

① Escribe los siguientes números como números complejos en su forma binómica:

a)  $\sqrt{-3} = \sqrt{-1} \cdot \sqrt{3} = i\sqrt{3} \Rightarrow z = 0 + i\sqrt{3}$

b)  $3 \Rightarrow z = 3 + 0i$

c)  $2 + \sqrt{-8} = 2 + \sqrt{-1} \cdot \sqrt{8} = 2 + i\sqrt{8} \Rightarrow z = 2 + 2i\sqrt{2}$

② Calcular los valores de  $a$  y  $b$ :

a)  $2 + 3bi = a - 1$

$$\left. \begin{array}{l} \text{Parte real} \rightarrow 2 = a - 1 \\ \text{Parte imaginaria} \rightarrow 3b = 0 \end{array} \right\} a = 3; b = 0$$

b)  $4a - 2b = 2 - ai$

$$\left. \begin{array}{l} \text{Parte real} \rightarrow 4a - 2b = 2 \\ \text{Parte imaginaria} \rightarrow 0 = -a \end{array} \right\} a = 0; b = -1$$

③ Calcular  $x$  e  $y$  para que el número complejo dado por  $z = -2x + \frac{1}{2}yi$  sea:

a) Un número real:

$$\text{Im}(z) = 0 \Rightarrow \frac{1}{2}y = 0 \Rightarrow y = 0 \quad \forall x \in \mathbb{R}.$$

b) Un número imaginario puro:

$$\operatorname{Re}(z) = 0 \Rightarrow -2x = 0 \Rightarrow x = 0 \quad \forall y \in \mathbb{R} - \{0\}$$

c) Un número complejo que no sea real ni imaginario puro:

$$\operatorname{Re}(z) \neq 0 \Rightarrow -2x \neq 0 \Rightarrow x \neq 0$$

$$\operatorname{Im}(z) \neq 0 \Rightarrow \frac{1}{2}y \neq 0 \Rightarrow y \neq 0$$

④ Calcular el opuesto y el conjugado de los siguientes números complejos:

a)  $z = \sqrt{2} - 3i$

$$\text{Conjugado} \Rightarrow \bar{z} = \sqrt{2} + 3i$$

$$\text{Opuesto} \Rightarrow -z = -(\sqrt{2} - 3i) = -\sqrt{2} + 3i$$

b)  $z = 3 - 2i$

$$\bar{z} = 3 + 2i \quad ; \quad -z = -3 + 2i$$

c)  $z = 6i$

$$\bar{z} = -6i \quad ; \quad -z = -6i$$

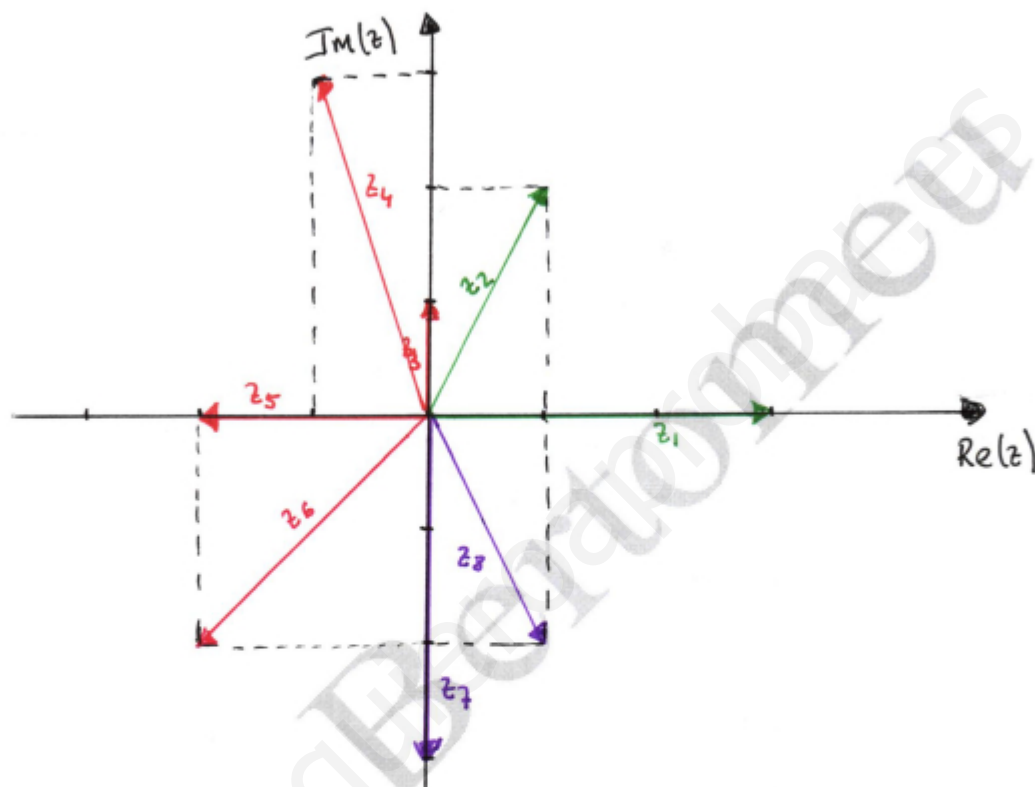
d)  $z = 7$

$$\bar{z} = 7 \quad ; \quad -z = -7$$

⑤ Representa los siguientes números complejos:

$$z_1 = 3; \quad z_2 = 1+2i; \quad z_3 = i; \quad z_4 = -1+3i; \quad z_5 = -2$$

$$z_6 = -2-2i; \quad z_7 = -3i; \quad z_8 = 1-2i$$



⑥ Efectúa las siguientes operaciones:

$$a) (-1-i) + (-4+5i) = -5+4i$$

$$b) \frac{-1-i}{-4+5i} = \frac{(-1-i) \cdot (-4-5i)}{(-4+5i) \cdot (-4-5i)} = \frac{4+5i+4i+5i^2}{(-4)^2 - (5i)^2} = \frac{4+9i-5}{16-25i^2} :$$

$$= \frac{-1+9i}{16+25} = \frac{-1+9i}{41}$$

$$c) (-1-i) \cdot (-4+5i) = 4-5i+4i-5i^2 = 9-i$$

$$\begin{aligned}
 d) \quad \frac{(-2+i)(1+3i)}{-1+2i} - 2i &= \frac{-2-6i+i+3i^2}{-1+2i} - 2i = \frac{-5-5i}{-1+2i} - 2i = \\
 &= \frac{-5-5i-2i(-1+2i)}{-1+2i} = \frac{-5-5i+2i-4i^2}{-1+2i} = \frac{-1-3i}{-1+2i} = \\
 &= \frac{(-1-3i)(-1-2i)}{(-1+2i)(-1-2i)} = \frac{1+2i+3i+6i^2}{(-1)^2-(2i)^2} = \frac{-5+5i}{1-4i^2} = -\frac{5+5i}{5} = -1+i
 \end{aligned}$$

⑦ Determina el inverso de los siguientes números complejos:

a)  $z=i$  ; b)  $z=3+4i$  ; c)  $z=-1-i$  ; d)  $4-3i=z$

a)  $z=i$

$$z^{-1} = \frac{1}{z} = \frac{1}{i} \cdot \frac{-i}{-i} = \frac{-i}{-i^2} = -i$$

b)  $z=3+4i$

$$z^{-1} = \frac{1}{z} = \frac{1}{3+4i} \cdot \frac{3-4i}{3-4i} = \frac{3-4i}{(3)^2-(4i)^2} = \frac{3-4i}{9-16i^2} = \frac{3-4i}{25}$$

c)  $z=-1-i$

$$z^{-1} = \frac{1}{z} = \frac{1}{-1-i} \cdot \frac{-1+i}{-1+i} = \frac{-1+i}{(-1)^2-(i)^2} = \frac{-1+i}{1-i^2} = \frac{-1+i}{2}$$

d)  $z=4-3i$

$$z^{-1} = \frac{1}{z} = \frac{1}{4-3i} \cdot \frac{4+3i}{4+3i} = \frac{4+3i}{4^2-(3i)^2} = \frac{4+3i}{16-9i^2} = \frac{4+3i}{25}$$

