdot y$

$\cos(y) \cdot y' = y + x \cdot y' \Rightarrow \cos(y) \cdot y' - xy' = y \Rightarrow$

$\Rightarrow y'(\cos y - x) = y \Rightarrow \boxed{y' = \frac{y}{\cos y - x}}$

p) $y^2 = \frac{x - y}{x + y}$

$2y \cdot y' = \frac{(1 - y')(x + y) - (x - y) \cdot (1 + y')}{(x + y)^2} \Rightarrow$

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$$\Rightarrow 2y(x+y)^2 \cdot y' = \cancel{x} + y - y'(x+y) - \cancel{x} + y - y'(x-y) \Rightarrow$$

$$\Rightarrow 2y(x+y)^2 \cdot y' + y'(x+y) + y'(x-y) = 2y \Rightarrow$$

$$\Rightarrow y'\left[2y(x+y)^2 + \cancel{x+y} + \cancel{x-y}\right] = 2y \Rightarrow$$

$$\Rightarrow \cancel{2}y'\left[y(x+y)^2 + x\right] = \cancel{2}y \Rightarrow \boxed{y' = \frac{y}{y(x+y)^2 + x}}$$

q) $\frac{y}{x} = 1 + \ln y$

$$\frac{y'x - y}{x^2} = \frac{1}{y} \cdot y' \Rightarrow y'xy - y^2 = x^2 \cdot y' \Rightarrow$$

$$\Rightarrow y'xy - x^2 \cdot y' = y^2 \Rightarrow y'(xy - x^2) = y^2 \Rightarrow \boxed{y' = \frac{y^2}{xy - x^2}}$$

r) $x^2 + y^2 - 2xy = 0$

$$2x + 2y \cdot y' - 2y - 2xy' = 0 \Rightarrow 2yy' - 2x \cdot y' = 2y - 2x$$

$$\Rightarrow \cancel{2}y'\cancel{(y-x)} = \cancel{2}\cancel{(y-x)} \Rightarrow \boxed{y' = 1}$$

s) $x = \cos(xy)$

$$1 = -\operatorname{sen}(x \cdot y) \cdot (y + x \cdot y') \Rightarrow 1 = -y\operatorname{sen}(xy) - xy'\operatorname{sen}(xy)$$

$$1 + y\operatorname{sen}(xy) = -x \cdot y' \cdot \operatorname{sen}(xy) \Rightarrow \boxed{y' = \frac{1 + y\operatorname{sen}(xy)}{-x\operatorname{sen}(xy)}}$$

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t) $x^3 + 3y^2 - 2ay = 0$

$3x^2 + 6y \cdot y' - 2ay' = 0 \Rightarrow 3x^2 = 2ay' - 6y \cdot y' \Rightarrow$

$\Rightarrow 3x^2 = y'(2a - 6y) \Rightarrow \boxed{y' = \frac{3x^2}{2a - 6y}}$

u) $e^{2y} - \ln x^3 = 3$

$e^{2y} \cdot 2y' - \frac{1}{x^3} \cdot 3x^2 = 0 \Rightarrow e^{2y} \cdot 2y' = \frac{3}{x} \Rightarrow \boxed{y' = \frac{3}{2x \cdot e^{2y}}}$

v) $x^3 + y^3 + xy = 0$

$3x^2 + 3y^2 \cdot y' + y + x \cdot y' = 0 \Rightarrow 3y^2 \cdot y' + x \cdot y' = -3x^2 - y$

$\Rightarrow y'(3y^2 + x) = -3x^2 - y \Rightarrow \boxed{y' = \frac{-3x^2 - y}{3y^2 + x}}$

w) $y^3 - 2xy + 7 = 3x + 1$

$3y^2 \cdot y' - 2y - 2xy' = 3 \Rightarrow 3y^2 \cdot y' - 2xy' = 3 + 2y$

$\Rightarrow y'(3y^2 - 2x) = 3 + 2y \Rightarrow \boxed{y' = \frac{3 + 2y}{3y^2 - 2x}}$

x) $(2y^2 + 3)^3 = 5x^3 - 3x$

$3(2y^2 + 3)^2 \cdot 4y \cdot y' = 15x^2 - 3 \Rightarrow \boxed{y' = \frac{15x^2 - 3}{12y(2y^2 + 3)^2}}$

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y) $\sqrt{x} + \sqrt{y} = 5$

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \cdot y' = 0 \Rightarrow \frac{1}{2\sqrt{y}} \cdot y' = \frac{-1}{2\sqrt{x}} \Rightarrow \boxed{y' = -\sqrt{\frac{y}{x}}}$$

z) $y^6 + y^4 - 2x^2 = 0$

$$6y^5 \cdot y' + 4y^3 \cdot y' - 4x = 0 \Rightarrow y'(6y^5 + 4y^3) = 4x$$

$$\Rightarrow \boxed{y' = \frac{4x}{6y^5 + 4y^3}}$$

(18) Encuentra las derivadas de las siguientes funciones:

a) $y = x^x$ b) $y = x^{\ln x}$ c) $y = (\ln x)^{2x}$

d) $y = x^{\cos x}$ e) $y = (2\sqrt{x})^{\ln x}$ f) $y = (\operatorname{sen} x)^{\ln x}$

g) $y = (\ln x)^{\operatorname{tg} x}$ h) $y = x^{\operatorname{sen} x}$ i) $y = (\operatorname{tg} x)^{\operatorname{arctg}(x)}$

j) $y = (\sqrt{x})^x$ k) $y = x^{\sqrt{x}}$ l) $y = (x - \operatorname{sen} x)^{\sqrt{x}}$

m) $y = (\operatorname{sen} x)^{2x}$ n) $y = \sqrt[x]{\cos x}$ o) $y = (\operatorname{arcsen} x)^{\operatorname{sen} x}$

p) $y = (1 + x^2)^x$ q) $y = (\operatorname{sen} x)^{\cos x}$ r) $y = \sqrt[x]{x^3}$

s) $y = \left(1 + \frac{1}{x}\right)^x$ t) $y = (\operatorname{tg} x)^x$

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a) $y = x^x \Rightarrow \ln y = \ln x^x \Rightarrow \ln y = x \cdot \ln x$

$\frac{1}{y} \cdot y' = \ln x + \cancel{x} \cdot \frac{1}{\cancel{x}} \Rightarrow y' = y(\ln x + 1) \Rightarrow$

$\Rightarrow \boxed{y' = x^x(\ln x + 1)}$

b) $y = x^{\ln x} \Rightarrow \ln y = \ln x^{\ln x} \Rightarrow \ln y = \ln x \cdot \ln x = (\ln x)^2$

$\frac{1}{y} \cdot y' = 2(\ln x) \cdot \frac{1}{x} \Rightarrow y' = y \cdot \frac{2\ln x}{x} \Rightarrow$

$\Rightarrow \boxed{y' = x^{\ln x} \cdot \frac{2\ln x}{x}}$

c) $y = (\ln x)^{2x} \Rightarrow \ln y = \ln\left((\ln x)^{2x}\right) \Rightarrow \ln y = 2x \cdot \ln(\ln|x|)$

$\frac{1}{y} \cdot y' = 2\ln(\ln|x|) + \cancel{2x} \cdot \frac{1}{\ln|x|} \cdot \frac{1}{\cancel{x}} \Rightarrow$

$\Rightarrow y' = y \cdot \left[2\ln(\ln|x|) + \frac{2}{\ln|x|}\right] = \boxed{(\ln x)^{2x} \cdot \left[2\ln(\ln|x|) + \frac{2}{\ln|x|}\right]}$

d) $y = x^{\cos x} \Rightarrow \ln y = \ln x^{\cos x} \Rightarrow \ln y = \cos x \cdot \ln x$

$\frac{1}{y} \cdot y' = -\operatorname{sen} x \cdot \ln x + \frac{\cos x}{x} \Rightarrow \boxed{y' = x^{\cos x} \cdot \left[-\operatorname{sen} x \cdot \ln|x| + \frac{\cos x}{x}\right]}$

e) $y = (2\sqrt{x})^{\ln x} \Rightarrow \ln y = \ln(2\sqrt{x})^{\ln x} \Rightarrow \ln y = \ln(x) \cdot \ln(2\sqrt{x})$

$\frac{1}{y} \cdot y' = \frac{1}{x} \cdot \ln(2\sqrt{x}) + \ln(x) \cdot \frac{1}{2\sqrt{x}} \cdot \frac{\cancel{2}}{\cancel{2}\sqrt{x}} \Rightarrow$

$\Rightarrow \boxed{y' = (2\sqrt{x})^{\ln x} \cdot \left[\frac{\ln(2\sqrt{x})}{x} + \frac{\ln(x)}{2x}\right]}$

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f) $y = (\operatorname{sen} x)^{\ln x} \Rightarrow \ln y = \ln (\operatorname{sen} x)^{\ln x} \Rightarrow \ln y = \ln x \cdot \ln(\operatorname{sen} x)$

$$\frac{1}{y}\cdot y' = \frac{1}{x}\cdot \ln(\operatorname{sen} x) + \ln x \cdot \frac{1}{\operatorname{sen} x}\cdot \cos x \Rightarrow$$

$$\boxed{y' = (\operatorname{sen} x)^{\ln x}\cdot\left[\frac{\ln(\operatorname{sen} x)}{x} + \frac{\ln x}{\operatorname{tg} x}\right]}$$

g) $y = (\ln x)^{\operatorname{tg} x} \Rightarrow \ln y = \ln\left((\ln x)^{\operatorname{tg} x}\right) \Rightarrow \ln y = \operatorname{tg} x \cdot \ln(\ln(x))$

$$\frac{1}{y}\cdot y' = \frac{1}{\cos^2 x}\cdot \ln(\ln(x)) + \operatorname{tg}(x)\cdot\frac{1}{\ln(x)}\cdot\frac{1}{x} \Rightarrow$$

$$\boxed{y' = (\ln x)^{\operatorname{tg} x}\cdot\left[\frac{\ln(\ln(x))}{\cos^2 x} + \frac{\operatorname{tg} x}{x\cdot\ln(x)}\right]}$$

h) $y = x^{\operatorname{sen} x} \Rightarrow \ln y = \ln x^{\operatorname{sen} x} \Rightarrow \ln y = \operatorname{sen} x \cdot \ln x$

$$\frac{1}{y}\cdot y' = \cos x\cdot\ln x + \operatorname{sen} x\cdot\frac{1}{x} \Rightarrow \boxed{y' = x^{\operatorname{sen} x}\cdot\left[\cos x\cdot\ln x + \frac{\operatorname{sen} x}{x}\right]}$$

i) $y = (\operatorname{tg} x)^{\operatorname{arctg} x} \Rightarrow \ln y = \ln(\operatorname{tg} x)^{\operatorname{arctg} x} \Rightarrow \ln y = \operatorname{arctg} x \cdot \ln(\operatorname{tg} x)$

$$\frac{1}{y}\cdot y' = \frac{1}{1+x^2}\cdot\ln(\operatorname{tg}(x)) + \operatorname{arctg} x\cdot\frac{1}{\operatorname{tg} x}\cdot\frac{1}{\cos^2 x} \Rightarrow$$

$$\Rightarrow \boxed{y' = (\operatorname{tg} x)^{\operatorname{arctg} x}\cdot\left[\frac{\ln(\operatorname{tg}(x))}{1+x^2} + \frac{\operatorname{arctg} x}{\operatorname{sen} x\cdot\cos x}\right]}$$

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j) $y = (\sqrt{x})^x \Rightarrow \ln y = \ln(\sqrt{x})^x \Rightarrow \ln y = x \cdot \ln(\sqrt{x})$

$$\frac{1}{y} \cdot y' = \ln\sqrt{x} + x \cdot \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \ln\sqrt{x} + \frac{1}{2} \Rightarrow$$

$$\Rightarrow \boxed{y' = (\sqrt{x})^x \cdot \left(\ln\sqrt{x} + \frac{1}{2}\right)}$$

k) $y = (x)^{\sqrt{x}} \Rightarrow \ln y = \ln(x)^{\sqrt{x}} \Rightarrow \ln y = \sqrt{x} \cdot \ln x$

$$\frac{1}{y} \cdot y' = \frac{1}{2\sqrt{x}} \cdot \ln x + \frac{\sqrt{x}}{x} \Rightarrow \boxed{y' = x^{\sqrt{x}} \cdot \left[\frac{\ln x}{2\sqrt{x}} + \frac{\sqrt{x}}{x}\right]}$$

l) $y = (x - \operatorname{sen} x)^{\sqrt{x}} \Rightarrow \ln y = \ln(x - \operatorname{sen} x)^{\sqrt{x}} \Rightarrow \ln y = \sqrt{x} \cdot \ln(x - \operatorname{sen} x)$

$$\frac{1}{y} \cdot y' = \frac{1}{2\sqrt{x}} \ln(x - \operatorname{sen} x) + \sqrt{x} \cdot \frac{1}{x - \operatorname{sen} x} \cdot (1 - \cos x) \Rightarrow$$

$$\Rightarrow \boxed{y' = (x - \operatorname{sen} x)^{\sqrt{x}} \cdot \left[\frac{\ln(x - \operatorname{sen} x)}{2\sqrt{x}} + \frac{\sqrt{x}\,(1 - \cos x)}{x - \operatorname{sen} x}\right]}$$

m) $y = (\operatorname{sen} x)^{2x} \Rightarrow \ln y = \ln(\operatorname{sen} x)^{2x} \Rightarrow \ln y = 2x \cdot \ln(\operatorname{sen} x)$

$$\frac{1}{y} \cdot y' = 2\ln(\operatorname{sen} x) + 2x \cdot \frac{1}{\operatorname{sen} x} \cdot \cos x \Rightarrow$$

$$\Rightarrow \boxed{y' = 2(\operatorname{sen} x)^{2x} \left[\ln(\operatorname{sen} x) + \frac{x}{\operatorname{tg} x}\right]}$$

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n) $y = \sqrt[x]{\cos x} \Rightarrow \ln y = \ln(\cos x)^{1/x} \Rightarrow \ln y = \frac{1}{x}\cdot \ln(\cos x)$

$$\frac{1}{y}\cdot y' = -\frac{1}{x^2}\cdot \ln(\cos x) + \frac{1}{x}\cdot\frac{1}{\cos x}\cdot \operatorname{sen} x \Rightarrow$$

$$\Rightarrow \boxed{y' = \frac{\sqrt[x]{\cos x}}{x}\cdot\left(\operatorname{tg} x - \frac{\ln(\cos x)}{x}\right)}$$

o) $y = (\operatorname{arcsen} x)^{\operatorname{sen} x} \Rightarrow \ln y = \ln(\operatorname{arcsen} x)^{\operatorname{sen} x} \Rightarrow \ln y = \operatorname{sen} x\cdot \ln(\operatorname{arcsen} x)$

$$\frac{1}{y}\cdot y' = \cos x\cdot \ln(\operatorname{arcsen}(x)) + \operatorname{sen} x\cdot\frac{1}{\operatorname{arcsen} x}\cdot\frac{1}{\sqrt{1-x^2}} \Rightarrow$$

$$\Rightarrow \boxed{y' = (\operatorname{arcsen} x)^{\operatorname{sen} x}\cdot\left[\cos x\cdot\ln(\operatorname{arcsen}(x)) + \frac{\operatorname{sen} x}{\operatorname{arcsen} x\cdot\sqrt{1-x^2}}\right]}$$

p) $y = (1+x^2)^x \Rightarrow \ln y = \ln(1+x^2)^x \Rightarrow \ln y = x\cdot\ln(1+x^2)$

$$\frac{1}{y}\cdot y' = \ln(1+x^2) + x\cdot\frac{1}{1+x^2}\cdot 2x \Rightarrow \boxed{y' = (1+x^2)^x\cdot\left(\ln(1+x^2) + \frac{2x^2}{1+x^2}\right)}$$

q) $y = (\operatorname{sen} x)^{\cos x} \Rightarrow \ln y = \ln(\operatorname{sen} x)^{\cos x} \Rightarrow \ln y = \cos x\cdot\ln(\operatorname{sen} x)$

$$\frac{1}{y}\cdot y' = -\operatorname{sen} x\cdot\ln(\operatorname{sen} x) + \cos x\cdot\frac{1}{\operatorname{sen} x}\cdot\cos x \Rightarrow$$

$$\Rightarrow \boxed{y' = (\operatorname{sen} x)^{\cos x}\cdot\left[-\operatorname{sen} x\cdot\ln(\operatorname{sen} x) + \frac{\cos x}{\operatorname{tg} x}\right]}$$

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r) $y = \sqrt[x]{x^3} \Rightarrow \ln y = \ln(x)^{3/x} \Rightarrow \ln y = \frac{3}{x}\cdot \ln x$

$$\frac{1}{y}\cdot y' = -\frac{3}{x^2}\ln x + \frac{3}{x}\cdot\frac{1}{x} \Rightarrow \boxed{y' = \sqrt[x]{x^3}\cdot\frac{3}{x^2}\cdot(1-\ln x)}$$

s) $y = \left(1+\frac{1}{x}\right)^x \Rightarrow \ln y = \ln\left(1+\frac{1}{x}\right)^x \Rightarrow \ln y = x\cdot\ln\left(1+\frac{1}{x}\right)$

$$\frac{1}{y}\cdot y' = \ln\left(1+\frac{1}{x}\right) + x\cdot\frac{1}{1+\frac{1}{x}}\cdot\frac{-1}{x^2} = \ln\left(1+\frac{1}{x}\right) + x\cdot\frac{1}{\frac{x+1}{x}}\cdot\frac{-1}{x^2}$$

$$\Rightarrow \boxed{y' = \left(1+\frac{1}{x}\right)^x\cdot\left[\ln\left(1+\frac{1}{x}\right) - \frac{1}{x+1}\right]}$$

t) $y = (\operatorname{tg} x)^x \Rightarrow \ln y = \ln(\operatorname{tg} x)^x \Rightarrow \ln y = x\cdot\ln(\operatorname{tg} x)$

$$\frac{1}{y}\cdot y' = \ln(\operatorname{tg} x) + x\cdot\frac{1}{\operatorname{tg} x}\cdot\frac{1}{\cos^2 x} \Rightarrow$$

$$\Rightarrow \boxed{y' = (\operatorname{tg} x)^x\cdot\left[\ln(\operatorname{tg} x) + \frac{x}{\operatorname{sen} x\cos x}\right]}$$

(19) Estudia la derivabilidad de las siguientes funciones:

a) $f(x) = x^3+3x^2-5$ b) $f(x) = \sqrt{x}$ c) $f(x) = \ln(x)$

d) $f(x) = \frac{1}{x}$ e) $f(x) = \frac{e^x}{x-3}$ f) $f(x) = \sqrt[3]{(x-2)}$

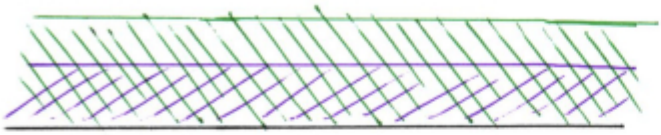
g) $f(x) = \operatorname{sen} x$ h) $f(x) = \frac{1}{\ln(x)}$ i) $f(x) = 3^x$

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a) $f(x) = x^3 + 3x^2 - 5 \longrightarrow Dom(f(x)) = \mathbb{R}$

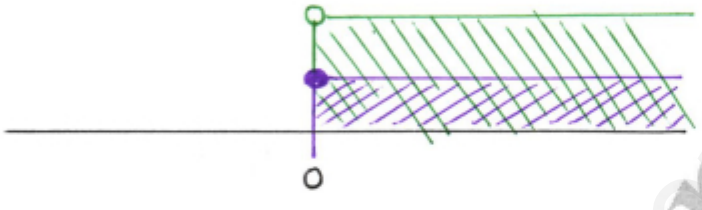
$f'(x) = 3x^2 + 6x \longrightarrow Dom(f'(x)) = \mathbb{R}$



$\Rightarrow f(x)$ es derivable en $\mathbb{R}$

b) $f(x) = \sqrt{x} \longrightarrow Dom(f(x)) = [0, +\infty)$

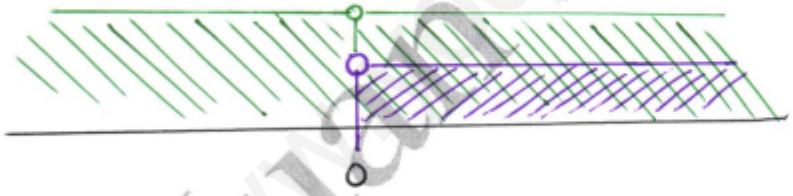
$f'(x) = \frac{1}{2\sqrt{x}} \longrightarrow Dom(f'(x)) = (0, +\infty)$



$\Rightarrow f(x)$ es derivable en $(0, +\infty)$

c) $f(x) = \ln(x) \longrightarrow Dom(f(x)) = (0, +\infty)$

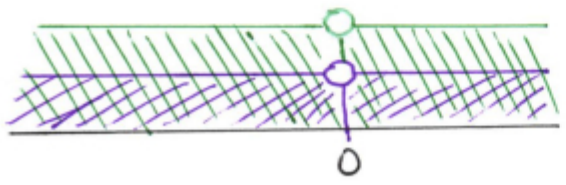
$f'(x) = \frac{1}{x} \longrightarrow Dom(f'(x)) = \mathbb{R} - \{0\}$



$\Rightarrow f(x)$ es derivable en $(0, +\infty)$

d) $f(x) = \frac{1}{x} \longrightarrow Dom(f(x)) = \mathbb{R} - \{0\}$

$f'(x) = \frac{-1}{x^2} \longrightarrow Dom(f'(x)) = \mathbb{R} - \{0\}$



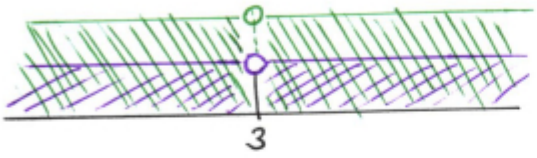
$\Rightarrow f(x)$ es derivable en $\mathbb{R} - \{0\}$

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e) $f(x) = \dfrac{e^x}{x-3} \longrightarrow Dom(f(x)) = \mathbb{R} - \{3\}$

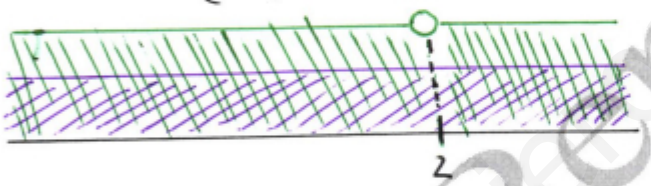
$f'(x) = \dfrac{e^x(x-3) - e^x}{(x-3)^2} = \dfrac{e^x(x-4)}{(x-3)^2} \longrightarrow Dom(f'(x)) = \mathbb{R} - \{3\}$



$\Rightarrow f(x)$ es derivable en $\mathbb{R} - \{3\}$

f) $f(x) = \sqrt[3]{(x-2)} \longrightarrow Dom(f(x)) = \mathbb{R}$

$f'(x) = \dfrac{1}{3\sqrt[3]{(x-2)^2}} \longrightarrow Dom(f'(x)) = \mathbb{R} - \{2\}$



$\Rightarrow f(x)$ es derivable en $\mathbb{R} - \{2\}$

g) $\left.\begin{array}{l} f(x) = \operatorname{sen} x \longrightarrow Dom(f(x)) = \mathbb{R} \\ f'(x) = \cos x \longrightarrow Dom(f'(x)) = \mathbb{R} \end{array}\right\}$ $f(x)$ es derivable en $\mathbb{R}$

h) $f(x) = \dfrac{1}{\ln x}$ $\left\{\begin{array}{l} \nearrow x > 0 \\ \searrow \ln x = 0 \Rightarrow x = e^0 = 1 \end{array}\right\} Dom(f(x)) = (0,1) \cup (1,+\infty)$

$f'(x) = \dfrac{-1}{x \cdot (\ln x)^2}$ $\left\{\begin{array}{l} \nearrow x > 0 \\ \searrow x \cdot (\ln x)^2 = 0 \left\{\begin{array}{l} \nearrow x = 0 \text{ (No sirve)} \\ \searrow \ln x = 0 \longrightarrow x = 1 \end{array}\right. \end{array}\right.$

$Dom(f'(x)) = (0,1) \cup (1,+\infty)$

$\Rightarrow f(x)$ es derivable en $(0,1) \cup (1,+\infty)$

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i) $f(x) = 3^x \longrightarrow Dom(f(x)) = \mathbb{R}$

$f'(x) = 3^x \cdot \ln 3 \longrightarrow Dom(f'(x)) = \mathbb{R}$

$\Big\}$ $f(x)$ es derivable en $\mathbb{R}$

(20) Estudia la continuidad y la derivabilidad de las siguientes funciones, haciendo además su representación gráfica:

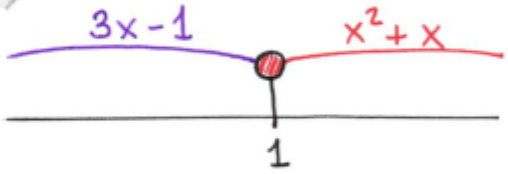
a) $f(x) = \begin{cases} 3x-1 & \text{si } x<1 \\ x^2+x & \text{si } x \geq 1 \end{cases}$

b) $f(x) = \begin{cases} -x^2 & \text{si } x<0 \\ x^2 & \text{si } x \geq 0 \end{cases}$

c) $f(x) = \begin{cases} 2x-1 & \text{si } x<3 \\ x^2-4 & \text{si } x \geq 3 \end{cases}$

d) $f(x) = \begin{cases} 3x-2 & \text{si } x \leq 2 \\ 3x+1 & \text{si } x>2 \end{cases}$

a) $f(x) = \begin{cases} 3x-1 & \text{si } x<1 \\ x^2+x & \text{si } x \geq 1 \end{cases}$



Si $x<1$ $\longrightarrow$ $f(x) = 3x-1 \longrightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, 1)$

Si $x>1$ $\longrightarrow$ $f(x) = x^2+x \longrightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(1, +\infty)$

Si $x=1$ Continuidad:

$f(1) = 1^2+1 = 2$

$\lim_{x\to 1} f(x) \longrightarrow \begin{cases} \lim_{x\to 1^-} (3x-1) = 2 \\ \lim_{x\to 1^+} (x^2+x) = 2 \end{cases} \Rightarrow \lim_{x\to 1} f(x) = 2$

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Como $f(1) = \lim_{x \to 1} f(x) \Rightarrow$ Continua en $x = 1$

Para la derivabilidad:

$$f'(x) = \begin{cases} 3 & \text{si } x < 1 \\ 2x+1 & \text{si } x \geq 1 \end{cases}$$

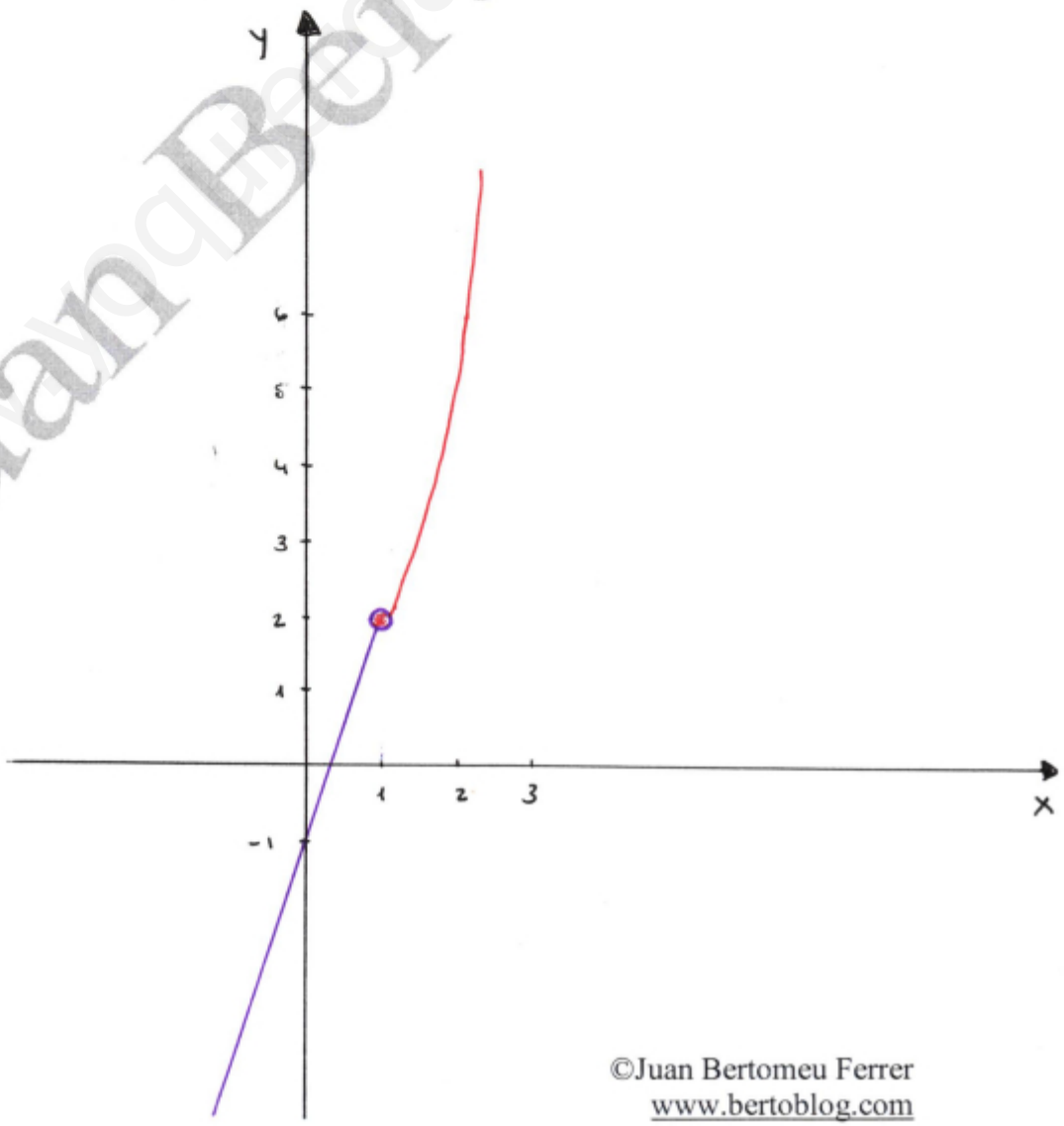
Si $x = 1$ Derivabilidad

$$\left.\begin{array}{l} f'(1^-) = 3 \\ f'(1^+) = 2 \cdot 1 + 1 = 3 \end{array}\right\} \text{Como } f'(1^-) = f'(1^+) \Rightarrow \text{Derivable en } x = 1$$

$\Rightarrow f(x)$ es continua y derivable en $\mathbb{R}$

x	$f(x) = 3x - 1$
0	-1
1	2 NO

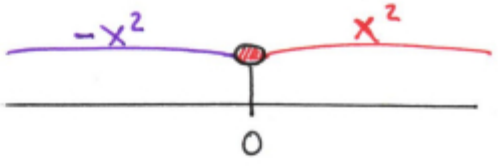
x	$f(x) = x^2 + x$
1	2 SI
2	6
3	12
...	...



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b) $f(x) = \begin{cases} -x^2 & \text{si } x < 0 \\ x^2 & \text{si } x \geq 0 \end{cases}$



Si $x<0$ → $f(x) = -x^2$ → Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(-\infty, 0)$

Si $x>0$ → $f(x) = x^2$ → Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(0, +\infty)$

Si $x=0$ Continuidad:

$f(0) = 0^2 = 0$

$$\lim_{x\to 0} f(x) \longrightarrow \left\{ \begin{array}{l} \lim\limits_{x\to 0^-} -x^2 = 0 \\ \lim\limits_{x\to 0^+} x^2 = 0 \end{array} \right\} \Rightarrow \lim_{x\to 0} f(x) = 0$$

Como $f(0) = \lim\limits_{x\to 0} f(x) \Rightarrow$ Continua en $x=0$

Para la derivabilidad:

$$f'(x) = \begin{cases} -2x & \text{si } x < 0 \\ 2x & \text{si } x \geq 0 \end{cases}$$

Si $x=0$ Derivabilidad:

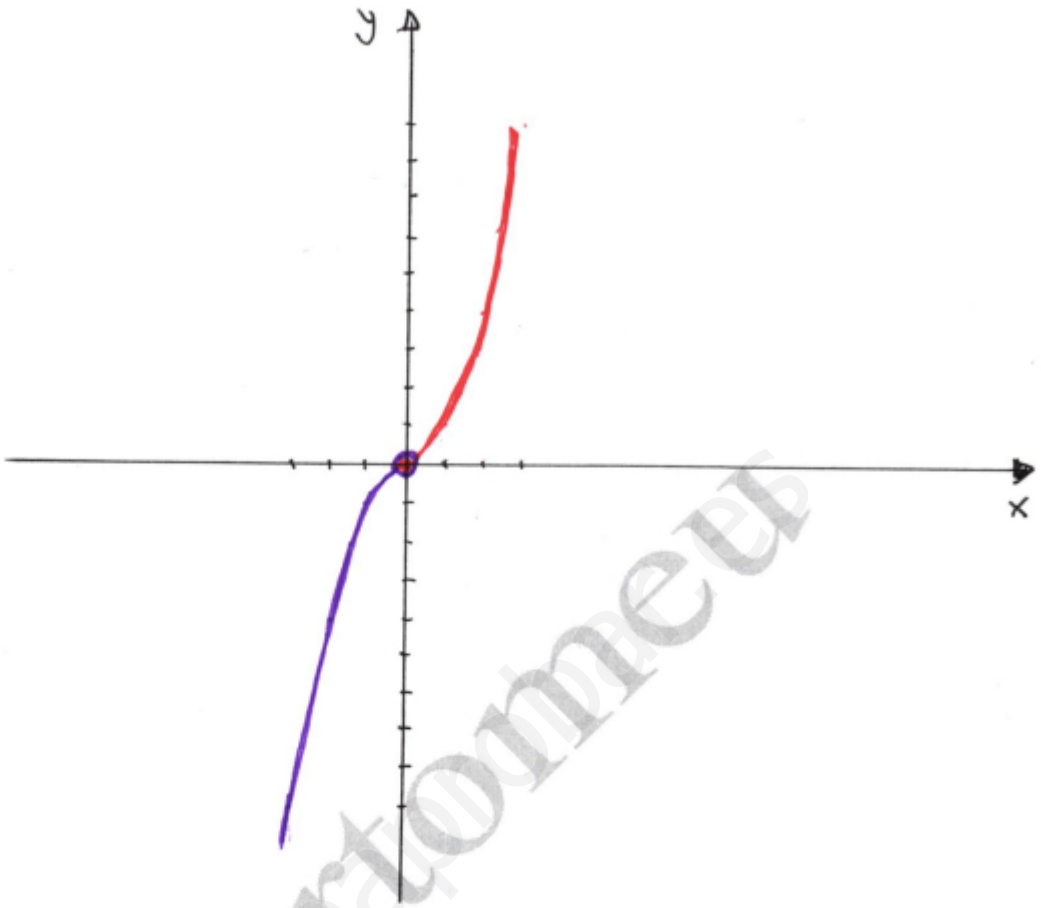
$$\left. \begin{array}{l} f'(0^-) = -2\cdot 0 = 0 \\ f'(0^+) = 2\cdot 0 = 0 \end{array} \right\} \text{Como } f'(0^-) = f'(0^+) \Rightarrow \text{Derivable en } x=0$$

$\Rightarrow f(x)$ es continua y derivable en $\mathbb{R}$

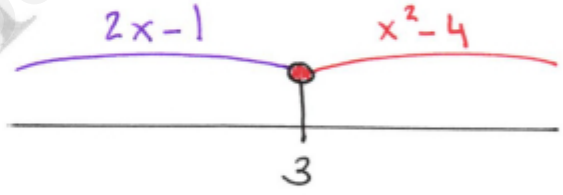
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x	$f(x) = -x^2$	
-3	-9	
-2	-4	
-1	-1	
0	0	NO
x	$f(x) = x^2$	
0	0	SI
1	1	
2	4	
3	9	
...	...	



c) $$f(x) = \begin{cases} 2x-1 & \text{si } x < 3 \\ x^2-4 & \text{si } x \geq 3 \end{cases}$$



Si $x < 3$ $\longrightarrow$ $f(x) = 2x-1$ $\Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, 3)$

Si $x > 3$ $\longrightarrow$ $f(x) = x^2-4$ $\Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(3, +\infty)$

Si $x = 3$ Continuidad:

$f(3) = 3^2 - 4 = 5$

$$\lim_{x \to 3} f(x) \longrightarrow \left\{ \begin{array}{l} \lim_{x \to 3^-} (2x-1) = 5 \\ \lim_{x \to 3^+} (x^2-4) = 5 \end{array} \right\} \Rightarrow \lim_{x \to 3} f(x) = 5$$