⑧ Expresar en forma polar:

a)  $z = 2 + i$  ; b)  $z = 2 - \sqrt{3}i$  ; c)  $z = -3 + 3i$  ; d)  $z = -2 - i$  ; e)  $z = 12$

a)  $z = 2 + i$

$$r = |z| = \sqrt{2^2 + 1^2} = \sqrt{5}$$

$$\Rightarrow z = \sqrt{5}_{26'565^\circ}$$

$$\alpha = \operatorname{arctg}\left(\frac{1}{2}\right) = 26'565^\circ$$

b)  $z = 2 - \sqrt{3}i$

$$r = |z| = \sqrt{2^2 + (\sqrt{3})^2} = \sqrt{7}$$

$$\Rightarrow z = \sqrt{7}_{319'1066^\circ}$$

$$\alpha = \operatorname{arctg}\left(-\frac{\sqrt{3}}{2}\right) = -40'8934^\circ$$

c)  $z = -3 + 3i$

$$r = |z| = \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$$

$$\Rightarrow z = 3\sqrt{2}_{135^\circ}$$

$$\alpha = \operatorname{arctg}\left(\frac{3}{-3}\right) = 135^\circ$$

Ojo con  
el cuadrante!!

d)  $z = -2 - i$

$$r = |z| = \sqrt{2^2 + 1^2} = \sqrt{5}$$

$$\Rightarrow z = \sqrt{5}_{206'565^\circ}$$

$$\alpha = \operatorname{arctg}\left(-\frac{1}{2}\right) = 206'565^\circ$$

e)  $z = 12$

$$r = |z| = \sqrt{12^2} = 12$$

$$\Rightarrow z = 12_{0^\circ}$$

$$\alpha = \operatorname{arctg}(0) = 0$$

9) Expresar en forma binómica y trigonométrica

a)  $z = 1_{120^\circ}$  ; b)  $z = 3_{240^\circ}$  ; c)  $z = 2_{\pi/3}$

$$a) z = 1_{120^\circ} = 1 \cdot (\cos(120) + i \cdot \text{sen}(120)) = -\frac{1}{2} + i \frac{\sqrt{3}}{2}$$

$$b) z = 3_{240^\circ} = 3 \cdot (\cos(240) + i \text{sen}(240)) = -\frac{3}{2} - i \frac{3\sqrt{3}}{2}$$

$$c) z = 2_{\pi/3} = 2 \cdot (\cos(\pi/3) + i \text{sen}(\pi/3)) = 1 + i\sqrt{3}$$

La calculadora en radianes!!

10) Expresar en forma polar y trigonométrica:

a)  $z = 3 - 4i$  ; b)  $z = 2 + 2i$  ; c)  $z = -\sqrt{3} + i$

a)  $z = 3 - 4i$

$$r = \sqrt{3^2 + 4^2} = 5$$

$$\Rightarrow z = 5_{306'87^\circ} = 5 (\cos(306'87) + i \text{sen}(306'87))$$

$$\alpha = \arctg\left(-\frac{4}{3}\right) = -53'13^\circ$$

b)  $z = 2 + 2i$

$$r = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$$

$$\Rightarrow z = 2\sqrt{2}_{45^\circ} = 2\sqrt{2} (\cos 45 + i \text{sen} 45)$$

$$\alpha = \arctg\left(\frac{2}{2}\right) = 45^\circ$$

c)  $z = -\sqrt{3} + i$

$$r = \sqrt{(\sqrt{3})^2 + 1^2} = 2$$

$$\Rightarrow z = 2_{150^\circ} = 2 (\cos 150 + i \text{sen} 150)$$

$$\alpha = \arctg\left(\frac{1}{-\sqrt{3}}\right) = 150^\circ$$

11) Efectúa las siguientes operaciones en forma polar:

a)  $4_{120^\circ} \cdot 3_{60^\circ}$ ; b)  $2_{230^\circ} \cdot 3_{130^\circ}$ ; c)  $\frac{3_{100^\circ}}{3_{40^\circ}}$ ; d)  $(2_{150^\circ})^2$

$$a) 4_{120^\circ} \cdot 3_{60^\circ} = (4 \cdot 3)_{120+60} = 12_{180^\circ}$$

$$b) 2_{230^\circ} \cdot 3_{130^\circ} = (2 \cdot 3)_{230+130} = 6_{360^\circ} = 6_{0^\circ}$$

$$c) \frac{3_{100^\circ}}{3_{40^\circ}} = \left(\frac{3}{3}\right)_{100-40} = 1_{60^\circ}$$

$$d) (2_{150^\circ})^2 = (2^2)_{2 \cdot 150} = 4_{300^\circ}$$

12) Se dan los números complejos  $z_1 = 1_{210^\circ}$  y  $z_2 = 3(\cos(-30^\circ) + i \operatorname{sen}(-30^\circ))$  y se pide calcular:

a)  $z_1 + z_2$

$$z_1 = 1_{210^\circ} = 1 \cdot (\cos(210^\circ) + i \operatorname{sen}(210^\circ)) = -\frac{\sqrt{3}}{2} - i \cdot \frac{1}{2}$$

$$z_2 = 3_{-30^\circ} = 3_{330^\circ} = 3 \cdot (\cos(330^\circ) + i \operatorname{sen}(330^\circ)) = 3 \left( \frac{\sqrt{3}}{2} - i \frac{1}{2} \right)$$

$$z_1 + z_2 = \left( -\frac{\sqrt{3}}{2} + \frac{3\sqrt{3}}{2} \right) + i \left( -\frac{1}{2} - \frac{3}{2} \right) = \sqrt{3} - 2i$$

b)  $\frac{z_1}{z_2} = \frac{1_{210^\circ}}{3_{330^\circ}} = \left( \frac{1}{3} \right)_{210-330} = \left( \frac{1}{3} \right)_{-120} = \left( \frac{1}{3} \right)_{240^\circ}$

$$c) \frac{z_1 \cdot \overline{z_2}}{z_2}$$

$$z_2 = 3 \left( \frac{\sqrt{3}}{2} - i \cdot \frac{1}{2} \right) \Rightarrow \overline{z_2} = 3 \left( \frac{\sqrt{3}}{2} + i \cdot \frac{1}{2} \right)$$

$$\overline{z_2} = \begin{cases} r = 3 \\ \alpha = \arctg \left( \frac{1/2}{\sqrt{3}/2} \right) = 30^\circ \end{cases} \Rightarrow \overline{z_2} = 3_{30^\circ}$$

$$\frac{z_1 \cdot \overline{z_2}}{z_2} = \frac{1_{210^\circ} \cdot 3_{30^\circ}}{3_{330^\circ}} = \frac{3_{240^\circ}}{3_{330^\circ}} = 1_{-90^\circ} = 1_{270^\circ} = -i$$

$$d) z_1^3 \cdot \overline{z_2}$$

$$z_1^3 \cdot \overline{z_2} = (1_{210^\circ})^3 \cdot 3_{30^\circ} = 1_{630^\circ} \cdot 3_{30^\circ} = 1_{270^\circ} \cdot 3_{30^\circ} = 3_{300^\circ}$$

⑬ Efectúa las operaciones:

$$a) (3-3i)^5$$

$$z = 3-3i \Rightarrow \begin{cases} r = \sqrt{3^2+3^2} = 3\sqrt{2} \\ \alpha = \arctg \left( \frac{-3}{3} \right) = -45^\circ \end{cases} \Rightarrow z = 3\sqrt{2}_{315^\circ}$$

$$\Rightarrow (3-3i)^5 = (3\sqrt{2}_{315^\circ})^5 = (3\sqrt{2})^5_{5 \cdot 315^\circ} = 972\sqrt{2}_{1575^\circ} = 972\sqrt{2}_{135^\circ}$$

$$b) (\sqrt{5}+2i)^8$$

$$z = \sqrt{5}+2i \Rightarrow \begin{cases} r = \sqrt{(\sqrt{5})^2+2^2} = 3 \\ \alpha = \arctg \left( \frac{2}{\sqrt{5}} \right) = 41'81'' \end{cases} \Rightarrow z = 3_{41'81''}$$

$$\Rightarrow (\sqrt{5}+2i)^8 = (3_{41'81''})^8 = (3^8)_{8 \cdot 41'81''} = 6561_{334'48''}$$

⑭ Utilizando la fórmula de De Moivre, expresar  $\cos(3\alpha)$  y  $\operatorname{sen}(3\alpha)$  en función de  $\cos\alpha$  y  $\operatorname{sen}\alpha$ .