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Como $f(3) = \lim_{x \to 3} f(x) \Rightarrow$ Continua en $x = 3$

Para la derivabilidad:

$$f'(x) = \begin{cases} 2 & \text{si } x < 3 \\ 2x & \text{si } x > 3 \end{cases}$$

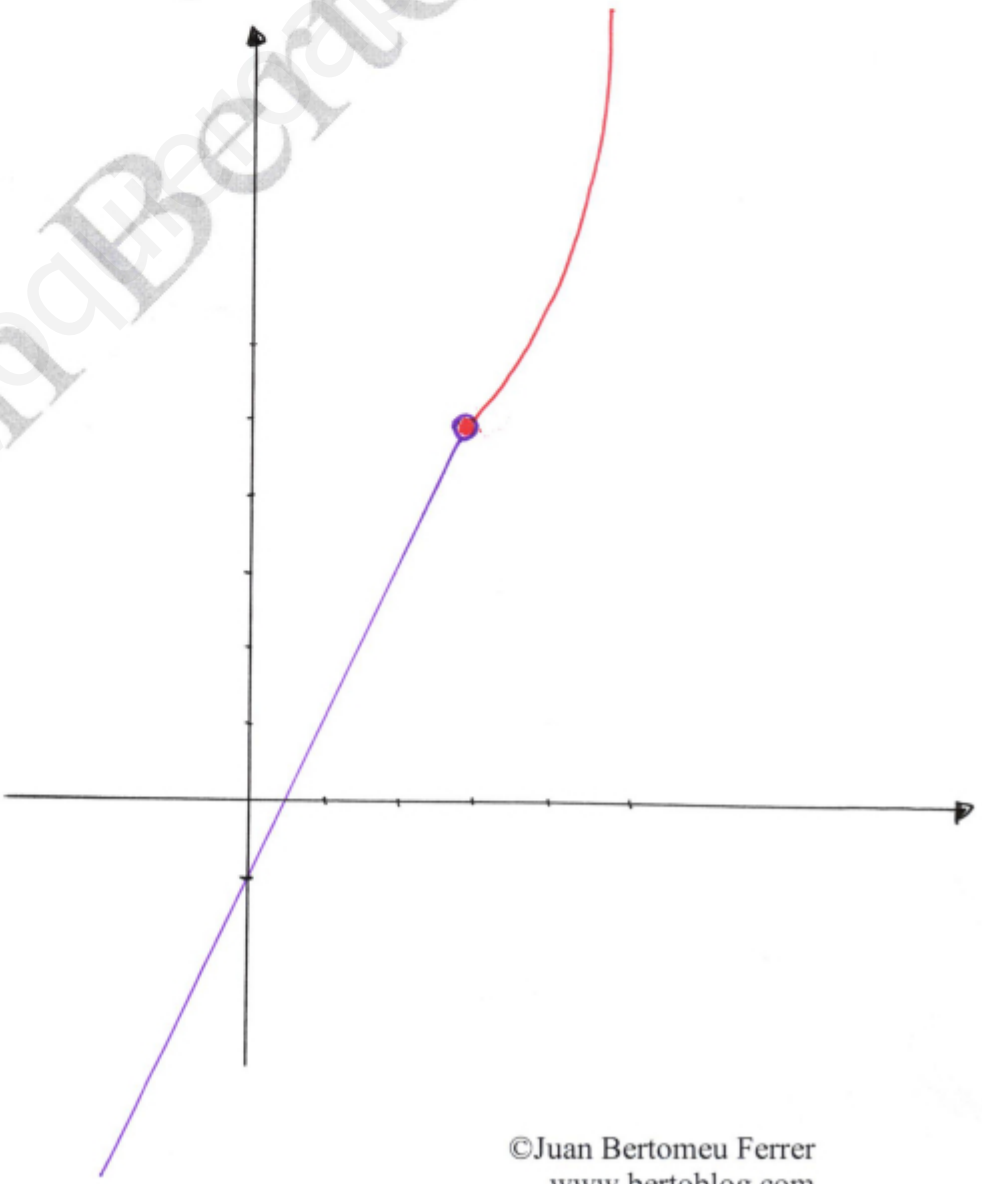
Si $x = 3$ Derivabilidad:

$$\left.\begin{array}{l} f'(3^-) = 2 \\ f'(3^+) = 2 \cdot 3 = 6 \end{array}\right\} \; f'(3^-) \neq f'(3^+) \Rightarrow \nexists f'(3) \Rightarrow \text{No derivable en } x = 3$$

$\Rightarrow f(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R} - \{3\}$

x	$f(x) = 2x - 1$
0	-1
3	5 NO

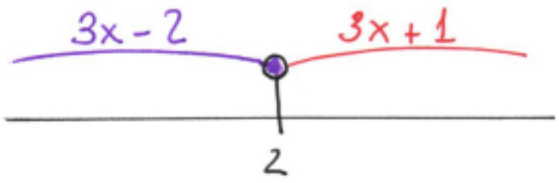
x	$f(x) = x^2 - 4$
3	5 SI
4	12
5	21
...	...



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d) $f(x) = \begin{cases} 3x-2 & \text{si } x \leq 2 \\ 3x+1 & \text{si } x > 2 \end{cases}$



Si $x < 2$ → $f(x) = 3x-2$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(-\infty, 2)$

Si $x > 2$ → $f(x) = 3x+1$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(2, +\infty)$

Si $x = 2$ Continuidad:

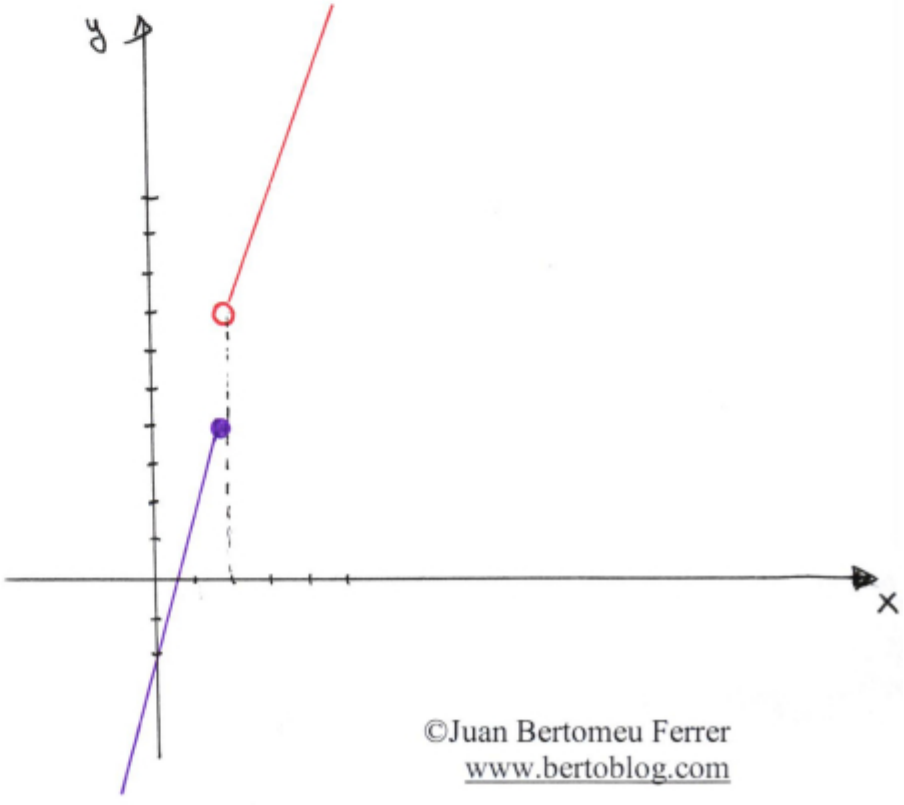
$f(2) = 3 \cdot 2 - 2 = 4$

$$\lim_{x \to 2} f(x) \longrightarrow \left\{ \begin{array}{l} \lim_{x \to 2^-} (3x-2) = 4 \\ \lim_{x \to 2^+} (3x+1) = 7 \end{array} \right\} \Rightarrow \nexists \lim_{x \to 2} f(x)$$

$f(x)$ presenta en $x=2$ una discontinuidad de SALTO FINITO y por tanto tampoco es derivable en $x=2$

x	$f(x) = 3x-2$
0	-2
2	4 SI

x	$f(x) = 3x+1$
2	7 NO
3	10



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(21) Estudia la continuidad y la derivabilidad de las siguientes funciones:

a) $f(x) = \begin{cases} x^2-1 & \text{si } x \le 1 \\ x-1 & \text{si } x > 1 \end{cases}$

b) $f(x) = \begin{cases} e^{-x} & \text{si } x \le 0 \\ 1-x & \text{si } x > 0 \end{cases}$

c) $f(x) = \begin{cases} \ln(x-1) & \text{si } x < 2 \\ 3x-6 & \text{si } x \ge 2 \end{cases}$

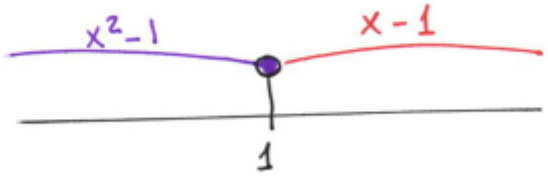
d) $f(x) = \begin{cases} x^2-2x+1 & \text{si } x < 2 \\ -2x+5 & \text{si } x \ge 2 \end{cases}$

e) $f(x) = \begin{cases} 2x-5 & \text{si } x < 3 \\ \sqrt{x-2} & \text{si } x \ge 3 \end{cases}$

f) $f(x) = \begin{cases} e^x+2 & \text{si } x \le 0 \\ x^2+x+3 & \text{si } x > 0 \end{cases}$

g) $f(x) = \begin{cases} 0 & \text{si } x < 0 \\ x^2 & \text{si } 0 \le x < 1 \\ x & \text{si } x \ge 1 \end{cases}$

a) $f(x) = \begin{cases} x^2-1 & \text{si } x \le 1 \\ x-1 & \text{si } x > 1 \end{cases}$



Si $x < 1$ → $f(x) = x^2-1$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(-\infty, 1)$

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Si $x>1$ $\rightarrow f(x) = x-1 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(1,+\infty)$

Si $x=1$ Continuidad:

$f(1) = 1^2 - 1 = 0$

$$\lim_{x\to 1} f(x) \rightarrow \left\{ \begin{array}{l} \lim\limits_{x\to 1^-} x^2-1 = 0 \\ \lim\limits_{x\to 1^+} x-1 = 0 \end{array} \right\} \Rightarrow \lim_{x\to 1} f(x) = 0$$

Como $f(1) = \lim\limits_{x\to 1} f(x) \Rightarrow$ Continua en $x=1$

Para la derivabilidad:

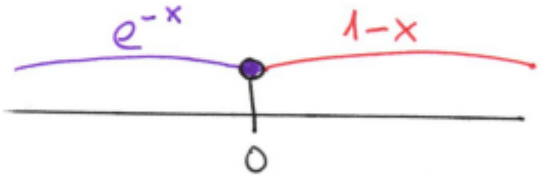
$$f'(x) = \begin{cases} 2x & si \quad x<1 \\ 1 & si \quad x>1 \end{cases}$$

Si $x=1$ Derivabilidad:

$$\left. \begin{array}{l} f'(1^-) = 2\cdot 1 = 2 \\ f'(1^+) = 1 \end{array} \right\} f'(1^-) \neq f'(1^+) \Rightarrow \nexists f'(1) \Rightarrow \text{No derivable en } x=1$$

$\Rightarrow f(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R} - \{1\}$

b) $$f(x) = \begin{cases} e^{-x} & si \quad x \leq 0 \\ 1-x & si \quad x>0 \end{cases}$$



Si $x<0$ $\rightarrow f(x) = e^{-x} \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty,0)$

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Si $x>0$ $\longrightarrow f(x) = 1-x \Rightarrow$ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(0,+\infty)$

Si $x=0$ Continuidad:

$$f(0) = e^{-0} = 1$$

$$\lim_{x\to 0} f(x) \longrightarrow \left\{ \begin{array}{l} \lim_{x\to 0^-} e^{-x} = 1 \\ \lim_{x\to 0^+} 1-x = 1 \end{array} \right\} \Rightarrow \lim_{x\to 0} f(x) = 1$$

Como $f(0) = \lim_{x\to 0} f(x) \Rightarrow$ Continua en $x=0$

Para la derivabilidad:

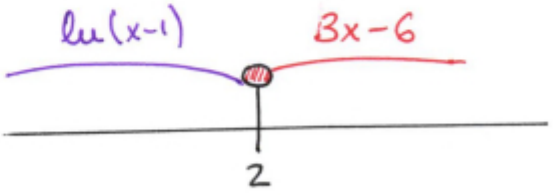
$$f'(x) = \begin{cases} -e^{-x} & \text{si } x \leq 0 \\ -1 & \text{si } x > 0 \end{cases}$$

Si $x=0$ Derivabilidad:

$$\left. \begin{array}{l} f'(0^-) = -e^{-0} = -1 \\ f'(0^+) = -1 \end{array} \right\} f'(0^-) = f'(0^+) \Rightarrow \text{Derivable en } x=0$$

$\Rightarrow f(x)$ es continua y derivable en $\mathbb{R}$

c) $$f(x) = \begin{cases} \ln(x-1) & \text{si } x < 2 \\ 3x-6 & \text{si } x \geq 2 \end{cases}$$



Si $x<2$ $\longrightarrow f(x) = \ln(x-1) \Rightarrow$ Continua y derivable en $(1,+\infty)$

↳ Continua y derivable en $(1,2)$

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Si $x > 2$ $\rightarrow f(x) = 3x - 6 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(2, +\infty)$

Si $x = 2$ Continuidad:

$f(2) = 3 \cdot 2 - 6 = 0$

$$\lim_{x \to 2} f(x) \longrightarrow \left\{ \begin{array}{l} \lim_{x \to 2^-} \ln(x-1) = 0 \\ \lim_{x \to 2^+} 3x - 6 = 0 \end{array} \right\} \Rightarrow \lim_{x \to 2} f(x) = 0$$

Como $f(2) = \lim_{x \to 2} f(x) \Rightarrow$ Continua en $x = 2$

Para la derivabilidad:

$$f'(x) = \begin{cases} \frac{1}{x-1} & \text{si } 1 < x < 2 \\ 3 & \text{si } x > 2 \end{cases}$$

Si $x = 2$ Derivabilidad:

$$\left. \begin{array}{l} f'(2^-) = \frac{1}{2-1} = 1 \\ f'(2^+) = 3 \end{array} \right\} f'(2^-) \neq f'(2^+) \Rightarrow \nexists f'(2) \Rightarrow$$ No derivable en $x = 2$

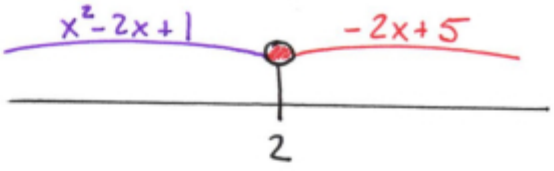
$\Rightarrow f(x)$ es continua en $(1, +\infty)$ y derivable en $(1,2) \cup (2, +\infty)$

$\nexists f(x)$ si $x \leq 1$

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d) $f(x) = \begin{cases} x^2-2x+1 & \text{si } x<2 \\ -2x+5 & \text{si } x \geq 2 \end{cases}$



Si $x<2$ → $f(x) = x^2-2x+1$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable $(-\infty, 2)$

Si $x>2$ → $f(x) = -2x+5$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(2, +\infty)$

Si $x=2$ Continuidad:

$f(2) = -2\cdot 2+5 = 1$

$$\lim_{x\to 2} f(x) \longrightarrow \left\{ \begin{array}{l} \lim_{x\to 2^-} (x^2-2x+1) = 1 \\ \lim_{x\to 2^+} (-2x+5) = 1 \end{array} \right\} \Rightarrow \lim_{x\to 2} f(x) = 1$$

Como $f(2) = \lim_{x\to 2} f(x)$ ⇒ Continua en $x=2$

Para la derivabilidad:

$$f'(x) = \begin{cases} 2x-2 & \text{si } x<2 \\ -2 & \text{si } x>2 \end{cases}$$

Si $x=2$ Derivabilidad:

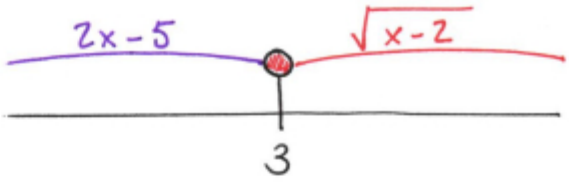
$$\left. \begin{array}{l} f'(2^-) = 2\cdot 2-2 = 2 \\ f'(2^+) = -2 \end{array} \right\} f'(2^-) \neq f'(2^+) \Rightarrow \nexists f'(2) \Rightarrow$$ No derivable en $x=2$

⇒ $f(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R}-\{2\}$

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e) $f(x) = \begin{cases} 2x-5 & \text{si } x < 3 \\ \sqrt{x-2} & \text{si } x \geq 3 \end{cases}$



Si $x < 3$ → $f(x) = 2x-5$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable $(-\infty, 3)$

Si $x > 3$ → $f(x) = \sqrt{x-2}$ ⇒ Continua en $[2, +\infty)$ y derivable en $(2, +\infty)$

↳ Continua y derivable en $(3, +\infty)$

Si $x = 3$ Continuidad:

$$f(3) = \sqrt{3-2} = 1$$

$$\lim_{x\to 3} f(x) \longrightarrow \left\{ \begin{array}{l} \lim_{x\to 3^-} (2x-5) = 1 \\ \lim_{x\to 3^+} \sqrt{x-2} = 1 \end{array} \right\} \Rightarrow \lim_{x\to 3} f(x) = 1$$

Como $f(3) = \lim_{x\to 3} f(x) \Rightarrow$ Continua en $x = 3$

Para la derivabilidad:

$$f'(x) = \begin{cases} 2 & \text{si } x < 3 \\ \dfrac{1}{2\sqrt{x-2}} & \text{si } x > 3 \end{cases}$$

Si $x = 3$ Derivabilidad:

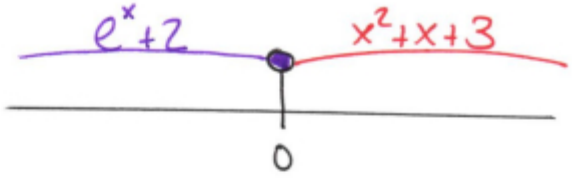
$$\left. \begin{array}{l} f'(3^-) = 2 \\ f'(3^+) = \dfrac{1}{2\cdot\sqrt{1}} = \dfrac{1}{2} \end{array} \right\} f'(3^-) \neq f'(3^+) \Rightarrow \nexists f'(3) \Rightarrow \text{No derivable en } x = 3$$