Consideremos un número complejo  $z$  de módulo la unidad

$$z = 1_\alpha = 1(\cos\alpha + i\operatorname{sen}\alpha)$$

Vamos a elevar a 3:

$$z^3 = (1_\alpha)^3 = 1_{3\alpha} = \cos(3\alpha) + i\operatorname{sen}(3\alpha) = (\cos\alpha + i\operatorname{sen}\alpha)^3$$

$$\Rightarrow \cos(3\alpha) + i\operatorname{sen}(3\alpha) = (\cos\alpha + i\operatorname{sen}\alpha)^3 \Rightarrow$$

$$\begin{array}{c} \Downarrow \\ \begin{array}{cccc} & & 1 & \\ & 1 & & 1 \\ 1 & & 2 & \\ 1 & 3 & 3 & 1 \end{array} \end{array} \quad (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$\Rightarrow \cos(3\alpha) + i\operatorname{sen}(3\alpha) = \cos^3\alpha + 3\cos^2\alpha \cdot i \cdot \operatorname{sen}\alpha + 3\cos\alpha \cdot (i\operatorname{sen}\alpha)^2 + (i\operatorname{sen}\alpha)^3$$

$$\Rightarrow \cos(3\alpha) + i\operatorname{sen}(3\alpha) = \cos^3\alpha + i \cdot 3\cos^2\alpha \operatorname{sen}\alpha + i^2 \cdot 3\cos\alpha \operatorname{sen}^2\alpha + i^3 \operatorname{sen}^3\alpha$$

$$\Rightarrow \cos(3\alpha) + i\operatorname{sen}(3\alpha) = \cos^3\alpha - 3\cos\alpha \operatorname{sen}^2\alpha + i(3\cos^2\alpha \operatorname{sen}\alpha - \operatorname{sen}^3\alpha)$$

Iguando parte real con parte real e imaginaria con imaginaria:

$$\left\{ \begin{array}{l} \text{Real} \Rightarrow \cos(3\alpha) = \cos^3\alpha - 3\cos\alpha \operatorname{sen}^2\alpha \\ \text{Imaginaria} \Rightarrow \operatorname{sen}(3\alpha) = 3\cos^2\alpha \operatorname{sen}\alpha - \operatorname{sen}^3\alpha \end{array} \right.$$

15) Calcula las raíces de:

$$a) \sqrt{3}_{150^\circ} = \sqrt{3}_{\frac{150 + k \cdot 360}{2}} \text{ con } k = 0, 1$$

$$\text{Si } k = 0 \Rightarrow \alpha = \frac{150}{2} = 75^\circ \Rightarrow \sqrt{3}_{75^\circ}$$

$$\text{Si } k = 1 \Rightarrow \alpha = \frac{150 + 360}{2} = 255^\circ \Rightarrow \sqrt{3}_{255^\circ}$$

$$b) \sqrt[4]{-i} = \sqrt[4]{1}_{270^\circ} = 1_{\frac{270 + k \cdot 360}{4}} \text{ con } k = 0, 1, 2, 3$$

$$\text{Si } k = 0 \Rightarrow \alpha = \frac{270}{4} = 67'5^\circ \Rightarrow 1_{67'5^\circ}$$

$$\text{Si } k = 1 \Rightarrow \alpha = \frac{270 + 360}{4} = 157'5^\circ \Rightarrow 1_{157'5^\circ}$$

$$\text{Si } k = 2 \Rightarrow \alpha = \frac{270 + 2 \cdot 360}{4} = 247'5^\circ \Rightarrow 1_{247'5^\circ}$$

$$\text{Si } k = 3 \Rightarrow \alpha = \frac{270 + 3 \cdot 360}{4} = 337'5^\circ \Rightarrow 1_{337'5^\circ}$$

$$c) \sqrt[4]{-8 + i8\sqrt{3}} \Rightarrow z = -8 + i \cdot 8\sqrt{3} = \begin{cases} r = \sqrt{8^2 + (8\sqrt{3})^2} = 16 \\ \alpha = \arctg\left(\frac{8\sqrt{3}}{-8}\right) = 120^\circ \end{cases} \Rightarrow z = 16_{120^\circ}$$

Ojo con el cuadrante!!

$$\Rightarrow \sqrt[4]{-8 + i8\sqrt{3}} = \sqrt[4]{16}_{120^\circ} = 2_{\frac{120 + k \cdot 360}{4}} = 2_{30^\circ + 90^\circ k} \text{ con } k = 0, 1, 2, 3$$

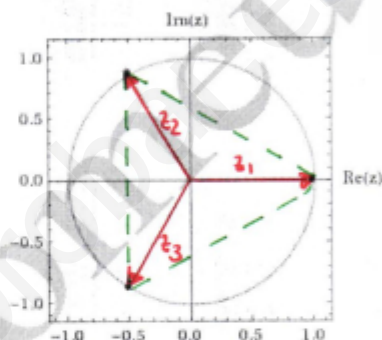
$$\Rightarrow \text{Las raíces son } \Rightarrow \begin{cases} k = 0 \rightarrow 2_{30^\circ} \\ k = 1 \rightarrow 2_{120^\circ} \\ k = 2 \rightarrow 2_{210^\circ} \\ k = 3 \rightarrow 2_{300^\circ} \end{cases}$$

①6) Resuelve las ecuaciones, y representa gráficamente las soluciones:

a)  $z^3 - 1 = 0$

$$z^3 = 1 \Rightarrow z = \sqrt[3]{1} = \sqrt[3]{1_0^\circ} = 1_{\frac{0+k \cdot 360}{3}} = 1_{0+120^\circ \cdot k} \text{ con } k=0,1,2$$

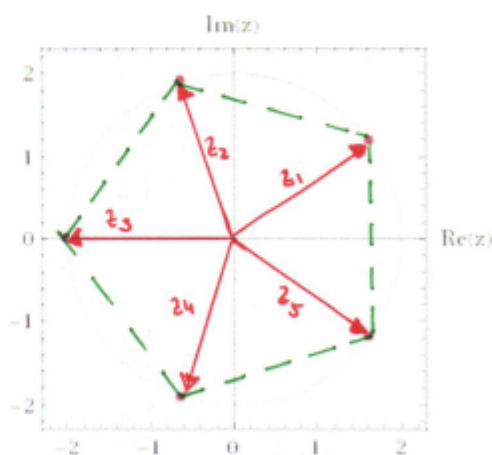
$$\Rightarrow \begin{cases} \text{Si } k=0 \rightarrow 1_0^\circ = 1 \\ \text{Si } k=1 \rightarrow 1_{120^\circ} = -\frac{1}{2} + i \cdot \frac{\sqrt{3}}{2} \\ \text{Si } k=2 \rightarrow 1_{240^\circ} = -\frac{1}{2} - i \cdot \frac{\sqrt{3}}{2} \end{cases}$$



b)  $z^5 + 32 = 0$

$$z^5 = -32 \Rightarrow z = \sqrt[5]{-32} = \sqrt[5]{32_{180^\circ}} = 2_{\frac{180+k \cdot 360}{5}} = 2_{36+72k} \quad k=0,1,2,3,4$$

$$\Rightarrow \begin{cases} \text{Si } k=0 \rightarrow 2_{36^\circ} = 1'618 + 1'176i \\ \text{Si } k=1 \rightarrow 2_{108^\circ} = -0'618 + 1'902i \\ \text{Si } k=2 \rightarrow 2_{180^\circ} = -2 \\ \text{Si } k=3 \rightarrow 2_{252^\circ} = -0'618 - 1'902i \\ \text{Si } k=4 \rightarrow 2_{324^\circ} = 1'618 - 1'176i \end{cases}$$



(17) Comprueba que si  $z$  y  $w$  son dos raíces sextas de 1, entonces también son raíces sextas de 1 los números  $z \cdot w$ ,  $\frac{z}{w}$ ,  $z^2$ ,  $z^3$ .

$$\text{Si } z \text{ es raíz sexta de } 1 \Rightarrow z^6 = 1$$

$$\text{Si } w \text{ es raíz sexta de } 1 \Rightarrow w^6 = 1$$

$$(z \cdot w)^6 = z^6 \cdot w^6 = 1 \cdot 1 = 1 \Rightarrow (z \cdot w) \text{ es raíz sexta de } 1$$

$$\left(\frac{z}{w}\right)^6 = \frac{z^6}{w^6} = \frac{1}{1} = 1 \Rightarrow \left(\frac{z}{w}\right) \text{ es raíz sexta de } 1$$

$$(z^2)^6 = (z^6)^2 = 1^2 = 1 \Rightarrow (z^2) \text{ es raíz sexta de } 1$$

$$(z^3)^6 = (z^6)^3 = 1^3 = 1 \Rightarrow (z^3) \text{ es raíz sexta de } 1$$

(18) El número complejo  $4+3i$  es una raíz cuarta de cierto número complejo  $z$ . Averigua las otras tres raíces cuartas de  $z$

$$4+3i \Rightarrow \begin{cases} r = \sqrt{4^2+3^2} = 5 \\ \alpha = \arctg\left(\frac{3}{4}\right) = 36'87^\circ \end{cases} \Rightarrow 5_{36'87^\circ}$$

y por tanto, las raíces que buscamos son:

$$5_{36'87^\circ} = 4+3i$$

$$5_{36'87^\circ+90^\circ} = 5_{126'87^\circ} = -3+4i$$

$$5_{126'87^\circ+90^\circ} = 5_{216'87^\circ} = -4-3i$$

$$5_{216'87^\circ+90^\circ} = 5_{306'87^\circ} = 3-4i$$

- (19) El producto de un número complejo  $z$  con su conjugado da como resultado  $48_{0^\circ}$ , y el cociente de dicho número  $z$  con su conjugado da como resultado un número complejo con argumento  $60^\circ$ . Averigua  $z$ .

$$\text{Sea } z = r_\alpha \Rightarrow \bar{z} = r_{-\alpha}$$

$$z \cdot \bar{z} = 48_{0^\circ} \Rightarrow r_\alpha \cdot r_{-\alpha} = (r \cdot r)_{\alpha - \alpha} = r^2_{0^\circ} = 48_{0^\circ} \Rightarrow r = \sqrt{48} = 4\sqrt{3}$$

$$\frac{z}{\bar{z}} = \frac{r_\alpha}{r_{-\alpha}} = \left(\frac{r}{r}\right)_{\alpha + \alpha} = 1_{2\alpha} \Rightarrow 2\alpha = 60^\circ \Rightarrow \alpha = 30^\circ$$

$$\Rightarrow z = 4\sqrt{3}_{30^\circ}$$

- (20) Averigua  $\text{sen } 15^\circ$  y  $\cos 15^\circ$  a partir del cociente  $\frac{1_{45^\circ}}{1_{30^\circ}}$

$$1_{45} = 1(\cos 45 + i \text{sen } 45) = \frac{\sqrt{2}}{2} + i \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2} + i\sqrt{2}}{2}$$

$$1_{30} = 1(\cos 30 + i \text{sen } 30) = \frac{\sqrt{3}}{2} + i \cdot \frac{1}{2} = \frac{\sqrt{3} + i}{2}$$

$$\frac{1_{45}}{1_{30}} = \frac{\frac{\sqrt{2} + i\sqrt{2}}{2}}{\frac{\sqrt{3} + i}{2}} = \frac{(\sqrt{2} + i\sqrt{2})(\sqrt{3} - i)}{(\sqrt{3} + i)(\sqrt{3} - i)} = \frac{\sqrt{6} - i\sqrt{2} + i\sqrt{6} - i^2 \cdot \sqrt{2}}{(\sqrt{3})^2 - (i)^2} =$$

$$= \frac{\sqrt{6} + \sqrt{2} + i(\sqrt{6} - \sqrt{2})}{4}$$

$$\frac{1_{45}}{1_{30}} = \left(\frac{1}{1}\right)_{45-30} = 1_{15^\circ} = 1(\cos 15 + i \text{sen } 15)$$

$$\Rightarrow \cos 15 + i \cdot \operatorname{sen} 15 = \frac{\sqrt{6} + \sqrt{2} + i(\sqrt{6} - \sqrt{2})}{4}$$

Iguando parte real con parte real e imaginaria con imaginaria

$$\left\{ \begin{array}{l} \text{Real} \Rightarrow \cos 15 = \frac{\sqrt{6} + \sqrt{2}}{4} \\ \text{Imaginaria} \Rightarrow \operatorname{sen} 15 = \frac{\sqrt{6} - \sqrt{2}}{4} \end{array} \right.$$

21) Resuelve la ecuación  $iz^3 + 3i - 2 = 1 + i$  y representa las soluciones.

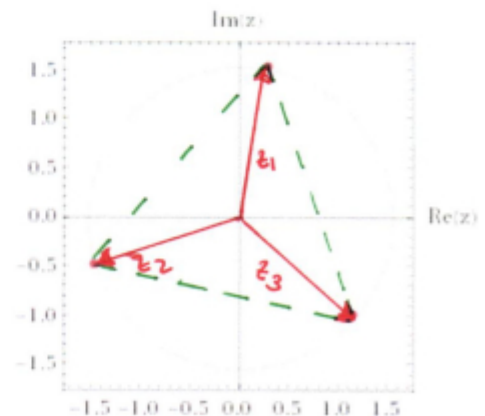
$$iz^3 + 3i - 2 = 1 + i \Rightarrow iz^3 = 3 - 2i \Rightarrow z^3 = \frac{3 - 2i}{i} \cdot \frac{-i}{-i} = -3i + 2i^2$$

$$\Rightarrow z^3 = -2 - 3i \Rightarrow \begin{cases} r = \sqrt{2^2 + 3^2} = \sqrt{13} \\ \alpha = \arctg\left(\frac{-3}{-2}\right) = 236'31'' \end{cases} \Rightarrow z^3 = \sqrt{13}_{236'31''}$$

Ojo con el cuadrante!!

$$\Rightarrow z = \sqrt[3]{\sqrt{13}_{236'31''}} = \sqrt[6]{13}_{\frac{236'31'' + k \cdot 360}{3}} = \sqrt[6]{13}_{78'77'' + k \cdot 120} \quad k = 0, 1, 2$$

$$\Rightarrow \left\{ \begin{array}{l} k = 0 \Rightarrow z_1 = \sqrt[6]{13}_{78'77''} \\ k = 1 \Rightarrow z_2 = \sqrt[6]{13}_{198'77''} \\ k = 2 \Rightarrow z_3 = \sqrt[6]{13}_{318'77''} \end{array} \right.$$



22) Resuelve las siguientes ecuaciones y sistemas en  $\mathbb{C}$

a)  $z^2 + 3z + 7 = 0$

$$z = \frac{-3 \pm \sqrt{9 - 28}}{2} = \frac{-3 \pm \sqrt{-19}}{2} = \frac{-3 \pm i\sqrt{19}}{2} \begin{cases} z_1 = -\frac{3}{2} + \frac{i\sqrt{19}}{2} \\ z_2 = -\frac{3}{2} - \frac{i\sqrt{19}}{2} \end{cases}$$

b)  $iz^3 - 27 = 0$

$$iz^3 = 27 \Rightarrow z^3 = \frac{27}{i} \cdot \frac{-i}{-i} = \frac{-27i}{-i^2} = -27i$$

$$\Rightarrow z = \sqrt[3]{-27i} = \sqrt[3]{27}_{270^\circ} = 3 \sqrt[3]{270 + k \cdot 360} = 3_{90^\circ + k \cdot 120^\circ} \quad k=0,1,2$$

$$\Rightarrow \begin{cases} \text{Si } k=0 \Rightarrow z_1 = 3_{90^\circ} = 3i \\ \text{Si } k=1 \Rightarrow z_2 = 3_{210^\circ} = 3 \left( -\frac{\sqrt{3}}{2} - i \cdot \frac{1}{2} \right) = -\frac{3\sqrt{3}}{2} - \frac{3i}{2} \\ \text{Si } k=2 \Rightarrow z_3 = 3_{330^\circ} = 3 \left( \frac{\sqrt{3}}{2} - i \cdot \frac{1}{2} \right) = \frac{3\sqrt{3}}{2} - \frac{3i}{2} \end{cases}$$