⇒ $f(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R} - \{3\}$

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f) $f(x) = \begin{cases} e^x + 2 & \text{si } x \leq 0 \\ x^2+x+3 & \text{si } x > 0 \end{cases}$



Si $x<0$ → $f(x) = e^x + 2$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable $(-\infty, 0)$

Si $x>0$ → $f(x) = x^2+x+3$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(0, +\infty)$

Si $x=0$ Continuidad:

$f(0) = e^0 + 2 = 3$

$\lim_{x\to 0} f(x) \longrightarrow \begin{cases} \lim_{x\to 0^-} e^x+2 = 3 \\ \lim_{x\to 0^+} (x^2+x+3) = 3 \end{cases} \Rightarrow \lim_{x\to 0} f(x) = 3$

Como $f(0) = \lim_{x\to 0} f(x)$ ⇒ Continua en $x=0$

Para la derivabilidad:

$$f'(x) = \begin{cases} e^x & \text{si } x \leq 0 \\ 2x+1 & \text{si } x > 0 \end{cases}$$

Si $x=0$ Derivabilidad:

$f'(0^-) = e^0 = 1$

$f'(0^+) = 2\cdot 0 + 1 = 1$

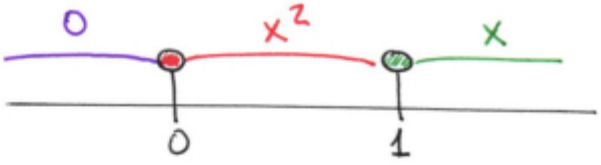
$\Rightarrow f'(0^-) = f'(0^+) \Rightarrow$ Derivable en $x=0$

⇒ $f(x)$ es continua y derivable en $\mathbb{R}$

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g) $f(x) = \begin{cases} 0 & \text{si } x<0 \\ x^2 & \text{si } 0 \le x < 1 \\ x & \text{si } x \ge 1 \end{cases}$



Si $x<0$ → $f(x)=0$ ⇒ Continua y derivable en $\mathbb{R}$
↳ Continua y derivable $(-\infty, 0)$

Si $0<x<1$ → $f(x)=x^2$ ⇒ Continua y derivable en $\mathbb{R}$
↳ Continua y derivable en $(0,1)$

Si $x>1$ → $f(x)=x$ ⇒ Continua y derivable en $\mathbb{R}$
↳ Continua y derivable en $(1,+\infty)$

Si $x=0$ Continuidad:

$f(0) = 0^2 = 0$

$\lim_{x\to 0} f(x) \longrightarrow \begin{cases} \lim_{x\to 0^-} 0 = 0 \\ \lim_{x\to 0^+} x^2 = 0 \end{cases} \Rightarrow \lim_{x\to 0} f(x) = 0$

Como $f(0) = \lim_{x\to 0} f(x)$ ⇒ Continua en $x=0$

Si $x=1$ Continuidad:

$f(1) = 1$

$\lim_{x\to 1} f(x) \longrightarrow \begin{cases} \lim_{x\to 1^-} x^2 = 1 \\ \lim_{x\to 1^+} x = 1 \end{cases} \Rightarrow \lim_{x\to 1} f(x) = 1$

Como $f(1) = \lim_{x\to 1} f(x)$ ⇒ Continua en $x=1$

Para la derivabilidad:

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$$f'(x) = \begin{cases} 0 & \text{si } x < 0 \\ 2x & \text{si } 0 \leq x < 1 \\ 1 & \text{si } x > 1 \end{cases}$$

Si $x = 0$ Derivabilidad:

$$\left.\begin{array}{l} f'(0^-) = 0 \\ f'(0^+) = 2 \cdot 0 = 0 \end{array}\right\} f'(0^-) = f'(0^+) \Rightarrow \text{Derivable en } x = 0$$

Si $x = 1$ Derivabilidad:

$$\left.\begin{array}{l} f'(1^-) = 2 \cdot 1 = 2 \\ f'(1^+) = 1 \end{array}\right\} f'(1^-) \neq f'(1^+) \Rightarrow \nexists f'(1) \Rightarrow \text{No derivable en } x = 1$$

$\Rightarrow$ $f(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R} - \{1\}$

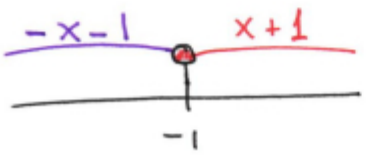
(22) Expresa las siguientes funciones como funciones definidas a trozos para estudiar su derivabilidad:

a) $f(x) = |x+1|$ b) $f(x) = |x^2 - 2x|$ c) $f(x) = 2x \cdot |x-4|$

d) $f(x) = |x+2| + |x-3|$ e) $f(x) = x^2 - 4 \cdot |x| + 3$ f) $f(x) = \dfrac{|x^2-1|}{1+|x|}$

a) $f(x) = |x+1|$

$x + 1 = 0 \Rightarrow x = -1$



$$f(x) = \begin{cases} -x-1 & \text{si } x < -1 \\ x+1 & \text{si } x \geq -1 \end{cases}$$

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Si $x<-1$ $\longrightarrow f(x) = -x-1 \Rightarrow$ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(-\infty, -1)$

Si $x>-1$ $\longrightarrow f(x) = x+1 \Rightarrow$ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(-1, +\infty)$

Si $x=-1$ Continuidad:

$$f(-1) = -1+1 = 0$$

$$\lim_{x\to -1} f(x) \longrightarrow \left\{ \begin{array}{l} \lim_{x\to -1^-} -x-1 = 0 \\ \lim_{x\to -1^+} x+1 = 0 \end{array} \right\} \Rightarrow \lim_{x\to -1} f(x) = 0$$

Como $f(-1) = \lim_{x\to -1} f(x) \Rightarrow$ Continua en $x=-1$

Para la derivabilidad:

$$f'(x) = \left\{ \begin{array}{ll} -1 & \text{si } x<-1 \\ 1 & \text{si } x>-1 \end{array} \right.$$

Si $x=-1$ Derivabilidad:

$$\left. \begin{array}{l} f'(-1^-) = -1 \\ f'(-1^+) = 1 \end{array} \right\} f'(-1^-) \neq f'(-1^+) \Rightarrow \nexists f'(-1) \Rightarrow \text{No derivable en } x=-1$$

$\Rightarrow f(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R} - \{-1\}$

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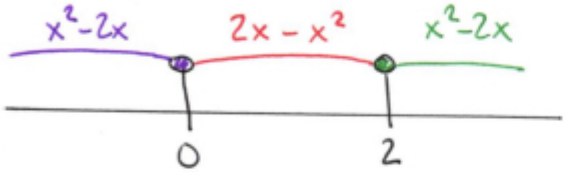
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b) $f(x) = |x^2-2x|$

$x^2-2x=0$

$x(x-2)=0 \begin{cases} x=0 \\ x=2 \end{cases}$

$$f(x) = \begin{cases} x^2-2x & \text{si} \quad x \leq 0 \\ 2x-x^2 & \text{si} \quad 0<x<2 \\ x^2-2x & \text{si} \quad x \geq 2 \end{cases}$$



Si $x<0$ $\rightarrow f(x) = x^2-2x \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, 0)$

Si $0<x<2$ $\rightarrow f(x) = 2x-x^2 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(0,2)$

Si $x>2$ $\rightarrow f(x) = x^2-2x \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(2,+\infty)$

Si $x=0$ Continuidad:

$f(0) = 0^2-2\cdot 0 = 0$

$\lim_{x\to 0} f(x) \rightarrow \begin{cases} \lim_{x\to 0^-} (x^2-2x) = 0 \\ \lim_{x\to 0^+} (2x-x^2) = 0 \end{cases} \Rightarrow \lim_{x\to 0} f(x) = 0$

Como $f(0) = \lim_{x\to 0} f(x) \Rightarrow$ Continua en $x=0$

Si $x=2$ Continuidad:

$f(2) = 2^2-2\cdot 2 = 0$

$\lim_{x\to 2} f(x) \rightarrow \begin{cases} \lim_{x\to 2^-} (2x-x^2) = 0 \\ \lim_{x\to 2^+} (x^2-2x) = 0 \end{cases} \Rightarrow \lim_{x\to 2} f(x) = 0$

Como $f(2) = \lim_{x\to 2} f(x) \Rightarrow$ Continua en $x=2$

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Para la derivabilidad:

$$f'(x) = \begin{cases} 2x-2 & \text{si } x<0 \\ 2-2x & \text{si } 0<x<2 \\ 2x-2 & \text{si } x>2 \end{cases}$$

Si $x=0$ Derivabilidad:

$$\left.\begin{array}{l} f'(0^-) = 2\cdot 0-2 = -2 \\ f'(0^+) = 2-2\cdot 0 = 2 \end{array}\right\} f'(0^-) \neq f'(0^+) \Rightarrow \nexists f'(0) \Rightarrow$$ No derivable en $x=0$

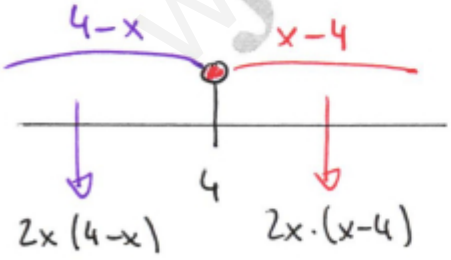
Si $x=2$ Derivabilidad:

$$\left.\begin{array}{l} f'(2^-) = 2-2\cdot 2 = -2 \\ f'(2^+) = 2\cdot 2-2 = 2 \end{array}\right\} f'(2^-) \neq f'(2^+) \Rightarrow \nexists f'(2) \Rightarrow$$ No derivable en $x=2$

$\Rightarrow f(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R}-\{0,2\}$

c) $f(x) = 2x\cdot|x-4|$

$x-4=0 \Rightarrow x=4$



$$\Rightarrow f(x) = \begin{cases} 8x-2x^2 & \text{si } x<4 \\ 2x^2-8x & \text{si } x\geq 4 \end{cases}$$

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Si $x<4$ $\rightarrow f(x) = 8x - 2x^2 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, 4)$

Si $x>4$ $\rightarrow f(x) = 2x^2 - 8x \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(4, +\infty)$

Si $x=4$ Continuidad:

$$f(4) = 2\cdot 4^2 - 8\cdot 4 = 0$$

$$\lim_{x\to 4} f(x) \longrightarrow \begin{cases} \lim\limits_{x\to 4^-} (8x-2x^2) = 0 \\ \lim\limits_{x\to 4^+} (2x^2-8x) = 0 \end{cases} \Rightarrow \lim_{x\to 4} f(x) = 0$$

Como $f(4) = \lim\limits_{x\to 4} f(x) \Rightarrow$ Continua en $x=4$

Para la derivabilidad:

$$f'(x) = \begin{cases} 8-4x & \text{si } x<4 \\ 4x-8 & \text{si } x>4 \end{cases}$$

Si $x=4$ Derivabilidad:

$$\left.\begin{aligned} f'(4^-) &= 8-4\cdot 4 = -8 \\ f'(4^+) &= 4\cdot 4 - 8 = 8 \end{aligned}\right\} f'(4^-) \neq f'(4^+) \Rightarrow \nexists f'(4) \Rightarrow$$ No derivable en $x=4$

$\Rightarrow f(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R} - \{4\}$

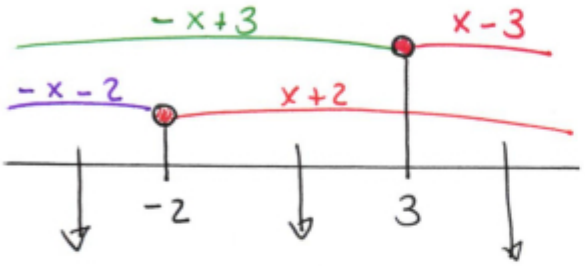
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d) $f(x) = |x+2| + |x-3|$

$x+2=0 \longrightarrow x=-2$

$x-3=0 \longrightarrow x=3$



$$\Rightarrow f(x) = \begin{cases} -2x+1 & \text{si } x < -2 \\ 5 & \text{si } -2 \le x < 3 \\ 2x-1 & \text{si } x \ge 3 \end{cases}$$

$-x-2$	$x+2$	$x-3$
$-x+3$	$-x+3$	$x+2$
$-2x+1$	5	$2x-1$

Si $x<-2 \Rightarrow f(x) = -2x+1 \Rightarrow$ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(-\infty, -2)$

Si $-2<x<3 \rightarrow f(x) = 5 \Rightarrow$ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(-2, 3)$

Si $x>3 \rightarrow f(x) = 2x-1 \Rightarrow$ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(3, +\infty)$

Si $x=-2$ Continuidad:

$$f(-2) = 5$$

$$\lim_{x \to -2} f(x) \longrightarrow \left\{ \begin{array}{l} \lim_{x \to -2^-} (-2x+1) = 5 \\ \lim_{x \to -2^+} 5 = 5 \end{array} \right\} \Rightarrow \lim_{x \to -2} f(x) = 5$$

Como $f(-2) = \lim_{x \to -2} f(x) \Rightarrow$ Continua en $x=-2$

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Si $x=3$ Continuidad:

$$f(3) = 2\cdot 3 - 1 = 5$$

$$\lim_{x\to 3} f(x) \longrightarrow \left\{ \begin{array}{l} \lim\limits_{x\to 3^-} 5 = 5 \\ \lim\limits_{x\to 3^+} (2x-1) = 5 \end{array} \right\} \Rightarrow \lim_{x\to 3} f(x) = 5$$

Como $f(3) = \lim\limits_{x\to 3} f(x) \Rightarrow$ Continua en $x=3$

Para la derivabilidad:

$$f'(x) = \left\{ \begin{array}{ll} -2 & \text{si } x<-2 \\ 0 & \text{si } -2<x<3 \\ 2 & \text{si } x>3 \end{array} \right.$$

Si $x=-2$ Derivabilidad:

$$\left. \begin{array}{l} f'(-2^-) = -2 \\ f'(-2^+) = 0 \end{array} \right\} f'(-2^-) \neq f'(-2^+) \Rightarrow \nexists f'(-2) \Rightarrow$$ No derivable en $x=-2$

Si $x=3$ Derivabilidad:

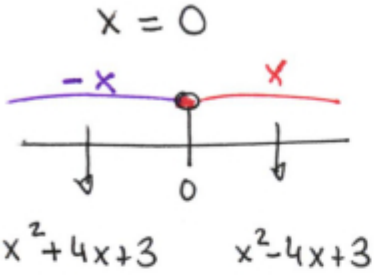
$$\left. \begin{array}{l} f'(3^-) = 0 \\ f'(3^+) = 2 \end{array} \right\} f'(3^-) \neq f'(3^+) \Rightarrow \nexists f'(3) \Rightarrow$$ No derivable en $x=3$

$\Rightarrow f(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R} - \{-2, 3\}$

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e) $f(x) = x^2 - 4\cdot|x| + 3$



$$\Rightarrow f(x) = \begin{cases} x^2+4x+3 & \text{si } x<0 \\ x^2-4x+3 & \text{si } x \geq 0 \end{cases}$$

Si $x<0$ $\rightarrow f(x) = x^2+4x+3 \Rightarrow$ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(-\infty, 0)$

Si $x>0$ $\rightarrow f(x) = x^2-4x+3 \Rightarrow$ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(0, +\infty)$

Si $x=0$ Continuidad:

$$f(0) = 0^2 - 4\cdot 0 + 3 = 3$$

$$\lim_{x\to 0} f(x) \longrightarrow \begin{cases} \lim_{x\to 0^-} (x^2+4x+3) = 3 \\ \lim_{x\to 0^+} (x^2-4x+3) = 3 \end{cases} \Rightarrow \lim_{x\to 0} f(x) = 3$$

Como $f(0) = \lim_{x\to 0} f(x) \Rightarrow$ Continua en $x=0$

Para la derivabilidad:

$$f'(x) = \begin{cases} 2x+4 & \text{si } x<0 \\ 2x-4 & \text{si } x>0 \end{cases}$$

Si $x=0$ Derivabilidad:

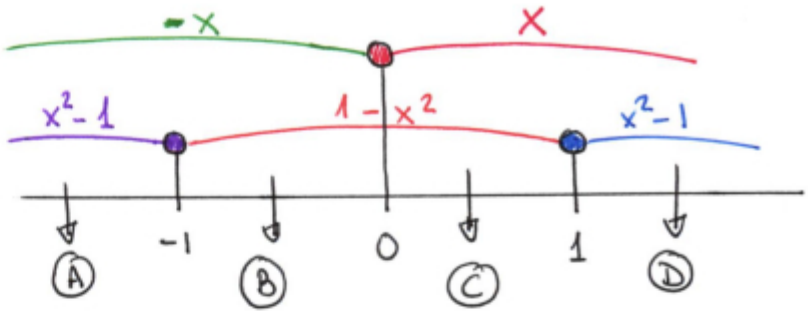
$$\left.\begin{array}{l} f'(0^-) = 2\cdot 0 + 4 = 4 \\ f'(0^+) = 2\cdot 0 - 4 = 4 \end{array}\right\} f'(0^-) \neq f'(0^+) \Rightarrow \nexists f'(0) \Rightarrow$$ No derivable en $x=0$

$\Rightarrow f(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R} - \{0\}$

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f) $f(x) = \dfrac{|x^2-1|}{1+|x|}$

$x = 0 \;;\; x^2 - 1 = 0 \begin{cases} \nearrow x = -1 \\ \searrow x = +1 \end{cases}$



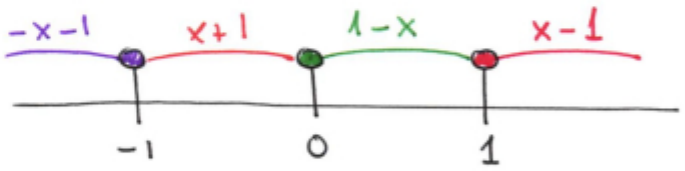
(A) $\dfrac{x^2-1}{1-x} = \dfrac{(x+1)(x-1)}{(1-x)} = (-1)\cdot(x+1) = -x-1$ (-1)