c)  $z^4 + 13z^2 + 36 = 0$  ;  $z^2 = t$

$$t^2 + 13t + 36 = 0 \rightarrow t = \frac{-13 \pm \sqrt{169 - 144}}{2} = \frac{-13 \pm 5}{2} \begin{cases} t_1 = -4 \\ t_2 = -9 \end{cases}$$

$$z^2 = -4 \Rightarrow z = \pm \sqrt{-4} \begin{cases} z_1 = +2i \\ z_2 = -2i \end{cases}$$

$$z^2 = -9 \Rightarrow z = \pm \sqrt{-9} \begin{cases} z_3 = +3i \\ z_4 = -3i \end{cases}$$

$$d) \quad z^4 - 8z = 0, \quad z(z^3 - 8) = 0 \quad \begin{cases} \boxed{z_1 = 0} \\ z^3 - 8 = 0 \end{cases}$$

$$z^3 - 8 = 0 \Rightarrow z^3 = 8 \Rightarrow z = \sqrt[3]{8} = \sqrt[3]{8_0^\circ} = 2_{\frac{0+k \cdot 360}{3}} = 2_{0+k \cdot 120^\circ}$$

$$\text{Si } k=0 \Rightarrow z_2 = 2_0^\circ = 2$$

$$\text{Si } k=1 \Rightarrow z_3 = 2_{120^\circ} = 2 \left( -\frac{1}{2} + i \frac{\sqrt{3}}{2} \right) = -1 + i\sqrt{3}$$

$$\text{Si } k=2 \Rightarrow z_4 = 2_{240^\circ} = 2 \left( -\frac{1}{2} - i \frac{\sqrt{3}}{2} \right) = -1 - i\sqrt{3}$$

$$e) \quad \begin{cases} z + 3w = 8 - 3i \\ 2z + w = 6 - i \end{cases} \quad -2E_1 \rightarrow \begin{aligned} -2z - 6w &= -16 + 6i \\ 2z + w &= 6 - i \\ \hline -5w &= -10 + 5i \end{aligned}$$

$$\boxed{w = 2 - i}$$

$$\rightarrow z = 8 - 3i - 3w$$

$$\boxed{z = 8 - 3i - 3(2 - i) = 8 - 3i - 6 + 3i = 2}$$

$$f) \quad \begin{cases} 2z - 5w = -5 + 2i \\ 4z - 3w = -3 - 10i \end{cases} \quad -2E_1 \rightarrow \begin{aligned} -4z + 10w &= +10 - 4i \\ 4z - 3w &= -3 - 10i \\ \hline 7w &= 7 - 14i \end{aligned}$$

$$\boxed{w = 1 - 2i}$$

$$\rightarrow 2z = -5 + 2i + 5w$$

$$2z = -5 + 2i + 5 - 10i$$

$$2z = -8i \Rightarrow \boxed{z = -4i}$$

23) Calcular el valor de  $k$  para que las siguientes expresiones sean números imaginarios puros:

$$a) \frac{3+ki}{k+2i} = \frac{(3+ki)(k-2i)}{(k+2i) \cdot (k-2i)} = \frac{3k - 6i + k^2i - 2ki^2}{(k)^2 - (2i)^2} =$$

$$= \frac{3k + (k^2-6)i + 2k}{k^2 - 4i^2} = \frac{5k + (k^2-6)i}{k^2+4} = \frac{5k}{k^2+4} + \frac{k^2-6}{k^2+4}i$$

Si tiene que ser imaginario puro  $\Rightarrow \operatorname{Re}(z) = 0$

$$\frac{5k}{k^2+4} = 0 \Rightarrow 5k = 0 \Rightarrow \boxed{k=0}$$

$$b) 3ki \cdot \frac{1-ki}{1-i} = \frac{3ki - 3k^2i^2}{1-i} = \frac{3k^2+3ki}{1-i} = \frac{(3k^2+3ki)(1+i)}{(1-i) \cdot (1+i)} =$$

$$= \frac{3k^2+3k^2i+3ki+3ki^2}{(1)^2 - (i)^2} = \frac{3k^2-3k + (3k^2+3k)i}{1-i^2} =$$

$$= \frac{3k^2-3k}{2} + \frac{3k^2+3k}{2}i$$

Si tiene que ser imaginario puro  $\Rightarrow \operatorname{Re}(z) = 0$

$$\frac{3k^2-3k}{2} = 0 \Rightarrow 3k(k-1) = 0 \rightarrow \begin{matrix} \cancel{k=0} \\ \boxed{k=1} \end{matrix} \quad \text{No sirve en este caso!!}$$

24) Calcular  $p$  y  $q$  para que se cumpla la igualdad:

$$(p+3i)(4+qi) = 15+9i$$

$$(p+3i)(4+qi) = 4p + pqi + 12i + 3qi^2 = (4p-3q) + (12+pq)i$$

$$\Rightarrow (4p-3q) + (12+pq)i = 15+9i \quad \begin{array}{l} \text{Re}(z) \rightarrow 4p-3q = 15 \\ \text{Im}(z) \rightarrow 12+pq = 9 \end{array}$$

$$\Rightarrow \left. \begin{array}{l} 4p-3q = 15 \\ pq = -3 \end{array} \right\} \begin{array}{l} \xrightarrow{\quad} 4p - 3 \cdot \left(-\frac{3}{p}\right) = 15 \Rightarrow \\ \quad \quad \quad q = -\frac{3}{p} \end{array}$$

$$\Rightarrow 4p^2 + 9 = 15p \Rightarrow 4p^2 - 15p + 9 = 0 \quad \begin{array}{l} \rightarrow p_1 = 3 \Rightarrow q_1 = -1 \\ \rightarrow p_2 = \frac{3}{4} \Rightarrow q_2 = -4 \end{array}$$

25) Comprueba que el número complejo  $z = 1-3i$  cumple que

$$\frac{z^2}{z} = z-5$$

$$\left. \frac{z^2}{z} = \frac{(1-3i)^2}{z} = \frac{1-6i+9i^2}{z} = \frac{-8-6i}{z} = -4-3i \right\} \Rightarrow \frac{z^2}{z} = z-5$$

$$z-5 = 1-3i-5 = -4-3i$$

26) La suma de dos números complejos  $z_1$  y  $z_2$  es  $3+2i$ , siendo la parte real de  $z_2$ ,  $\text{Re}(z_2)=2$ . Además la división  $\frac{z_1}{z_2}$  es un número imaginario puro. Calcula  $z_1$  y  $z_2$ .

$$\left. \begin{array}{l} z_1 = a + bi \\ z_2 = 2 + ci \end{array} \right\} z_1 + z_2 = (a+2) + (b+c)i = 3+2i \Rightarrow$$

$$\Rightarrow \begin{cases} a+2=3 \rightarrow a=1 \\ b+c=2 \rightarrow c=2-b \end{cases}$$

$$\frac{z_1}{z_2} = \frac{1+bi}{2+ci} = \frac{1+bi}{2+(2-b)i} = \frac{(1+bi)(2-(2-b)i)}{(2+(2-b)i) \cdot (2-(2-b)i)} =$$

$$= \frac{2 - (2-b)i + 2bi - 2bi^2 + b^2i^2}{(2)^2 - ((2-b)i)^2} = \frac{2+2b-b^2 + (3b-2)i}{4 - (4-4b+b^2)i^2} =$$

$$= \frac{2+2b-b^2}{b^2-4b+8} + \frac{3b-2}{b^2-4b+8} i$$

Como  $\frac{z_1}{z_2}$  es imaginario puro  $\Rightarrow \text{Re}\left(\frac{z_1}{z_2}\right) = 0$

$$\frac{2+2b-b^2}{b^2-4b+8} = 0 \Rightarrow -b^2+2b+2=0 \begin{cases} \rightarrow b=1+\sqrt{3} \Rightarrow c=1-\sqrt{3} \\ \rightarrow b=1-\sqrt{3} \Rightarrow c=1+\sqrt{3} \end{cases}$$

$$\Rightarrow \begin{cases} z_1 = 1 + (1+\sqrt{3})i ; z_2 = 2 + (1-\sqrt{3})i \\ z_1 = 1 + (1-\sqrt{3})i ; z_2 = 2 + (1+\sqrt{3})i \end{cases}$$

27) Determina dos números complejos sabiendo que su suma es 3 y su cociente es  $1i$

$$\left. \begin{array}{l} z_1 = a+bi \\ z_2 = c+di \end{array} \right\} \begin{array}{l} z_1 + z_2 = 3 \\ \text{y} \quad \frac{z_1}{z_2} = 1i \end{array}$$

$$a+bi+c+di=3$$

$$(a+c)+(b+d)i=3 \quad \begin{array}{l} \rightarrow a+c=3 \\ \rightarrow b+d=0 \end{array}$$

$$\frac{a+bi}{c+di} = \frac{(a+bi)(c-di)}{(c+di)(c-di)} = \frac{ac-adi+cbi-bdi^2}{c^2-d^2i^2} =$$

$$= \frac{ac+bd}{c^2+d^2} + \frac{cb-ad}{c^2+d^2} \cdot i = 1i$$

$$\rightarrow \frac{ac+bd}{c^2+d^2} = 0 \Rightarrow ac+bd=0$$

$$\rightarrow \frac{cb-ad}{c^2+d^2} = 1 \Rightarrow cb-ad=c^2+d^2$$

$$\Rightarrow \left\{ \begin{array}{l} a+c=3 \rightarrow a=3-c \\ b+d=0 \rightarrow b=-d \\ ac+bd=0 \rightarrow (3-c)c-d^2=0 \rightarrow d^2=(3-c)c \\ cb-ad=c^2+d^2 \rightarrow -cd-(3-c)d=c^2+(3-c)c \end{array} \right.$$

$$-cd - (3-c)d = c^2 + 3c - c^2$$

$$-cd - (3-c)d = 3c$$

$$d(-c-3+c) = 3c \Rightarrow -3d = 3c \Rightarrow \boxed{c = -d}$$

$$d^2 = (3-c)c$$

$$d^2 = (3+d)(-d) \Rightarrow d^2 = -3d - d^2 \Rightarrow 2d^2 + 3d = 0$$

$$d(2d+3) = 0 \begin{cases} \rightarrow d=0 \Rightarrow c=0 \Rightarrow b=0 \Rightarrow a=3 \text{ No sirve} \\ \rightarrow d = -\frac{3}{2} \Rightarrow c = \frac{3}{2} \Rightarrow b = \frac{3}{2} \Rightarrow a = \frac{3}{2} \end{cases}$$

Los números pedidos son:

$$z_1 = \frac{3}{2} + \frac{3}{2}i \quad \text{y} \quad z_2 = \frac{3}{2} - \frac{3}{2}i$$

(28) Escribe una ecuación de segundo grado  $x^2 + bx + c = 0$  (con  $b$  y  $c$  números reales) que tenga por solución al número complejo:

a)  $1-i$

Si  $z_1 = 1-i$  es solución de la ecuación, también será solución su conjugado  $\bar{z}_1 = 1+i$ .

$$x^2 + bx + c = (x - z_1)(x - \bar{z}_1) = x^2 - (z_1 + \bar{z}_1)x + z_1 \cdot \bar{z}_1$$

$$z_1 + \bar{z}_1 = (1-i) + (1+i) = 2$$

$$z_1 \cdot \bar{z}_1 = (1-i)(1+i) = 1 - i^2 = 2$$

$$\Rightarrow x^2 - 2x + 2 = 0$$

b)  $3-2i$

Si  $z_1 = 3-2i$  es solución de la ecuación, también será solución su conjugado  $\bar{z}_1 = 3+2i$

$$x^2 + bx + c = (x - z_1)(x - \bar{z}_1) = x^2 - (z_1 + \bar{z}_1)x + z_1 \bar{z}_1$$

$$z_1 + \bar{z}_1 = (3-2i) + (3+2i) = 6$$

$$\Rightarrow x^2 - 6x + 13 = 0$$

$$z_1 \cdot \bar{z}_1 = (3-2i) \cdot (3+2i) = 9 - 4i^2 = 13$$

(29) Calcular los números complejos  $p$  y  $q$  para que la ecuación  $x^3 + px^2 + qx - 2 = 0$  tenga como solución a los números complejos  $z_1 = i$  y  $z_2 = 2i$ . ¿Cuál es la tercera solución de la ecuación?

$$x^3 + px^2 + qx - 2 = (x - z_1)(x - z_2)(x - z_3)$$

$$(x - z_1)(x - z_2) = (x - i)(x - 2i) = x^2 - 2xi - xi + 2i^2 = x^2 - 3xi - 2$$

$$x^3 + px^2 + qx - 2 = (x^2 - 3xi - 2)(x - z_3)$$

$$x^3 + px^2 + qx - 2 = x^3 - x^2 z_3 - 3x^2 i + 3x z_3 i - 2x + 2z_3$$

$$x^3 + px^2 + qx - 2 = x^3 + (-3i - z_3) \cdot x^2 + (+3z_3i - 2)x + 2z_3$$

$$\begin{aligned} \rightarrow p &= -3i - z_3 & \rightarrow p &= 1 - 3i \\ \rightarrow q &= +3z_3i - 2 & \rightarrow q &= -2 - 3i \\ \rightarrow -2 &= 2z_3 & \Rightarrow z_3 &= -1 \end{aligned}$$

③ Resuelve las siguientes ecuaciones en  $\mathbb{C}$

a)  $x^2 + ix + 1 = 0$

$$x = \frac{-i \pm \sqrt{i^2 - 4}}{2} = \frac{-i \pm \sqrt{-5}}{2} = \frac{-i \pm i\sqrt{5}}{2}$$

$$\begin{aligned} \rightarrow x_1 &= -\frac{1+\sqrt{5}}{2} \cdot i \\ \rightarrow x_2 &= -\frac{1-\sqrt{5}}{2} \cdot i \end{aligned}$$

b)  $ix^2 + 7x - 12i = 0$

$$x = \frac{-7 \pm \sqrt{49 + 48i^2}}{2i} = \frac{-7 \pm 1}{2i}$$

$$\begin{aligned} \rightarrow x_1 &= -\frac{6}{2i} = -\frac{3}{i} \cdot \frac{-i}{-i} = 3i \\ \rightarrow x_2 &= -\frac{8}{2i} = -\frac{4}{i} \cdot \frac{-i}{-i} = 4i \end{aligned}$$

c)  $ix^3 + 27 = 0$

$$x^3 = -\frac{27}{i} \cdot \frac{-i}{-i} = 27i \Rightarrow x = \sqrt[3]{27i} = \sqrt[3]{27_{90^\circ}} = 3_{\frac{90+k \cdot 360}{3}} = 3_{30+k \cdot 120}$$

$$\Rightarrow 3_{30+k \cdot 120} \begin{cases} k=0 \rightarrow x_1 = 3_{30^\circ} \\ k=1 \rightarrow x_2 = 3_{150^\circ} \\ k=2 \rightarrow x_3 = 3_{270^\circ} \end{cases}$$

31) Calcula el valor de "b" para que se verifique la igualdad

$$4_{72^\circ} \cdot (1+bi) = 8_{132^\circ}$$

$$4_{72^\circ} \cdot (1+bi) = 8_{132^\circ} \Rightarrow 1+bi = \frac{8_{132^\circ}}{4_{72^\circ}} = 2_{60^\circ}$$

$$\rightarrow r = \sqrt{1^2 + b^2} = 2 \Rightarrow b^2 = 3 \begin{cases} b = -\sqrt{3} \rightarrow \text{No sirve} \\ b = +\sqrt{3} \end{cases}$$

$$\rightarrow \operatorname{tg} 60 = \frac{b}{1} \Rightarrow b = \sqrt{3}$$

32) Comprueba que las soluciones de la ecuación  $z^6 - 64 = 0$  dan cero cuando las sumamos todas.