(B) $\dfrac{1-x^2}{1-x} = \dfrac{(1-x)(1+x)}{(1-x)} = x+1$

(D) $\dfrac{x^2-1}{1+x} = \dfrac{(x+1)(x-1)}{(1+x)} = x-1$

(C) $\dfrac{1-x^2}{1+x} = \dfrac{(1-x)(1+x)}{(1+x)} = 1-x$

$$f(x) = \begin{cases} -x-1 & \text{si } x \le -1 \\ x+1 & \text{si } -1 < x < 0 \\ 1-x & \text{si } 0 \le x < 1 \\ x-1 & \text{si } x \ge 1 \end{cases}$$



Si $x < -1$ → $f(x) = -x-1$ ⇒ Continua y derivable en $\mathbb{R}$
↳ Continua y derivable en $(-\infty, -1)$

Si $-1 < x < 0$ → $f(x) = x+1$ ⇒ Continua y derivable en $\mathbb{R}$
↳ Continua y derivable en $(-1, 0)$

Si $0 < x < 1$ → $f(x) = 1-x$ ⇒ Continua y derivable en $\mathbb{R}$
↳ Continua y derivable en $(0, 1)$

Si $x > 1$ → $f(x) = x-1$ ⇒ Continua y derivable en $\mathbb{R}$
↳ Continua y derivable en $(1, +\infty)$

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Si $x=-1$ Continuidad:

$$f(-1) = -(-1)-1 = 0$$

$$\lim_{x\to -1} f(x) \longrightarrow \left\{ \begin{array}{l} \lim_{x\to -1^-} (-x-1) = 0 \\ \lim_{x\to -1^+} (x+1) = 0 \end{array} \right\} \Rightarrow \lim_{x\to -1} f(x) = 0$$

Como $f(-1) = \lim_{x\to -1} f(x) \Rightarrow$ Continua en $x=-1$

Si $x=0$ Continuidad:

$$f(0) = 1-0 = 1$$

$$\lim_{x\to 0} f(x) \longrightarrow \left\{ \begin{array}{l} \lim_{x\to 0^-} (x+1) = 1 \\ \lim_{x\to 0^+} (1-x) = 1 \end{array} \right\} \Rightarrow \lim_{x\to 0} f(x) = 1$$

Como $f(0) = \lim_{x\to 0} f(x) \Rightarrow$ Continua en $x=0$

Si $x=1$ Continuidad:

$$f(1) = 1-1 = 0$$

$$\lim_{x\to 1} f(x) \longrightarrow \left\{ \begin{array}{l} \lim_{x\to 1^-} (1-x) = 0 \\ \lim_{x\to 1^+} (x-1) = 0 \end{array} \right\} \Rightarrow \lim_{x\to 1} f(x) = 0$$

Como $f(1) = \lim_{x\to 1} f(x) \Rightarrow$ Continua en $x=1$

Para la derivabilidad:

$$f'(x) = \left\{ \begin{array}{ll} -1 & \text{si } x<-1 \\ 1 & \text{si } -1<x<0 \\ -1 & \text{si } 0<x<1 \\ 1 & \text{si } x>1 \end{array} \right.$$

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Si $x = -1$) Derivabilidad:

$$\left.\begin{array}{l} f'(-1^-) = -1 \\ f'(-1^+) = 1 \end{array}\right\} f'(-1^-) \neq f'(-1^+) \Rightarrow \nexists f'(-1) \Rightarrow$$

No derivable en $x = -1$

Si $x = 0$) Derivabilidad:

$$\left.\begin{array}{l} f'(0^-) = 1 \\ f'(0^+) = -1 \end{array}\right\} f'(0^-) \neq f'(0^+) \Rightarrow \nexists f'(0) \Rightarrow$$

No derivable en $x = 0$

Si $x = 1$) Derivabilidad:

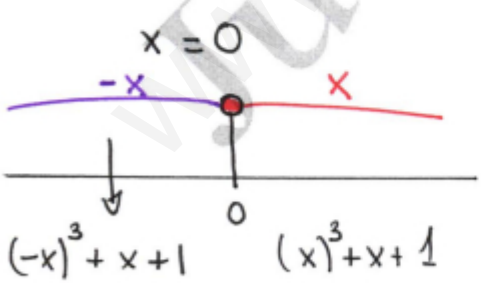
$$\left.\begin{array}{l} f'(1^-) = -1 \\ f'(1^+) = 1 \end{array}\right\} f'(1^-) \neq f'(1^+) \Rightarrow \nexists f'(1) \Rightarrow$$

No derivable en $x = 1$

$\Rightarrow f(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R} - \{-1, 0, 1\}$

(23) Estudia la continuidad de las funciones $f(x) = |x|^3 + x + 1$ y $g(x) = |x|^3 + |x| + 1$. Estudia también la derivabilidad

$$f(x) = |x|^3 + x + 1$$



$$\Rightarrow f(x) = \begin{cases} -x^3 + x + 1 & \text{si } x < 0 \\ x^3 + x + 1 & \text{si } x \geq 0 \end{cases}$$

Si $x < 0$ $\rightarrow f(x) = -x^3 + x + 1 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, 0)$

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Si $x>0 \longrightarrow f(x) = x^3+x+1 \Rightarrow$ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(0,+\infty)$

Si $x=0$: Continuidad:

$$f(0) = 0^3+0+1 = 1$$

$$\lim_{x\to 0} f(x) \longrightarrow \left\{ \begin{array}{l} \lim_{x\to 0^-} (-x^3+x+1) = 1 \\ \lim_{x\to 0^+} (x^3+x+1) = 1 \end{array} \right\} \Rightarrow \lim_{x\to 0} f(x) = 1$$

Como $f(0) = \lim_{x\to 0} f(x) \Rightarrow$ Continua en $x=0$

Para la derivabilidad:

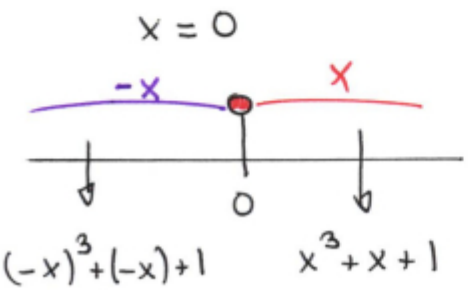
$$f'(x) = \begin{cases} -3x^2+1 & \text{si } x<0 \\ 3x^2+1 & \text{si } x\geq 0 \end{cases}$$

Si $x=0$: Derivabilidad:

$$\left. \begin{array}{l} f'(0^-) = -3\cdot 0+1 = 1 \\ f'(0^+) = 3\cdot 0+1 = 1 \end{array} \right\} f'(0^-) = f'(0^+) \Rightarrow \text{Derivable en } x=0$$

$\Rightarrow f(x)$ es continua y derivable en $\mathbb{R}$

$$g(x) = |x|^3+|x|+1$$



$$\Rightarrow g(x) = \begin{cases} -x^3-x+1 & \text{si } x<0 \\ x^3+x+1 & \text{si } x\geq 0 \end{cases}$$

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Si $x<0$ $\rightarrow g(x) = -x^3 - x + 1 \Rightarrow$ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(-\infty, 0)$

Si $x>0$ $\rightarrow g(x) = x^3 + x + 1 \Rightarrow$ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(0, +\infty)$

Si $x=0$ Continuidad:

$g(0) = 0^3 + 0 + 1 = 1$

$$\lim_{x\to 0} g(x) \longrightarrow \left\{ \begin{array}{l} \lim_{x\to 0^-} (-x^3 - x + 1) = 1 \\ \lim_{x\to 0^+} (x^3 + x + 1) = 1 \end{array} \right\} \lim_{x\to 0} g(x) = 1$$

Como $g(0) = \lim_{x\to 0} g(x) \Rightarrow$ Continua en $x=0$

Para la derivabilidad:

$$g'(x) = \begin{cases} -3x^2 - 1 & \text{si } x<0 \\ 3x^2 + 1 & \text{si } x>0 \end{cases}$$

Si $x=0$ Derivabilidad:

$$\left. \begin{array}{l} g'(0^-) = -1 \\ g'(0^+) = +1 \end{array} \right\} g'(0^-) \neq g'(0^+) \Rightarrow \nexists\, g'(0) \Rightarrow$$

No derivable en $x=0$

$\Rightarrow g(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R} - \{0\}$

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(24) Estudia la continuidad y la derivabilidad de las siguientes funciones:

a) $f(x) = \begin{cases} 1-x^2 & \text{si } x \le 0 \\ \cos x & \text{si } x > 0 \end{cases}$

b) $f(x) = \begin{cases} 2\operatorname{sen} x + 1 & \text{si } x < 0 \\ -x^3 + 2x + 1 & \text{si } x \ge 0 \end{cases}$

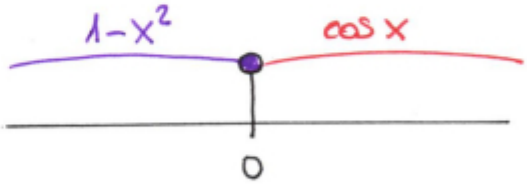
c) $f(x) = \begin{cases} \dfrac{x^2+2x-3}{1-x^2} & \text{si } -2 \le x < 1 \\ 1 - 3\sqrt{2x-1} & \text{si } 1 \le x \le 5 \end{cases}$

d) $f(x) = \begin{cases} x(x-2) & \text{si } x \le 2 \\ \sqrt[3]{x-2} & \text{si } x > 2 \end{cases}$

e) $f(x) = \begin{cases} 2x^2+5x+6 & \text{si } x \le -1 \\ 2 - \dfrac{1}{x} & \text{si } -1 < x < 2 \\ \dfrac{5}{4} + 2^{-x} & \text{si } x \ge 2 \end{cases}$

f) $f(x) = \begin{cases} -1 & \text{si } x < -4 \\ x+2 & \text{si } -4 \le x < 2 \\ \dfrac{8}{x} & \text{si } x \ge 2 \end{cases}$

a) $f(x) = \begin{cases} 1-x^2 & \text{si } x \le 0 \\ \cos x & \text{si } x > 0 \end{cases}$



Si $x < 0$ → $f(x) = 1-x^2$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(-\infty, 0)$

Si $x > 0$ → $f(x) = \cos x$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(0, +\infty)$

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Si $x = 0$: Continuidad:

$$f(0) = 1 - 0^2 = 1$$

$$\lim_{x\to 0} f(x) \longrightarrow \left\{ \begin{array}{l} \lim_{x\to 0^-} (1-x^2) = 1 \\ \lim_{x\to 0^+} (\cos x) = 1 \end{array} \right\} \Rightarrow \lim_{x\to 0} f(x) = 1$$

Como $f(0) = \lim_{x\to 0} f(x) \Rightarrow$ Continua en $x = 0$

Para la derivabilidad:

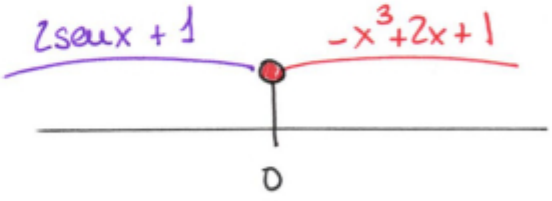
$$f'(x) = \begin{cases} -2x & \text{si } x < 0 \\ -\operatorname{sen} x & \text{si } x > 0 \end{cases}$$

Si $x = 0$: Derivabilidad:

$$\left. \begin{array}{l} f'(0^-) = -2\cdot 0 = 0 \\ f'(0^+) = -\operatorname{sen}(0) = 0 \end{array} \right\} f'(0^-) = f'(0^+) \Rightarrow \text{Derivable en } x = 0$$

$\Rightarrow f(x)$ es continua y derivable en $\mathbb{R}$.

b) $$f(x) = \begin{cases} 2\operatorname{sen} x + 1 & \text{si } x < 0 \\ -x^3 + 2x + 1 & \text{si } x \geq 0 \end{cases}$$



Si $x < 0 \longrightarrow f(x) = 2\operatorname{sen} x + 1 \Rightarrow$ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(-\infty, 0)$

Si $x > 0 \longrightarrow f(x) = -x^3 + 2x + 1 \Rightarrow$ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(0, +\infty)$

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Si $x=0$ Continuidad:

$$f(0) = -0^3+2\cdot 0+1 = 1$$

$$\lim_{x\to 0} f(x) \Rightarrow \left\{ \begin{array}{l} \lim\limits_{x\to 0^-} (2\operatorname{sen} x+1) = 1 \\ \lim\limits_{x\to 0^+} (-x^3+2x+1) = 1 \end{array} \right\} \Rightarrow \lim_{x\to 0} f(x) = 1$$

Como $f(0) = \lim\limits_{x\to 0} f(x) \Rightarrow$ Continua en $x=0$

Para la derivabilidad:

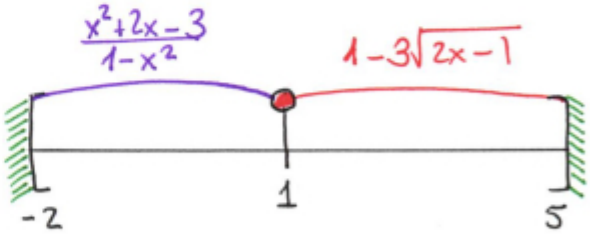
$$f'(x) = \begin{cases} 2\cos x & \text{si } x<0 \\ -3x^2+2 & \text{si } x\geq 0 \end{cases}$$

Si $x=0$ Derivabilidad:

$$\left. \begin{array}{l} f'(0^-) = 2\cos 0 = 2 \\ f'(0^+) = -3\cdot 0+2 = 2 \end{array} \right\} f'(0^-) = f'(0^+) \Rightarrow \text{Derivable en } x=0$$

$\Rightarrow f(x)$ es continua y derivable en $\mathbb{R}$

c) $$f(x) = \begin{cases} \dfrac{x^2+2x-3}{1-x^2} & \text{si } -2\leq x<1 \\ 1-3\sqrt{2x-1} & \text{si } 1\leq x\leq 5 \end{cases}$$



Si $-2\leq x<1 \rightarrow f(x) = \dfrac{x^2+2x-3}{1-x^2} \Rightarrow$ Continua y derivable en $\mathbb{R}-\{-1,1\}$

↳ Continua y derivable en $[-2,-1)\cup(-1,1)$

SI QUE AFECTA — NO AFECTA

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En $x=-1$ hay una discontinuidad:

$$\lim_{x\to -1}\frac{x^2+2x-3}{1-x^2}=\left[\frac{-4}{0}\right]\longrightarrow\begin{cases}\lim\limits_{x\to -1^-}\dfrac{x^2+2x-3}{1-x^2}=+\infty\\ \lim\limits_{x\to -1^+}\dfrac{x^2+2x-3}{1-x^2}=-\infty\end{cases}\Rightarrow\nexists\lim_{x\to -1}\frac{x^2+2x-3}{1-x^2}$$

En $x=-1$ hay una discontinuidad de salto infinito y por tanto

$f(x)$ no es derivable en $x=-1$

Si $1<x\leq 5$ $\to f(x)=1-3\sqrt{2x-1}\Rightarrow$ Continua en $[1/2,+\infty)$ y derivable en $(1/2,+\infty)$

$\hookrightarrow$ Continua y derivable en $(1,5]$

Si $x=1$ Continuidad:

$f(1)=1-3\sqrt{1}=-2$

$$\lim_{x\to 1}f(x)\longrightarrow\begin{cases}\lim\limits_{x\to 1^-}\dfrac{x^2+2x-3}{1-x^2}=\left[\dfrac{0}{0}\right]=\lim\limits_{x\to 1^-}\dfrac{(x-1)(x+3)}{(1-x)(1+x)}=\\ \quad=\lim\limits_{x\to 1^-}(-1)\cdot\left(\dfrac{x+3}{1+x}\right)=-2\\ \lim\limits_{x\to 1^+}(1-3\sqrt{2x-1})=-2\end{cases}\Rightarrow\lim_{x\to 1}f(x)=-2$$

Como $f(1)=\lim\limits_{x\to 1}f(x)\Rightarrow$ Continua en $x=1$

Para la derivabilidad:

$$f'(x)=\begin{cases}\dfrac{2}{(1+x)^2} & \text{si } -2\leq x<1 \text{ con } x\neq -1\\ \dfrac{-3}{\sqrt{2x-1}} & \text{si } 1<x\leq 5\end{cases}$$

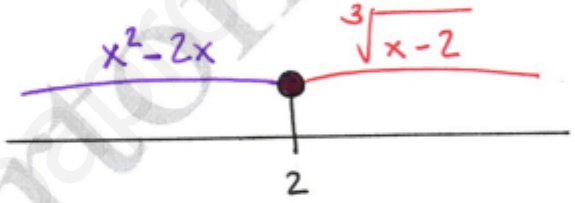
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Si $x = 1$ Derivabilidad:

$$\left.\begin{array}{l} f'(1^-) = \dfrac{2}{(1+1)^2} = \dfrac{1}{2} \\ f'(1^+) = \dfrac{-3}{\sqrt{1}} = -3 \end{array}\right\} f'(1^-) \neq f'(1^+) \Rightarrow \nexists f'(1) \Rightarrow$$ No derivable en $x = 1$

$$\Rightarrow \begin{cases} f(x) \text{ es continua en } [-2,-1) \cup (-1,5] \\ f(x) \text{ es derivable en } [-2,-1) \cup (-1,1) \cup (1,5] \end{cases}$$

d) $$f(x) = \begin{cases} x(x-2) & \text{si } x \leq 2 \\ \sqrt[3]{x-2} & \text{si } x > 2 \end{cases}$$



Si $x < 2$ $\rightarrow f(x) = x^2-2x \Rightarrow$ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(-\infty, 2)$

Si $x > 2$ $\rightarrow f(x) = \sqrt[3]{x-2} \Rightarrow$ Continua en $\mathbb{R}$ y derivable en $\mathbb{R} - \{2\}$ (NO AFECTA)

↳ Continua y derivable en $(2, +\infty)$

Si $x = 2$ Continuidad:

$$f(2) = 2^2 - 2\cdot 2 = 0$$

$$\lim_{x\to 2} f(x) \rightarrow \left\{\begin{array}{l} \lim\limits_{x\to 2^-} (x^2-2x) = 0 \\ \lim\limits_{x\to 2^+} (\sqrt[3]{x-2}) = 0 \end{array}\right\} \Rightarrow \lim_{x\to 2} f(x) = 0$$

Como $f(2) = \lim\limits_{x\to 2} f(x) \Rightarrow$ Continua en $x = 2$

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Para la derivabilidad:

$$f'(x) = \begin{cases} 2x-2 & \text{si } x<2 \\ \dfrac{1}{3\sqrt[3]{(x-2)^2}} & \text{si } x>2 \end{cases}$$