$$z^6 - 64 = 0 \Rightarrow z^6 = 64 \Rightarrow z = \sqrt[6]{64} = \sqrt[6]{64_{0^\circ}} = 2_{\frac{0^\circ + k \cdot 360^\circ}{6}} = 2_{0^\circ + k \cdot 60^\circ}$$

$$\Rightarrow \begin{cases} k=0 \rightarrow z_1 = 2_{0^\circ} = 2 \\ k=1 \rightarrow z_2 = 2_{60^\circ} = 2(\cos 60 + i \sin 60) = 1 + i\sqrt{3} \\ k=2 \rightarrow z_3 = 2_{120^\circ} = 2(\cos 120 + i \sin 120) = -1 + i\sqrt{3} \\ k=3 \rightarrow z_4 = 2_{180^\circ} = -2 \\ k=4 \rightarrow z_5 = 2_{240^\circ} = 2(\cos 240 + i \sin 240) = -1 - i\sqrt{3} \\ k=5 \rightarrow z_6 = 2_{300^\circ} = 2(\cos 300 + i \sin 300) = 1 - i\sqrt{3} \end{cases}$$

$$\Rightarrow z_1 + z_2 + z_3 + z_4 + z_5 + z_6 = 2 + 1 + i\sqrt{3} - 1 + i\sqrt{3} - 2 - 1 - i\sqrt{3} + 1 - i\sqrt{3} = 0$$

- ③③ Determina el valor de  $k$  para que  $\frac{2+i}{k-i}$  sea un número real. ¿Cuál es ese número real?

$$\frac{2+i}{k-i} = \frac{(2+i)(k+i)}{(k-i)(k+i)} = \frac{2k+2i+i^2+k^2-i^2}{k^2-i^2} = \frac{2k-1+(2+k)i}{k^2+1} =$$

$$= \frac{2k-1}{k^2+1} + \frac{2+k}{k^2+1} \cdot i$$

Si es un número real  $\Rightarrow \text{Im}(z)=0$

$$\frac{2+k}{k^2+1} = 0 \Rightarrow 2+k=0 \Rightarrow \boxed{k=-2}$$

siendo el número real  $z = \frac{2 \cdot (-2) - 1}{2^2 + 1} = \frac{-5}{5} = -1$

- ③④ Calcula el valor de " $a$ " para que el número  $z = \frac{6-2i}{1+ai}$  sea

a) Un número imaginario puro:

$$z = \frac{6-2i}{1+ai} \cdot \frac{1-ai}{1-ai} = \frac{6-6ai-2i+2ai^2}{1-a^2i^2} = \frac{6-2a+(-2-6a)i}{1+a^2} =$$

$$= \frac{6-2a}{1+a^2} + \frac{-2-6a}{1+a^2} \cdot i$$

$\Rightarrow$  Si es imaginario puro  $\Rightarrow \text{Re}(z)=0$

$$\frac{6-2a}{1+a^2} = 0 \Rightarrow 6-2a=0 \Rightarrow \boxed{a=3}$$

b) Un número real:

Si es un número real  $\Rightarrow \text{Im}(z) = 0$

$$\frac{-2-6a}{1+a^2} = 0 \Rightarrow -2-6a=0 \Rightarrow \boxed{a=-3}$$

c) El número complejo  $2-4i$ :

$$\frac{6-2a}{1+a^2} + \frac{-2-6a}{1+a^2} i = 2-4i$$

$$\xrightarrow{\text{Re}(z)} \frac{6-2a}{1+a^2} = 2 \Rightarrow 6-2a = 2+2a^2 \Rightarrow 2a^2+2a-4=0 \begin{cases} \rightarrow a=1 \\ \rightarrow a=-2 \end{cases}$$

$$\xrightarrow{\text{Im}(z)} \frac{-2-6a}{1+a^2} = -4 \Rightarrow 2+6a = 4+4a^2 \Rightarrow 4a^2-6a+2=0 \begin{cases} \rightarrow a=1 \\ \rightarrow a=1/2 \end{cases}$$

El valor de "a" que satisface ambas ecuaciones es  $\boxed{a=1}$

d) Un número complejo con módulo 1:

$$r = \sqrt{(\text{Re}(z))^2 + (\text{Im}(z))^2} \Rightarrow 1 = \sqrt{\left(\frac{6-2a}{1+a^2}\right)^2 + \left(\frac{-2-6a}{1+a^2}\right)^2} \Rightarrow$$

$$\Rightarrow 1 = \sqrt{\frac{(6-2a)^2 + (-2-6a)^2}{(1+a^2)^2}} \Rightarrow 1 = \frac{1}{1+a^2} \cdot \sqrt{(6-2a)^2 + (-2-6a)^2} \Rightarrow$$

$$\Rightarrow 1+a^2 = \sqrt{(6-2a)^2 + (-2-6a)^2} \Rightarrow (1+a^2)^2 = (6-2a)^2 + (-2-6a)^2 \Rightarrow$$

$$\Rightarrow 1 + 2a^2 + a^4 = 36 - 24a + 4a^2 + 4 + 24a + 36a^2$$

$$\Rightarrow a^4 - 38a^2 - 39 = 0 \quad ; \quad a^2 = t$$

$$\Rightarrow t^2 - 38t - 39 = 0 \quad \begin{cases} t = 39 \\ t = -1 \end{cases}$$

$$a^2 = 39 \Rightarrow a = \pm \sqrt{39}$$

$$a^2 = -1 \Rightarrow a = \pm i \quad (\text{donde } a = \pm i \text{ no es válida})$$

e) Un número complejo con argumento  $\frac{7\pi}{4}$

$$\operatorname{tg} \alpha = \frac{\operatorname{Im}(z)}{\operatorname{Re}(z)} \Rightarrow \operatorname{tg}\left(\frac{7\pi}{4}\right) = \frac{\frac{-2-6a}{1+a^2}}{\frac{6-2a}{1+a^2}} \Rightarrow -1 = \frac{-2-6a}{6-2a}$$

$$\Rightarrow -6 + 2a = -2 - 6a \Rightarrow 8a = 4 \Rightarrow a = \frac{1}{2}$$

(35) Calcular el valor de "a" para que  $z = a + \frac{\sqrt{3}}{2}i$  verifique que  $z^2 = \bar{z}$

$$z^2 = \left(a + \frac{\sqrt{3}}{2}i\right)^2 = a^2 + ia\sqrt{3} + \frac{3}{4}i^2 = \left(a^2 - \frac{3}{4}\right) + ia\sqrt{3}$$

$$\bar{z} = a - \frac{\sqrt{3}}{2}i$$

$$\Rightarrow z^2 = \bar{z} \quad \left. \begin{array}{l} \text{Real} \rightarrow a^2 - \frac{3}{4} = a \rightarrow a^2 - a - \frac{3}{4} = 0 \\ \text{Im} \rightarrow a \cdot \sqrt{3} = -\frac{\sqrt{3}}{2} \rightarrow a = -\frac{1}{2} \end{array} \right\} \begin{array}{l} a = 3/2 \\ a = -1/2 \end{array} \quad \boxed{a = -1/2}$$

③⑥ Averigua "x" sabiendo que  $\frac{x i}{1+3i} - \frac{2x}{4-i} = 1$

$$x \left( \frac{i}{1+3i} - \frac{2}{4-i} \right) = 1$$

$$x \cdot \frac{i(4-i) - 2(1+3i)}{(1+3i)(4-i)} = 1 \Rightarrow x \cdot \frac{4i - i^2 - 2 - 6i}{4 - i + 12i - 3i^2} = 1 \Rightarrow$$

$$\Rightarrow x \cdot \frac{-1 - 2i}{7 + 11i} = 1 \Rightarrow x = \frac{7 + 11i}{-1 - 2i} \Rightarrow$$

$$\Rightarrow \boxed{x = \frac{(7 + 11i)(-1 + 2i)}{(-1 - 2i)(-1 + 2i)} = \frac{-7 + 14i - 11i + 22i^2}{(-1)^2 - (2i)^2} = \frac{-29 + 3i}{5}}$$

③⑦ Calcula "c" sabiendo que la representación gráfica de  $z = \frac{12+ci}{-5+2i}$  está sobre la bisectriz del primer cuadrante

$$z = \frac{(12+ci)(-5-2i)}{(-5+2i)(-5-2i)} = \frac{-60 - 24i - 5ci - 2ci^2}{25 - 4i^2} = \frac{-60 + 2c + (-24 - 5c)i}{29}$$

Si está sobre la bisectriz, su argumento será  $\alpha = 45^\circ$

$$\operatorname{tg} \alpha = \frac{\operatorname{Im}(z)}{\operatorname{Re}(z)} \Rightarrow \operatorname{tg} 45^\circ = \frac{-24 - 5c}{-60 + 2c} \Rightarrow 1 = \frac{-24 - 5c}{-60 + 2c} \Rightarrow$$

$$\Rightarrow -60 + 2c = -24 - 5c \Rightarrow 7c = 36 \Rightarrow \boxed{c = \frac{36}{7}}$$

38) Calcular "k" sabiendo que la representación gráfica de  $z = \frac{2+i}{k+i}$  está sobre la bisectriz del primer cuadrante:

$$z = \frac{2+i}{k+i} \cdot \frac{k-i}{k-i} = \frac{2k-2i+ki-i^2}{k^2-i^2} = \frac{2k+1+(k-2)i}{k^2+1} =$$

$$= \frac{2k+1}{k^2+1} + \frac{k-2}{k^2+1} i$$

Si está sobre la bisectriz, su argumento será  $\alpha = 45^\circ$

$$\operatorname{tg} \alpha = \frac{\operatorname{Im}(z)}{\operatorname{Re}(z)} \Rightarrow \operatorname{tg} 45 = \frac{\frac{k-2}{k^2+1}}{\frac{2k+1}{k^2+1}} \Rightarrow 1 = \frac{k-2}{2k+1} \Rightarrow$$

$$\Rightarrow 2k+1 = k-2 \Rightarrow \boxed{k = -3}$$

39) Halla el valor de "k" para que  $z = \frac{2-ki}{k-i}$  sea:

a) un número imaginario puro

b) un número real

$$z = \frac{2-ki}{k-i} \cdot \frac{k+i}{k+i} = \frac{2k+2i-k^2i-ki^2}{k^2-i^2} = \frac{3k+(2-k^2)i}{k^2+1} =$$

$$= \frac{3k}{k^2+1} + \frac{2-k^2}{k^2+1} i$$

a) Si tiene que ser imaginario puro  $\Rightarrow \operatorname{Re}(z) = 0$

$$\frac{3k}{k^2+1} = 0 \Rightarrow 3k=0 \Rightarrow \boxed{k=0}$$

b) Si tiene que ser un número real  $\Rightarrow \operatorname{Im}(z) = 0$

$$\frac{2-k^2}{k^2+1} = 0 \Rightarrow 2-k^2=0 \begin{cases} \rightarrow \boxed{k=\sqrt{2}} \\ \rightarrow \boxed{k=-\sqrt{2}} \end{cases}$$

40) Calcular las raíces indicadas, expresando los resultados en forma binómica:

$$A) \sqrt[4]{\frac{(2+i)(-1+3i)^2}{(\sqrt{2}-i\sqrt{2})^3}}$$

$$z_1 = 2+i \rightarrow \begin{cases} r = \sqrt{2^2+1^2} = \sqrt{5} \\ \alpha = \arctg\left(\frac{1}{2}\right) = 26'565^\circ \end{cases} \Rightarrow z_1 = \sqrt{5}_{26'565^\circ}$$

$$z_2 = -1+3i \rightarrow \begin{cases} r = \sqrt{1^2+3^2} = \sqrt{10} \\ \alpha = \arctg\left(\frac{3}{-1}\right) = 108'435^\circ \end{cases} \Rightarrow z_2 = \sqrt{10}_{108'435^\circ}$$

Ojo con el cuadrante!!

$$z_3 = \sqrt{2}-i\sqrt{2} \rightarrow \begin{cases} r = \sqrt{(\sqrt{2})^2+(\sqrt{2})^2} = 2 \\ \alpha = \arctg\left(\frac{-\sqrt{2}}{\sqrt{2}}\right) = 315^\circ \end{cases} \Rightarrow z_3 = 2_{315^\circ}$$

$$\Rightarrow \sqrt[4]{\frac{(2+i)(-1+3i)^2}{(\sqrt{2}-i\sqrt{2})^3}} = \sqrt[4]{\frac{\sqrt{5}_{26'56.5} \cdot (\sqrt{10}_{108'43.5})^2}{(2_{31.5})^3}} =$$

$$= \sqrt[4]{\frac{\sqrt{5}_{26'56.5} \cdot 10_{216'87}}{8_{22.5}}} = \sqrt[4]{\frac{10\sqrt{5}_{243'43.5}}{8_{22.5}}} = \sqrt[4]{\left(\frac{5\sqrt{5}}{4}\right)_{18'43.5}} =$$

$$= \left(\sqrt[4]{\frac{5\sqrt{5}}{4}}\right)_{\frac{18'43.5^\circ + k \cdot 360^\circ}{4}} = \left(\sqrt[4]{\frac{5\sqrt{5}}{4}}\right)_{4'61^\circ + k \cdot 90^\circ} \text{ con } k = 0, 1, 2, 3$$

$$\Rightarrow \left\{ \begin{array}{l} k=0 \rightarrow z_1 = \left(\sqrt[4]{\frac{5\sqrt{5}}{4}}\right)_{4'61^\circ} = 1'293 (\cos 4'61^\circ + i \operatorname{sen} 4'61^\circ) = \\ \quad \quad \quad = 1'2888 + 0'1039i \\ k=1 \rightarrow z_2 = \left(\sqrt[4]{\frac{5\sqrt{5}}{4}}\right)_{94'61^\circ} = 1'293 (\cos 94'61^\circ + i \operatorname{sen} 94'61^\circ) = \\ \quad \quad \quad = -0'1039 + 1'2888i \\ k=2 \rightarrow z_3 = \left(\sqrt[4]{\frac{5\sqrt{5}}{4}}\right)_{184'61^\circ} = 1'293 (\cos 184'61^\circ + i \operatorname{sen} 184'61^\circ) = \\ \quad \quad \quad = -1'2888 - 0'1039i \\ k=3 \rightarrow z_4 = \left(\sqrt[4]{\frac{5\sqrt{5}}{4}}\right)_{274'61^\circ} = 1'293 (\cos 274'61^\circ + i \operatorname{sen} 274'61^\circ) = \\ \quad \quad \quad = 0'1039 - 1'2888i \end{array} \right.$$

$$B) \sqrt[4]{\frac{(2-3i)-(-1+3i)}{(-\sqrt{3}-i)^4}} = \sqrt[4]{\frac{3-6i}{(-\sqrt{3}-i)^4}} = \textcircled{*}$$

$$z_1 = 3-6i \rightarrow \begin{cases} r = \sqrt{3^2+6^2} = \sqrt{45} = 3\sqrt{5} \\ \alpha = \arctg\left(-\frac{6}{3}\right) = 296'565^\circ \end{cases} \Rightarrow z_1 = 3\sqrt{5}_{296'565^\circ}$$

$$z_2 = -\sqrt{3}-i \rightarrow \begin{cases} r = \sqrt{(\sqrt{3})^2+1^2} = 2 \\ \alpha = \arctg\left(\frac{-1}{-\sqrt{3}}\right) = 210^\circ \end{cases} \Rightarrow z_2 = 2_{210^\circ}$$

ojo con el cuadrante!!

$$\textcircled{*} = \sqrt[4]{\frac{3\sqrt{5}_{296'565}}{(2_{210})^4}} = \sqrt[4]{\frac{3\sqrt{5}_{296'565}}{16_{120}}} = \sqrt[4]{\left(\frac{3\sqrt{5}}{16}\right)_{176'565}} = \left(\sqrt[4]{\frac{3\sqrt{5}}{16}}\right)_{\frac{176'565+k \cdot 360}{4}}$$

$$= \left(\sqrt[4]{\frac{3\sqrt{5}}{16}}\right)_{44'14^\circ + k \cdot 90^\circ} \text{ con } k = 0, 1, 2, 3$$

$$k=0 \rightarrow z_1 = \left(\sqrt[4]{\frac{3\sqrt{5}}{16}}\right)_{44'14^\circ} = 0'805(\cos 44'14 + i \sin 44'14) = 0'578 + 0'561i$$

$$\Rightarrow k=1 \rightarrow z_2 = \left(\sqrt[4]{\frac{3\sqrt{5}}{16}}\right)_{134'14^\circ} = -0'561 + 0'578i$$

$$k=2 \rightarrow z_3 = \left(\sqrt[4]{\frac{3\sqrt{5}}{16}}\right)_{224'14^\circ} = -0'578 - 0'561i$$

$$k=3 \rightarrow z_4 = \left(\sqrt[4]{\frac{3\sqrt{5}}{16}}\right)_{314'14^\circ} = 0'561 - 0'578i$$