Si $x=2$ Derivabilidad:

$$f'(2^-) = 2\cdot 2 - 2 = 2$$

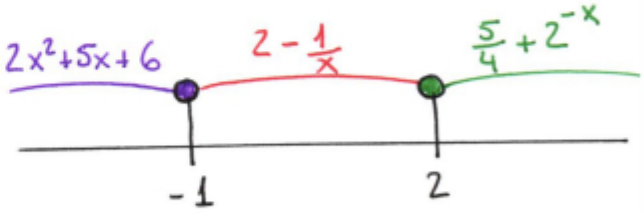
$$f'(2^+) = \lim_{h\to 0^+} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0^+} \frac{\sqrt[3]{2+h-2}-0}{h} =$$

$$= \lim_{h\to 0^+} \frac{\sqrt[3]{h}}{h}\cdot\frac{\sqrt[3]{h^2}}{\sqrt[3]{h^2}} = \lim_{h\to 0^+} \frac{h}{h\sqrt[3]{h^2}} = \lim_{h\to 0^+} \frac{1}{\sqrt[3]{h^2}} = +\infty$$

$\Rightarrow f'(2^-) \neq f'(2^+) \Rightarrow \nexists f'(2) \Rightarrow$ No derivable en $x=2$

$\Rightarrow f(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R}-\{2\}$

e) $$f(x) = \begin{cases} 2x^2+5x+6 & \text{si } x\leq -1 \\ 2-\dfrac{1}{x} & \text{si } -1<x<2 \\ \dfrac{5}{4}+2^{-x} & \text{si } x\geq 2 \end{cases}$$



Si $x<-1$ $\longrightarrow f(x) = 2x^2+5x+6 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable $(-\infty,-1)$

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Si $-1<x<2$ $\rightarrow f(x)=2-\frac{1}{x}$ $\Rightarrow$ Continua y derivable en $\mathbb{R}-\{0\}$ (SI QUE AFECTA)

↳ Continua y derivable en $(-1,0)\cup(0,2)$

En $x=0$ hay una discontinuidad:

$$\lim_{x\to 0} 2-\frac{1}{x}=\lim_{x\to 0}\frac{2x-1}{x}=\left[\frac{-1}{0}\right]\rightarrow\begin{cases}\lim\limits_{x\to 0^-}\frac{2x-1}{x}=+\infty\\ \lim\limits_{x\to 0^+}\frac{2x-1}{x}=-\infty\end{cases}$$

En $x=0$ hay una discontinuidad de salto infinito y por lo tanto, $f(x)$ no será derivable en $x=0$

Si $x>2$ $\rightarrow f(x)=\frac{5}{4}+2^{-x}$ $\Rightarrow$ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(2,+\infty)$

Si $x=-1$: Continuidad:

$$f(-1)=2\cdot(-1)^2+5\cdot(-1)+6=3$$

$$\lim_{x\to -1} f(x)\rightarrow\begin{cases}\lim\limits_{x\to -1^-}(2x^2+5x+6)=3\\ \lim\limits_{x\to -1^+}\left(2-\frac{1}{x}\right)=3\end{cases}\Rightarrow\lim_{x\to -1} f(x)=3$$

Como $f(-1)=\lim\limits_{x\to -1} f(x)$ $\Rightarrow$ Continua en $x=-1$

Si $x=2$ Continuidad:

$$f(2)=\frac{5}{4}+2^{-2}=\frac{3}{2}$$

$$\lim_{x\to 2} f(x)\rightarrow\begin{cases}\lim\limits_{x\to 2^-}\left(2-\frac{1}{x}\right)=\frac{3}{2}\\ \lim\limits_{x\to 2^+}\left(\frac{5}{4}+2^{-x}\right)=\frac{3}{2}\end{cases}\Rightarrow\lim_{x\to 2} f(x)=\frac{3}{2}$$

Como $f(2)=\lim\limits_{x\to 2} f(x)$ $\Rightarrow$ Continua en $x=2$

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Para la derivabilidad:

$$f'(x) = \begin{cases} 4x+5 & \text{si } x \leq -1 \\ \frac{1}{x^2} & \text{si } -1 < x < 2 \text{ con } x \neq 0 \\ -2^{-x} \cdot \ln 2 & \text{si } x > 2 \end{cases}$$

Si $x = -1$, Derivabilidad:

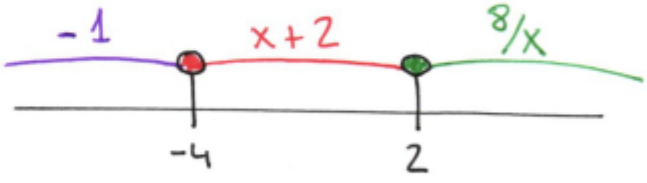
$$\left.\begin{array}{l} f'(-1^-) = 4\cdot(-1)+5 = 1 \\ f'(-1^+) = \frac{1}{(-1)^2} = 1 \end{array}\right\} f'(-1^-) = f'(-1^+) \Rightarrow \text{Derivable en } x = -1$$

Si $x = 2$, Derivabilidad:

$$\left.\begin{array}{l} f'(2^-) = \frac{1}{2^2} = \frac{1}{4} \\ f'(2^+) = -\frac{1}{4}\cdot \ln 2 \end{array}\right\} f'(2^-) \neq f'(2^+) \Rightarrow \nexists f'(2) \Rightarrow \text{No derivable en } x = 2$$

$\Rightarrow f(x)$ es continua en $\mathbb{R} - \{0\}$ y derivable en $\mathbb{R} - \{0, 2\}$

f) $$f(x) = \begin{cases} -1 & \text{si } x < -4 \\ x+2 & \text{si } -4 \leq x < 2 \\ \frac{8}{x} & \text{si } x \geq 2 \end{cases}$$



Si $x < -4$ → $f(x) = -1$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(-\infty, -4)$

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Si $-4<x<2$ $\longrightarrow f(x) = x+2 \Rightarrow$ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(-4,2)$

Si $x>2$ $\longrightarrow f(x) = \frac{8}{x} \Rightarrow$ Continua y derivable en $\mathbb{R}-\{0\}$

↳ NO AFECTA

↳ Continua y derivable en $(2,+\infty)$

Si $x=-4$ Continuidad:

$f(-4) = -4+2 = -2$

$$\lim_{x\to -4} f(x) \longrightarrow \left\{ \begin{array}{l} \lim_{x\to -4^-} -1 = -1 \\ \lim_{x\to -4^+} (x+2) = -2 \end{array} \right\} \Rightarrow \nexists \lim_{x\to -4} f(x)$$

$f(x)$ presenta en $x=-4$ una discontinuidad de salto finito y por tanto no será derivable en $x=-4$

Si $x=2$ Continuidad:

$f(2) = \frac{8}{2} = 4$

$$\lim_{x\to 2} f(x) \longrightarrow \left\{ \begin{array}{l} \lim_{x\to 2^-} (x+2) = 4 \\ \lim_{x\to 2^+} \frac{8}{x} = 4 \end{array} \right\} \Rightarrow \lim_{x\to 2} f(x) = 4$$

Como $f(2) = \lim_{x\to 2} f(x) \Rightarrow$ Continua en $x=2$

Para la derivabilidad:

$$f'(x) = \left\{ \begin{array}{ll} 0 & \text{si } x<-4 \\ 1 & \text{si } -4<x<2 \\ -\frac{8}{x^2} & \text{si } x>2 \end{array} \right.$$

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Si $x=2$ Derivabilidad:

$$\left.\begin{array}{l} f'(2^-) = 1 \\ f'(2^+) = \frac{-8}{2^2} = -2 \end{array}\right\} f'(2^-) \neq f'(2^+) \Rightarrow \nexists f'(2) \Rightarrow$$ No derivable en $x=2$

$\Rightarrow f(x)$ es continua en $\mathbb{R}-\{-4\}$ y derivable en $\mathbb{R}-\{-4, 2\}$

(25) Calcula el valor de los parámetros para que las siguientes funciones sean derivables en $\mathbb{R}$:

a) $f(x) = \begin{cases} ax^2+3x & \text{si } x \leq 2 \\ x^2-5x-4 & \text{si } x > 2 \end{cases}$

b) $f(x) = \begin{cases} x^3-x & \text{si } x \leq 0 \\ ax+b & \text{si } x > 0 \end{cases}$

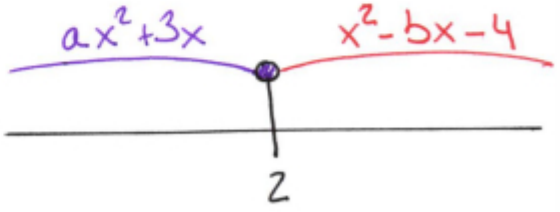
c) $f(x) = \begin{cases} x^2-5x+a & \text{si } x \leq 1 \\ -x^2+bx & \text{si } x > 1 \end{cases}$

d) $f(x) = \begin{cases} ax^2+bx-3 & \text{si } x < 1 \\ 2bx-4 & \text{si } x \geq 1 \end{cases}$

e) $f(x) = \begin{cases} (x-1)^3 & \text{si } x \leq 0 \\ ax+b & \text{si } x > 0 \end{cases}$

f) $f(x) = \begin{cases} ax^2+3x & \text{si } x < -2 \\ x^2-bx-4 & \text{si } x \geq -2 \end{cases}$

a) $f(x) = \begin{cases} ax^2+3x & \text{si } x \leq 2 \\ x^2-bx-4 & \text{si } x > 2 \end{cases}$



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Si $x<2$ $\rightarrow f(x) = ax^2+3x \Rightarrow$ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(-\infty, 2)$

Si $x>2$ $\rightarrow f(x) = x^2-5x-4 \Rightarrow$ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(2, +\infty)$

Si $x=2$ Continuidad:

$$f(2) = a\cdot 2^2+3\cdot 2 = 4a+6$$

$$\lim_{x\to 2} f(x) \longrightarrow \left\{ \begin{array}{l} \lim\limits_{x\to 2^-} (ax^2+3x) = 4a+6 \\ \lim\limits_{x\to 2^+} (x^2-5x-4) = -2b \end{array} \right\} 4a+6 = -2b$$

Para la derivabilidad:

$$f'(x) = \left\{ \begin{array}{ll} 2ax+3 \text{ si } x\leq 2 & f'(2^-) = 4a+3 \\ 2x-5 \text{ si } x>2 & f'(2^+) = 4-5 \end{array} \right\} 4a+3 = 4-5$$

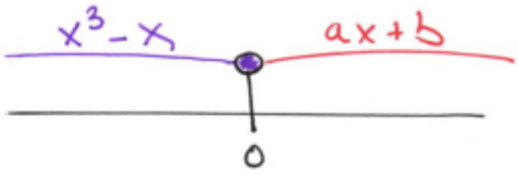
$f(x)$ será continua y derivable en $x=2$ (y por tanto en $\mathbb{R}$) cuando se verifique:

$$\left. \begin{array}{l} 4a+2b = -6 \\ 4a+b = 1 \end{array} \right\} E_2-E_1 \quad \boxed{b=-7} \Rightarrow \boxed{a=2}$$

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b) $f(x) = \begin{cases} x^3 - x & \text{si } x \leq 0 \\ ax+b & \text{si } x > 0 \end{cases}$



Si $x<0$ → $f(x) = x^3 - x$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable $(-\infty, 0)$

Si $x>0$ → $f(x) = ax+b$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(0, +\infty)$

Si $x=0$: Continuidad:

$f(0) = 0^3 - 0 = 0$

$\lim_{x\to 0} f(x) \longrightarrow \left\{ \begin{array}{l} \lim_{x\to 0^-} (x^3-x) = 0 \\ \lim_{x\to 0^+} (ax+b) = b \end{array} \right\} \; b = 0$

Para la derivabilidad:

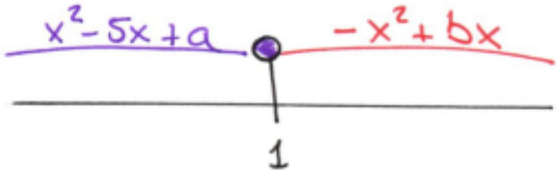
$$f'(x) = \begin{cases} 3x^2 - 1 & \text{si } x \leq 0 \\ a & \text{si } x > 0 \end{cases} \quad \left\{ \begin{array}{l} f'(0^-) = -1 \\ f'(0^+) = a \end{array} \right\} \; a = -1$$

$f(x)$ será continua y derivable en $x=0$ (y por tanto en $\mathbb{R}$) cuando se tenga $\boxed{a=-1}$ y $\boxed{b=0}$.

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c) $f(x) = \begin{cases} x^2-5x+a & \text{si } x \le 1 \\ -x^2+bx & \text{si } x > 1 \end{cases}$



Si $x<1$ → $f(x) = x^2-5x+a$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(-\infty, 1)$

Si $x>1$ → $f(x) = -x^2+bx$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(1, +\infty)$

Si $x=1$ Continuidad:

$f(1) = 1^2-5\cdot 1+a = a-4$

$$\lim_{x\to 1} f(x) \longrightarrow \left\{ \begin{array}{l} \lim\limits_{x\to 1^-} (x^2-5x+a) = a-4 \\ \lim\limits_{x\to 1^+} (-x^2+bx) = b-1 \end{array} \right\} \; a-4 = b-1$$

Para la derivabilidad:

$$f'(x) = \begin{cases} 2x-5 & \text{si } x \le 1 \\ -2x+b & \text{si } x > 1 \end{cases} \quad \left. \begin{array}{l} f'(1^-) = -3 \\ f'(1^+) = b-2 \end{array} \right\} \; -3 = b-2$$

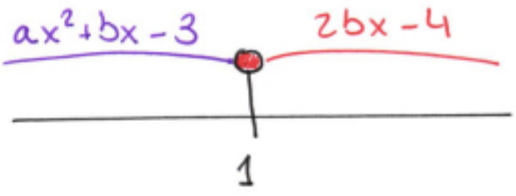
$f(x)$ será continua y derivable en $x=1$ (y por tanto en $\mathbb{R}$) cuando se verifique:

$$\left. \begin{array}{l} a-b=3 \\ b=-1 \end{array} \right\} \Rightarrow \boxed{b=-1} \text{ y } \boxed{a=2}$$

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d) $f(x) = \begin{cases} ax^2+bx-3 & \text{si } x<1 \\ 2bx-4 & \text{si } x \geq 1 \end{cases}$



Si $x<1$ → $f(x) = ax^2+bx-3$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(-\infty, 1)$

Si $x>1$ → $f(x) = 2bx-4$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(1, +\infty)$

Si $x=1$ Continuidad:

$f(1) = 2b-4$

$\lim_{x\to 1} f(x) \longrightarrow \begin{cases} \lim_{x\to 1^-} (ax^2+bx-3) = a+b-3 \\ \lim_{x\to 1^+} (2bx-4) = 2b-4 \end{cases} \Rightarrow a+b-3 = 2b-4$

Para la derivabilidad:

$$f'(x) = \begin{cases} 2ax+b & \text{si } x<1 \\ 2b & \text{si } x \geq 1 \end{cases} \qquad \begin{matrix} f'(1^-) = 2a+b \\ f'(1^+) = 2b \end{matrix} \Big\} \Rightarrow 2a+b = 2b$$

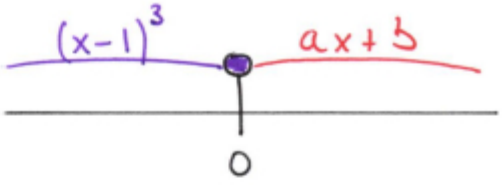
$f(x)$ será continua y derivable en $x=1$ (y por tanto en $\mathbb{R}$) cuando se verifique:

$$\left.\begin{matrix} a-b = -1 \\ 2a-b = 0 \end{matrix}\right\} E_2 - E_1 \rightarrow \boxed{a=1} \Rightarrow \boxed{b=2}$$

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e) $f(x) = \begin{cases} (x-1)^3 & \text{si } x \leq 0 \\ ax+b & \text{si } x > 0 \end{cases}$



Si $x<0$ → $f(x) = (x-1)^3$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(-\infty, 0)$

Si $x>0$ → $f(x) = ax+b$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(0, +\infty)$

Si $x=0$ Continuidad:

$f(0) = (0-1)^3 = -1$

$\lim_{x\to 0} f(x) \longrightarrow \left\{ \begin{array}{l} \lim_{x\to 0^-} (x-1)^3 = -1 \\ \lim_{x\to 0^+} (ax+b) = b \end{array} \right\} \; b = -1$

Para la derivabilidad:

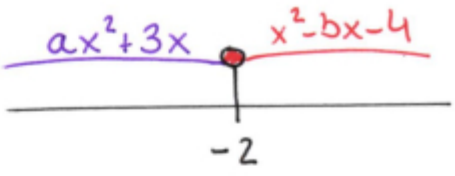
$$f'(x) = \left\{ \begin{array}{lll} 3(x-1)^2 & \text{si } x \leq 0 & f'(0^-) = 3 \\ a & \text{si } x > 0 & f'(0^+) = a \end{array} \right\} \; a = 3$$

$f(x)$ será continua y derivable en $x=0$ (y por tanto en $\mathbb{R}$) cuando se tenga $\boxed{a=3}$ y $\boxed{b=-1}$

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f) $f(x) = \begin{cases} ax^2+3x & \text{si } x<-2 \\ x^2-bx-4 & \text{si } x \geq -2 \end{cases}$



Si $x<-2$ $\longrightarrow f(x) = ax^2+3x \Rightarrow$ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(-\infty, -2)$

Si $x>-2$ $\longrightarrow f(x) = x^2-bx-4 \Rightarrow$ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(-2, +\infty)$

Si $x=-2$ Continuidad:

$f(-2) = (-2)^2 - b\cdot(-2) - 4 = 2b$

$$\lim_{x\to -2} f(x) \longrightarrow \left\{ \begin{array}{l} \lim\limits_{x\to -2^-} (ax^2+3x) = 4a-6 \\ \lim\limits_{x\to -2^+} (x^2-bx-4) = 2b \end{array} \right\} \quad 4a-6 = 2b$$

Para la derivabilidad:

$$f'(x) = \begin{cases} 2ax+3 & \text{si } x<-2 \\ 2x-b & \text{si } x \geq -2 \end{cases} \qquad \left. \begin{array}{l} f'(-2^-) = -4a+3 \\ f'(-2^+) = -4-b \end{array} \right\} -4a+3 = -4-b$$

$f(x)$ será continua y derivable en $x=-2$ (y por tanto en $\mathbb{R}$) cuando se verifique:

$$\left. \begin{array}{r} 4a-2b = 6 \\ -4a+b = -7 \end{array} \right\} E_1+E_2 \longrightarrow -b = -1 \Rightarrow \boxed{b=1} \Rightarrow \boxed{a=2}$$

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(26) Determina, si es posible, el valor de los parámetros para que las siguientes funciones sean continuas y derivables en $\mathbb{R}$:

a) $f(x) = \begin{cases} \cos x & \text{si } x \leq 0 \\ x^2 + a & \text{si } x > 0 \end{cases}$

b) $f(x) = \begin{cases} x^2 + ax & \text{si } x \leq -3 \\ \dfrac{x}{x+4} & \text{si } x > -3 \end{cases}$

c) $f(x) = \begin{cases} 2x + e^{1-x} & \text{si } x < 1 \\ 1 + \sqrt{x+m} & \text{si } x \geq 1 \end{cases}$

d) $f(x) = \begin{cases} ax^2 + bx + 1 & \text{si } x < 0 \\ a^2 - \operatorname{sen} x & \text{si } x \geq 0 \end{cases}$

e) $f(x) = \begin{cases} 1 + a \cdot \sqrt{x+1} \cdot \ln(x+1) & \text{si } x > 0 \\ (x-1)^2 & \text{si } x \leq 0 \end{cases}$

f) $f(x) = \begin{cases} (x+b)^2 & \text{si } x < -1 \\ \dfrac{ax}{\sqrt{x+2}} & \text{si } x \geq -1 \end{cases}$

g) $f(x) = \begin{cases} \ln(e + \operatorname{sen} x) & \text{si } x < 0 \\ x^3 + ax + b & \text{si } x \geq 0 \end{cases}$

h) $f(x) = \begin{cases} e^{a(x-\pi)} & \text{si } x < \pi \\ 2a + b \cdot \operatorname{sen}(x-\pi) & \text{si } x \geq \pi \end{cases}$

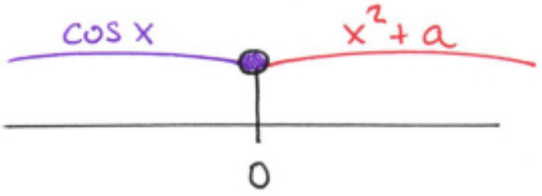
i) $f(x) = \begin{cases} 2e^x + x + a & \text{si } x \leq 0 \\ x^2 + b(x+1) & \text{si } 0 < x \leq 2 \\ x^4 - 3a & \text{si } x > 2 \end{cases}$

j) $f(x) = \begin{cases} -x^3 + 7x + c & \text{si } x \leq -2 \\ ax^2 + 3x + 11 & \text{si } -2 < x \leq \dfrac{3}{2} \\ 18\sqrt{2x+1} + b & \text{si } x > \dfrac{3}{2} \end{cases}$

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a) $f(x) = \begin{cases} \cos x & \text{si } x \leq 0 \\ x^2+a & \text{si } x > 0 \end{cases}$



Si $x<0$ → $f(x) = \cos x$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(-\infty, 0)$

Si $x>0$ → $f(x) = x^2+a$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(0, +\infty)$

Si $x = 0$ Continuidad:

$f(0) = \cos 0 = 1$

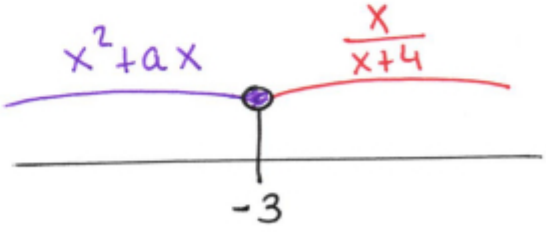
$\lim_{x\to 0} f(x) \longrightarrow \begin{cases} \lim_{x\to 0^-} (\cos x) = 1 \\ \lim_{x\to 0^+} (x^2+a) = a \end{cases}$ $\boxed{a = 1}$

Para la derivabilidad:

$$f'(x) = \begin{cases} -\operatorname{sen} x & \text{si } x \leq 0 \\ 2x & \text{si } x > 0 \end{cases} \qquad \begin{cases} f'(0^-) = \operatorname{sen} 0 = 0 \\ f'(0^+) = 2\cdot 0 = 0 \end{cases} \Rightarrow f'(0^-) = f'(0^+)$$

Si $\boxed{a = 1}$ $f(x)$ es continua y derivable en $\mathbb{R}$

b) $f(x) = \begin{cases} x^2+ax & \text{si } x \leq -3 \\ \dfrac{x}{x+4} & \text{si } x > -3 \end{cases}$



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Si $x<-3 \longrightarrow f(x) = x^2+ax \Rightarrow$ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(-\infty, -3)$

Si $x>-3 \longrightarrow f(x) = \dfrac{x}{x+4} \Rightarrow$ Continua y derivable en $\mathbb{R}-\{-4\}$ (NO AFECTA)

↳ Continua y derivable en $(-3, +\infty)$

Si $x=-3$: Continuidad:

$$f(-3) = (-3)^2+a\cdot(-3) = 9-3a$$

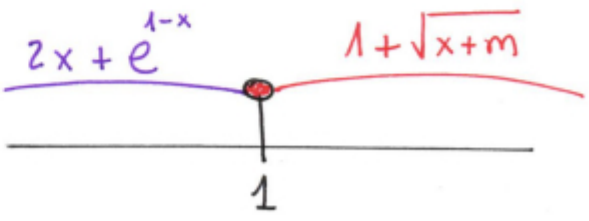
$$\lim_{x\to -3} f(x) \longrightarrow \left\{ \begin{array}{l} \lim\limits_{x\to -3^-} (x^2+ax) = 9-3a \\ \lim\limits_{x\to -3^+} \dfrac{x}{x+4} = -3 \end{array} \right\} \; 9-3a=-3 \Rightarrow a=4$$

Para la derivabilidad:

$$f'(x) = \begin{cases} 2x+4 & \text{si } x<-3 \\ \dfrac{4}{(x+4)^2} & \text{si } x>-3 \end{cases} \quad \left. \begin{array}{l} f'(-3^-) = -2 \\ f'(-3^+) = 4 \end{array} \right\} \; f'(-3^-) \neq f'(-3^+)$$

Por lo tanto, para $a=4$ la función será continua en $\mathbb{R}$ pero no derivable en $x=-3$ (derivable en $\mathbb{R}-\{-3\}$)

c) $$f(x) = \begin{cases} 2x+e^{1-x} & \text{si } x<1 \\ 1+\sqrt{x+m} & \text{si } x\geq 1 \end{cases}$$



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Si $x < 1$ $\longrightarrow f(x) = 2x + e^{1-x} \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, 1)$

Si $x > 1$ $\longrightarrow f(x) = 1 + \sqrt{x+m} \Rightarrow$ Continua en $[-m, +\infty)$ y derivable en $(-m, +\infty)$

$m > -1 \longrightarrow$ continua y derivable en $(1, +\infty)$

Si $x = 1$ Continuidad:

$$f(1) = 1 + \sqrt{1+m}$$

$$\lim_{x \to 1} f(x) \longrightarrow \left\{ \begin{array}{l} \lim_{x \to 1^-} (2x + e^{1-x}) = 3 \\ \lim_{x \to 1^+} (1 + \sqrt{x+m}) = 1 + \sqrt{1+m} \end{array} \right\} \; 3 = 1 + \sqrt{1+m}$$

$$2 = \sqrt{1+m} \Rightarrow 4 = 1 + m \Rightarrow \boxed{m = 3}$$

Veamos la derivabilidad para ese valor:

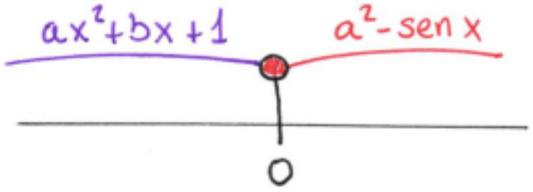
$$f'(x) = \left\{ \begin{array}{ll} 2 - e^{1-x} \text{ si } x < 1 & f'(1^-) = 1 \\ \dfrac{1}{2\sqrt{x+3}} \text{ si } x > 1 & f'(1^+) = \dfrac{1}{4} \end{array} \right\} \; f'(1^-) \neq f'(1^+)$$

Por lo tanto, para $m = 3$ la función será continua en $x = 1$ (y por tanto en $\mathbb{R}$) pero no será derivable en $x = 1$.

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d) $f(x) = \begin{cases} ax^2+bx+1 & \text{si } x<0 \\ a^2-\text{sen}\,x & \text{si } x\geq 0 \end{cases}$



Si $x<0$ $\rightarrow f(x) = ax^2+bx+1 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty, 0)$

Si $x>0$ $\rightarrow f(x) = a^2-\text{sen}x \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(0, +\infty)$

Si $x=0$: Continuidad:

$f(0) = a^2-\text{sen}(0) = a^2$

$$\lim_{x\to 0} f(x) \rightarrow \left\{ \begin{array}{l} \lim\limits_{x\to 0^-} (ax^2+bx+1) = 1 \\ \lim\limits_{x\to 0^+} (a^2-\text{sen}(x)) = a^2 \end{array} \right\} \quad a^2 = 1$$

Para la derivabilidad:

$$f'(x) = \left\{ \begin{array}{ll} 2ax+b & \text{si } x<0 \quad f'(0^-) = b \\ -\cos x & \text{si } x\geq 0 \quad f'(0^+) = -1 \end{array} \right\} \quad b = -1$$

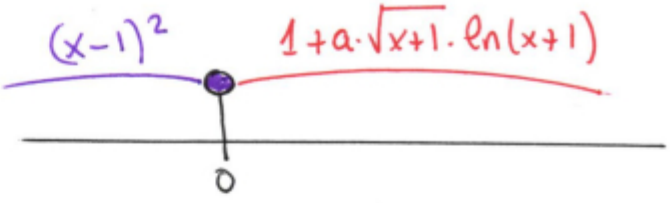
Para que sea continua y derivable en $\mathbb{R}$ se tendrá que cumplir:

$$a^2 = 1 \left\{ \begin{array}{l} \rightarrow a = -1 \\ \rightarrow a = +1 \end{array} \right. \quad \text{y} \quad b = -1$$

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e) $f(x) = \begin{cases} 1 + a\cdot\sqrt{x+1}\cdot \ln(x+1) & \text{si } x > 0 \\ (x-1)^2 & \text{si } x \le 0 \end{cases}$



Si $x < 0 \rightarrow f(x) = (x-1)^2 \Rightarrow$ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(-\infty, 0)$

Si $x > 0 \rightarrow f(x) = 1 + a\cdot\sqrt{x+1}\cdot\ln(x+1) \Rightarrow$ Continua y derivable en $(-1, +\infty)$

↳ Continua y derivable en $(0, +\infty)$

Si $x = 0$ Continuidad:

$f(0) = (0-1)^2 = 1$

$$\lim_{x\to 0} f(x) \rightarrow \left\{ \begin{array}{l} \lim\limits_{x\to 0^-} (x-1)^2 = 1 \\ \lim\limits_{x\to 0^+} \left(1 + a\cdot\sqrt{x+1}\cdot\ln(x+1)\right) = 1 \end{array} \right\} \Rightarrow \lim_{x\to 0} f(x) = 1$$

Como $f(0) = \lim\limits_{x\to 0} f(x) \Rightarrow$ Continua en $x = 0$ $(\forall a \in \mathbb{R})$

Para la derivabilidad:

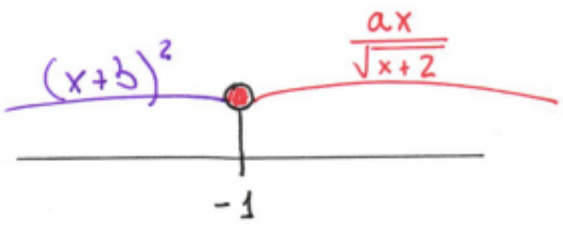
$$f'(x) = \begin{cases} 2(x-1) & \text{si } x \le 0 \\ \dfrac{a}{\sqrt{x+1}}\left(\dfrac{1}{2}\ln(x+1) + 1\right) & \text{si } x > 0 \end{cases} \qquad \begin{array}{l} f'(0^-) = -2 \\ f'(0^+) = \dfrac{a}{\sqrt{1}}\cdot 1 \end{array} \Bigg\} \; a = -2$$

$\Rightarrow f(x)$ es continua en $\mathbb{R}$ $\forall a \in \mathbb{R}$ y será derivable en $\mathbb{R}$ cuando sea $a = -2$

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f) $f(x) = \begin{cases} (x+b)^2 & \text{si } x < -1 \\ \dfrac{ax}{\sqrt{x+2}} & \text{si } x \geq -1 \end{cases}$



Si $x < -1$ → $f(x) = (x+b)^2$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(-\infty, -1)$

Si $x > -1$ → $f(x) = \dfrac{ax}{\sqrt{x+2}}$ ⇒ Continua y derivable en $(-2, +\infty)$

↳ Continua y derivable en $(-1, +\infty)$

Si $x = -1$ Continuidad:

$$f(-1) = \frac{a\cdot(-1)}{\sqrt{1}} = -a$$

$$\lim_{x\to -1} f(x) \longrightarrow \left\{ \begin{array}{l} \lim\limits_{x\to -1^-} (x+b)^2 = (b-1)^2 \\ \lim\limits_{x\to -1^+} \dfrac{ax}{\sqrt{x+2}} = -a \end{array} \right\} (b-1)^2 = -a$$

Para la derivabilidad:

$$f'(x) = \begin{cases} 2(x+b) & \text{si } x < -1 \\ \dfrac{ax+4a}{2\sqrt{(x+2)^3}} & \text{si } x \geq -1 \end{cases} \quad \begin{array}{l} f'(-1^-) = 2(b-1) \\ f'(-1^+) = \dfrac{3a}{2} \end{array} \Bigg\} 2(b-1) = \frac{3a}{2}$$

Para que sea continua y derivable en $\mathbb{R}$ se tendrá que cumplir:

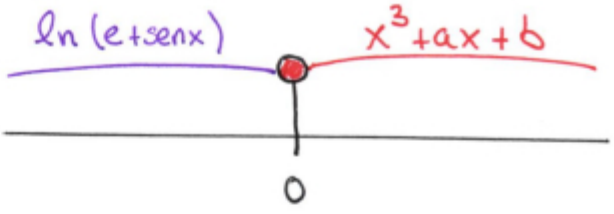
$$\left.\begin{array}{l} (b-1)^2 = -a \\ 2(b-1) = \dfrac{3a}{2} \end{array}\right\} \left(\frac{3a}{4}\right)^2 = -a \Rightarrow \frac{9a^2}{16} + a = 0 \Rightarrow a\left(\frac{9a}{16} + 1\right) = 0$$

$a = 0 \Rightarrow b = 1$

$a = -\frac{16}{9} \Rightarrow b = -1/3$

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g) $f(x) = \begin{cases} \ln(e+\operatorname{sen}x) & \text{si } x<0 \\ x^3+ax+b & \text{si } x\geq 0 \end{cases}$



Si $x<0$ → $f(x) = \ln(e+\operatorname{sen}x)$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(-\infty, 0)$

Si $x>0$ → $f(x) = x^3+ax+b$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(0, +\infty)$

Si $x=0$ Continuidad:

$f(0) = 0^3+0\cdot a+b = b$

$$\lim_{x\to 0} f(x) \longrightarrow \left\{ \begin{array}{l} \lim\limits_{x\to 0^-} \ln(e+\operatorname{sen}x) = 1 \\ \lim\limits_{x\to 0^+} (x^3+ax+b) = b \end{array} \right\} \; b = 1$$

Para la derivabilidad:

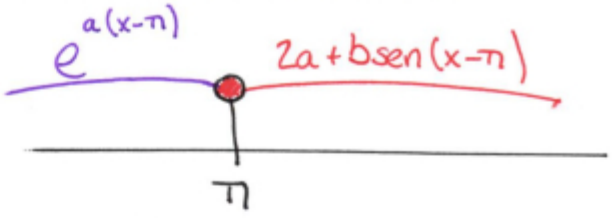
$$f'(x) = \begin{cases} \dfrac{\cos x}{e+\operatorname{sen}x} & \text{si } x<0 \\ 3x^2+a & \text{si } x\geq 0 \end{cases} \qquad \left\{ \begin{array}{l} f'(0^-) = 1/e \\ f'(0^+) = a \end{array} \right\} \; a = \frac{1}{e}$$

$f(x)$ será continua y derivable en $x=0$ (y por tanto en $\mathbb{R}$) cuando se tenga $a = \frac{1}{e}$ y $b = 1$

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h) $f(x) = \begin{cases} e^{a(x-\pi)} & \text{si } x < \pi \\ 2a + b\,\text{sen}(x-\pi) & \text{si } x \geq \pi \end{cases}$



Si $x < \pi$ $\rightarrow f(x) = e^{a(x-\pi)}$ $\Rightarrow$ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(-\infty, \pi)$

Si $x > \pi$ $\rightarrow f(x) = 2a + b\,\text{sen}(x-\pi)$ $\Rightarrow$ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(\pi, +\infty)$

Si $x = \pi$: Continuidad:

$$f(\pi) = 2a + b \cdot \text{sen}\,0 = 2a$$

$$\lim_{x\to\pi} f(x) \rightarrow \left\{ \begin{array}{l} \lim\limits_{x\to\pi^-} e^{a(x-\pi)} = e^0 = 1 \\ \lim\limits_{x\to\pi^+} (2a + b\,\text{sen}(x-\pi)) = 2a \end{array} \right\} \; 2a = 1$$

Para la derivabilidad:

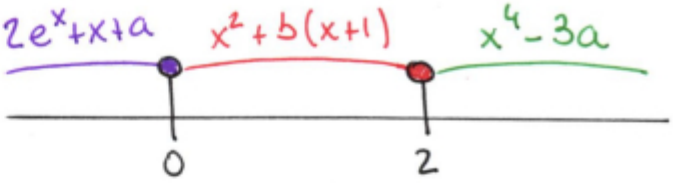
$$f'(x) = \begin{cases} a \cdot e^{a(x-\pi)} & \text{si } x < \pi \\ b \cdot \cos(x-\pi) & \text{si } x \geq \pi \end{cases} \quad \left\{ \begin{array}{l} f'(\pi^-) = a \\ f'(\pi^+) = b \end{array} \right\} \; a = b$$

$f(x)$ será continua y derivable en $x = \pi$ (y por tanto en $\mathbb{R}$) cuando se tenga $a = b = \frac{1}{2}$

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i) $f(x) = \begin{cases} 2e^x + x + a & \text{si } x \leq 0 \\ x^2 + b(x+1) & \text{si } 0 < x \leq 2 \\ x^4 - 3a & \text{si } x > 2 \end{cases}$



Si $x<0$ → $f(x) = 2e^x + x + a$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(-\infty, 0)$

Si $0<x<2$ → $f(x) = x^2 + b(x+1)$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(0,2)$

Si $x>2$ → $f(x) = x^4 - 3a$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(2, +\infty)$

Si $x=0$ Continuidad:

$f(0) = 2\cdot e^0 + 0 + a = a+2$

$$\lim_{x\to 0} f(x) \longrightarrow \left\{ \begin{array}{l} \lim\limits_{x\to 0^-} (2e^x + x + a) = a+2 \\ \lim\limits_{x\to 0^+} (x^2 + b(x+1)) = b \end{array} \right\} \; a+2 = b$$

Si $x=2$ Continuidad:

$f(2) = 2^2 + b\cdot(2+1) = 3b+4$

$$\lim_{x\to 2} f(x) \longrightarrow \left\{ \begin{array}{l} \lim\limits_{x\to 2^-} (x^2 + b(x+1)) = 3b+4 \\ \lim\limits_{x\to 2^+} (x^4 - 3a) = 16-3a \end{array} \right\} \; 3b+4 = 16-3a$$

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La función será continua en $x=0$ y en $x=2$ (y por tanto en $\mathbb{R}$) cuando se verifique:

$$\left.\begin{array}{l} a-b=-2 \\ 3a+3b=12 \end{array}\right\} \left.\begin{array}{l} a-b=-2 \\ a+b=4 \end{array}\right\} 2a=2 \Rightarrow a=1 \Rightarrow b=3$$

Y para esos valores, estudiemos la derivabilidad:

$$f'(x)=\begin{cases} 2e^x+1 & \text{si } x\leq 0 \\ 2x+3 & \text{si } 0<x<2 \\ 4x^3 & \text{si } x>2 \end{cases}$$

Si $x=0$ Derivabilidad:

$$\left.\begin{array}{l} f'(0^-)=2\cdot e^0+1=3 \\ f'(0^+)=2\cdot 0+3=3 \end{array}\right\} f'(0^-)=f'(0^+) \Rightarrow \text{Derivable en } x=0$$

Si $x=2$ Derivabilidad:

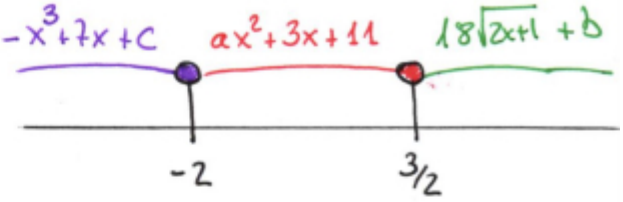
$$\left.\begin{array}{l} f'(2^-)=2\cdot 2+3=7 \\ f'(2^+)=4\cdot 2^3=32 \end{array}\right\} f'(2^-)\neq f'(2^+) \Rightarrow \nexists f'(2) \Rightarrow \text{No derivable en } x=2$$

$\Rightarrow$ Para $a=1$ y $b=3$ $f(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R}-\{2\}$

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j) $$f(x)=\begin{cases} -x^3+7x+c & \text{si } x\le -2 \\ ax^2+3x+11 & \text{si } -2<x\le 3/2 \\ 18\sqrt{2x+1}+b & \text{si } x>\frac{3}{2} \end{cases}$$



Si $x<-2$ $\rightarrow f(x)=-x^3+7x+c \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-\infty,-2)$

Si $-2<x<3/2$ $\rightarrow f(x)=ax^2+3x+11 \Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-2,3/2)$

Si $x>3/2$ $\rightarrow f(x)=18\sqrt{2x+1}+b \Rightarrow$ Continua en $[-1/2,+\infty)$ y derivable en $(-1/2,+\infty)$

$\hookrightarrow$ Continua y derivable en $(3/2,+\infty)$

Si $x=-2$ Continuidad:

$$f(-2)=-(-2)^3+7\cdot(-2)+c=c-6$$

$$\lim_{x\to -2} f(x) \rightarrow \left.\begin{cases} \lim\limits_{x\to -2^-}(-x^3+7x+c)=c-6 \\ \lim\limits_{x\to -2^+} ax^2+3x+11=4a+5 \end{cases}\right\} \; c-6=4a+5$$

Si $x=3/2$ Continuidad:

$$f(3/2)=a\cdot\left(\frac{3}{2}\right)^2+3\cdot\frac{3}{2}+11=\frac{9a}{4}+\frac{31}{2}$$

$$\lim_{x\to 3/2} f(x) \rightarrow \left.\begin{cases} \lim\limits_{x\to 3/2^-}(ax^2+3x+11)=\frac{9a}{4}+\frac{31}{2} \\ \lim\limits_{x\to 3/2^+} 18\sqrt{2x+1}+b=36+b \end{cases}\right\} \; \frac{9a}{4}+\frac{31}{2}=36+b$$

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Para la derivabilidad:

$$f'(x) = \begin{cases} -3x^2+7 & \text{si } x \leq -2 \\ 2ax+3 & \text{si } -2 < x \leq 3/2 \\ \dfrac{18}{\sqrt{2x+1}} & \text{si } x > 3/2 \end{cases}$$

Si $x = -2$ Derivabilidad:

$$\left.\begin{array}{l} f'(-2^-) = -3\cdot(-2)^2+7 = -5 \\ f'(-2^+) = 2a\cdot(-2)+3 = 3-4a \end{array}\right\} \; -5 = 3-4a$$

Si $x = 3/2$ Derivabilidad:

$$\left.\begin{array}{l} f'(3/2^-) = 2a\cdot\dfrac{3}{2}+3 = 3a+3 \\ f'(3/2^+) = \dfrac{18}{\sqrt{2\cdot\frac{3}{2}+1}} = 9 \end{array}\right\} \; 3a+3 = 9$$

$f(x)$ será continua y derivable en $\mathbb{R}$ cuando se verifique que:

$$\left.\begin{array}{l} -4a+c = 11 \\ 9a-4b = 82 \\ 3-4a = -5 \\ 3a+3 = 9 \end{array}\right.$$

$-4a+c = 11 \Rightarrow -8+c = 11 \Rightarrow c = 19$

$9a-4b = 82 \Rightarrow 18-4b = 82 \Rightarrow b = -16$

$3-4a = -5 \Rightarrow 4a = 8 \Rightarrow a = 2$

$3a+3 = 9 \Rightarrow 3a = 6 \Rightarrow a = 2$

$\Rightarrow f(x)$ es continua y derivable en $\mathbb{R}$ si $a = 2$, $b = -16$, y $c = 19$

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(27) Determina el valor de los parámetros para que las siguientes funciones sean continuas en $\mathbb{R}$. Para los valores obtenidos, estudia la derivabilidad:

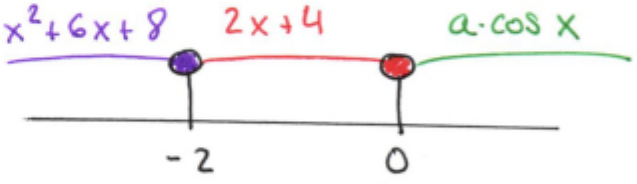
a) $f(x) = \begin{cases} x^2+6x+8 & \text{si } x \le -2 \\ 2x+4 & \text{si } -2 < x \le 0 \\ a\cdot\cos x & \text{si } x > 0 \end{cases}$

b) $f(x) = \begin{cases} \operatorname{sen} x & \text{si } x \le 0 \\ x - ax^2 & \text{si } x > 0 \end{cases}$

c) $f(x) = \begin{cases} 3x+2 & \text{si } x < 0 \\ x^2 + 2a\cos x & \text{si } 0 \le x < \pi \\ ax^2+b & \text{si } x \ge \pi \end{cases}$

d) $f(x) = \begin{cases} 2x+a & \text{si } x < -1 \\ ax+b & \text{si } -1 \le x < 0 \\ 3x^2+2 & \text{si } x \ge 0 \end{cases}$

a) $f(x) = \begin{cases} x^2+6x+8 & \text{si } x \le -2 \\ 2x+4 & \text{si } -2 < x \le 0 \\ a\cdot\cos x & \text{si } x > 0 \end{cases}$



Si $x < -2$ → $f(x) = x^2+6x+8$ ⇒ Continua y derivable en $\mathbb{R}$
↳ Continua y derivable en $(-\infty, -2)$

Si $-2 < x < 0$ → $f(x) = 2x+4$ ⇒ Continua y derivable en $\mathbb{R}$
↳ Continua y derivable en $(-2, 0)$

Si $x > 0$ → $f(x) = a\cdot\cos x$ ⇒ Continua y derivable en $\mathbb{R}$
↳ Continua y derivable en $(0, +\infty)$

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Si $x = -2$ Continuidad:

$$f(-2) = (-2)^2 + 6\cdot(-2) + 8 = 0$$

$$\lim_{x\to -2} f(x) \longrightarrow \left\{ \begin{array}{l} \lim\limits_{x\to -2^-} (x^2+6x+8) = 0 \\ \\ \lim\limits_{x\to -2^+} (2x+4) = 0 \end{array} \right\} \Rightarrow \lim_{x\to -2} f(x) = 0$$

Como $f(-2) = \lim\limits_{x\to -2} f(x) \Rightarrow$ Continua en $x = -2$

Si $x = 0$ Continuidad:

$$f(0) = 2\cdot 0 + 4 = 4$$

$$\lim_{x\to 0} f(x) \longrightarrow \left\{ \begin{array}{l} \lim\limits_{x\to 0^-} (2x+4) = 4 \\ \\ \lim\limits_{x\to 0^+} a\cdot\cos x = a \end{array} \right\} a = 4$$

Si $a = 4 \Rightarrow \lim\limits_{x\to 0} f(x) = 4 = f(0) \Rightarrow$ Continua en $x = 0$, y por tanto en $\mathbb{R}$

Y para $a = 4$, veamos la derivabilidad:

$$f'(x) = \left\{ \begin{array}{ll} 2x + 6 & \text{si } x \leq -2 \\ 2 & \text{si } -2 < x < 0 \\ -4\,\text{sen}\,x & \text{si } x > 0 \end{array} \right.$$

Si $x = -2$ Derivabilidad:

$$\left. \begin{array}{l} f'(-2^-) = -4+6 = 2 \\ f'(-2^+) = 2 \end{array} \right\} f'(-2^-) = f'(-2^+) \Rightarrow \text{Derivable en } x = -2$$

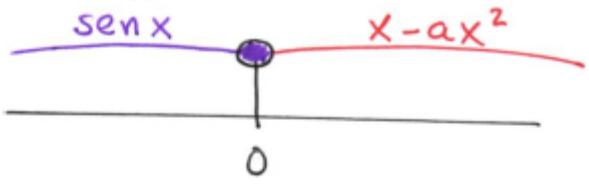
Si $x = 0$ Derivabilidad:

$$\left. \begin{array}{l} f'(0^-) = 2 \\ f'(0^+) = -4\cdot\text{sen}\,0 = 0 \end{array} \right\} f'(0^-) \neq f'(0^+) \Rightarrow \nexists f'(0) \Rightarrow \text{No derivable en } x = 0$$

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b) $f(x) = \begin{cases} \operatorname{sen} x & \text{si } x \leq 0 \\ x - ax^2 & \text{si } x > 0 \end{cases}$



Si $x<0$ → $f(x) = \operatorname{sen} x$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(-\infty, 0)$

Si $x>0$ → $f(x) = x - ax^2$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(0, +\infty)$

Si $x = 0$ Continuidad:

$f(0) = \operatorname{sen}(0) = 0$

$$\lim_{x\to 0} f(x) \longrightarrow \left\{ \begin{array}{l} \lim\limits_{x\to 0^-} \operatorname{sen} x = 0 \\ \lim\limits_{x\to 0^+} x - ax^2 = 0 \end{array} \right\} \Rightarrow \lim_{x\to 0} f(x) = 0$$

Como $f(0) = \lim\limits_{x\to 0} f(x)$, $f(x)$ es continua en $x=0$ (y en $\mathbb{R}$) $\forall a \in \mathbb{R}$

Veamos la derivabilidad:

$$f'(x) = \begin{cases} \cos x & \text{si } x \leq 0 \\ 1 - 2ax & \text{si } x > 0 \end{cases}$$

Si $x = 0$ Derivabilidad:

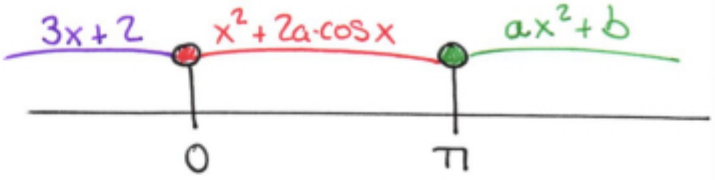
$$\left. \begin{array}{l} f'(0^-) = \cos 0 = 1 \\ f'(0^+) = 1 - 2a0 = 1 \end{array} \right\} \Rightarrow f'(0^-) = f'(0^+) \Rightarrow \text{Derivable en } x = 0$$

⇒ $f(x)$ es continua y derivable en $\mathbb{R}$ $\forall a \in \mathbb{R}$

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c) $f(x) = \begin{cases} 3x+2 & \text{si } x<0 \\ x^2+2a\cos x & \text{si } 0 \le x < \pi \\ ax^2+b & \text{si } x \ge \pi \end{cases}$



Si $x<0$ → $f(x) = 3x+2$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(-\infty, 0)$

Si $0<x<\pi$ → $f(x) = x^2+2a\cos x$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(0, \pi)$

Si $x>\pi$ → $f(x) = ax^2+b$ ⇒ Continua y derivable en $\mathbb{R}$

↳ Continua y derivable en $(\pi, +\infty)$

Si $x = 0$: Continuidad:

$f(0) = 0^2+2a\cdot\cos 0 = 2a$

$$\lim_{x\to 0} f(x) \longrightarrow \left\{ \begin{array}{l} \lim\limits_{x\to 0^-} (3x+2) = 2 \\ \lim\limits_{x\to 0^+} (x^2+2a\cos x) = 2a \end{array} \right\} \quad 2 = 2a \Rightarrow a = 1$$

Si $x = \pi$: Continuidad:

$f(\pi) = a\pi^2 + b$

$$\lim_{x\to \pi} f(x) \longrightarrow \left\{ \begin{array}{l} \lim\limits_{x\to \pi^-} (x^2+2a\cos x) = \pi^2 - 2a \\ \lim\limits_{x\to \pi^+} (ax^2+b) = a\pi^2+b \end{array} \right\} \quad \pi^2 - 2a = a\pi^2 + b$$

$f(x)$ será continua en $\mathbb{R}$ si:

$$\left. \begin{array}{l} a = 1 \\ b = \pi^2 - 2a - a\pi^2 \end{array} \right\} \Rightarrow a = 1 \text{ y } b = -2$$

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Y para esos valores, estudiemos la derivabilidad:

$$f'(x) = \begin{cases} 3 & \text{si } x < 0 \\ 2x - 2\operatorname{sen} x & \text{si } 0 < x < \pi \\ 2x & \text{si } x \geq \pi \end{cases}$$

Si $x = 0$: Derivabilidad:

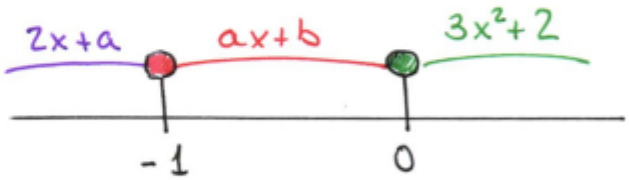
$$\left.\begin{array}{l} f'(0^-) = 3 \\ f'(0^+) = 2\cdot 0 - 2\operatorname{sen} 0 = 0 \end{array}\right\} f'(0^-) \neq f'(0^+) \Rightarrow \nexists f'(0) \Rightarrow \text{No derivable en } x = 0$$

Si $x = \pi$: Derivabilidad:

$$\left.\begin{array}{l} f'(\pi^-) = 2\pi - 2\cdot \operatorname{sen}\pi = 2\pi \\ f'(\pi^+) = 2\pi \end{array}\right\} f'(\pi^-) = f'(\pi^+) \Rightarrow \text{Derivable en } x = \pi$$

$\Rightarrow$ Para $a = 1$ y $b = 2$, $f(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R} - \{0\}$

d) $$f(x) = \begin{cases} 2x + a & \text{si } x < -1 \\ ax + b & \text{si } -1 \leq x < 0 \\ 3x^2 + 2 & \text{si } x \geq 0 \end{cases}$$



Si $x < -1$ $\rightarrow$ $f(x) = 2x + a$ $\Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua en $(-\infty, -1)$ y derivable

Si $-1 < x < 0$ $\rightarrow$ $f(x) = ax + b$ $\Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(-1, 0)$

Si $x > 0$ $\rightarrow$ $f(x) = 3x^2 + 2$ $\Rightarrow$ Continua y derivable en $\mathbb{R}$

$\hookrightarrow$ Continua y derivable en $(0, +\infty)$

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Si $x=-1$ Continuidad:

$$f(-1) = a\cdot(-1)+b = b-a$$

$$\lim_{x\to -1} f(x) \longrightarrow \left\{ \begin{array}{l} \lim\limits_{x\to -1^-} (2x+a) = a-2 \\ \lim\limits_{x\to -1^+} (ax+b) = b-a \end{array} \right\} \quad a-2 = b-a$$

Si $x=0$ Continuidad:

$$f(0) = 3\cdot 0^2+2 = 2$$

$$\lim_{x\to 0} f(x) \longrightarrow \left\{ \begin{array}{l} \lim\limits_{x\to 0^-} (ax+b) = b \\ \lim\limits_{x\to 0^+} (3x^2+2) = 2 \end{array} \right\} \quad b=2$$

$f(x)$ será continua en $\mathbb{R}$ si:

$$\left. \begin{array}{l} 2a-b = 2 \\ b = 2 \end{array} \right\} \Rightarrow a=2 \text{ y } b=2$$

Y para esos valores, estudiemos la derivabilidad:

$$f'(x) = \begin{cases} 2 & \text{si } x<-1 \\ 2 & \text{si } -1 \leq x < 0 \\ 6x & \text{si } x>0 \end{cases}$$

donde vemos que $f'(-1^-) = f'(-1^+) = 2 \Rightarrow$ Derivable en $x=-1$

y donde igualmente vemos que $\left. \begin{array}{l} f'(0^-) = 2 \\ f'(0^+) = 0 \end{array} \right\} \Rightarrow \nexists f'(0)$

$\Rightarrow$ Para $a=2$ y $b=2$, $f(x)$ es continua en $\mathbb{R}$ y derivable en $\mathbb{R}-\{0\}$

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