OPCIÓN B

B1) Encuentra los valores de $t \in \mathbb{R}$ para los que el determinante de la matriz AB vale 0, siendo

$$A = \begin{pmatrix} 2 & -1 & 3 \\ 0 & t & 2 \\ 0 & 1+t & 3 \end{pmatrix} \quad \text{y} \quad B = \begin{pmatrix} 2+t & -1 & 0 \\ 1 & t & 0 \\ 4 & 7 & t \end{pmatrix}$$

(2 puntos)

$$|AB| = 0 \Rightarrow \begin{cases} |A| = 0 \Rightarrow |A| = \begin{vmatrix} 2 & -1 & 3 \\ 0 & t & 2 \\ 0 & 1+t & 3 \end{vmatrix} = 2 \cdot \begin{vmatrix} t & 2 \\ 1+t & 3 \end{vmatrix} = 2 \cdot (3t - 2 - 2t) = 2 \cdot (t-2) \Rightarrow 2 \cdot (t-2) = 0 \Rightarrow t = 2 \\ |B| = 0 \Rightarrow |A| = \begin{vmatrix} 2+t & -1 & 0 \\ 1 & t & 0 \\ 4 & 7 & t \end{vmatrix} = t \cdot \begin{vmatrix} 2+t & -1 \\ 1 & t \end{vmatrix} = t \cdot (2t + t^2 + 1) = t \cdot (t+1)^2 \Rightarrow \begin{cases} t+1 = 0 \Rightarrow t = -1 \\ t = 0 \end{cases} \end{cases}$$

B2) Dados los puntos $P \equiv (1,2,-1)$, $Q \equiv (2,-1,1)$ y $R \equiv (3,1,2)$, encuentra todos los posibles puntos S tales que P, Q, R y S son los vértices de un paralelogramo.

(3 puntos)

Supongamos que **PS** es paralelo a **QR**

$$\begin{cases} \overrightarrow{QR} = (3,1,2) - (2,-1,1) = (1,2,1) \\ \overrightarrow{PS} = (x,y,z) - (1,2,-1) = (x-1, y-2, z+1) \end{cases} \Rightarrow (x-1, y-2, z+1) = \pm (1,2,1) \Rightarrow \begin{cases} \begin{cases} x-1 = 1 \Rightarrow x = 2 \\ y-2 = 2 \Rightarrow y = 4 \\ z+1 = 1 \Rightarrow z = 0 \end{cases} \\ \begin{cases} x-1 = -1 \Rightarrow x = 0 \\ y-2 = -2 \Rightarrow y = 0 \\ z+1 = -1 \Rightarrow z = -2 \end{cases} \end{cases}$$

$$\begin{cases} S_1 = (2,4,0) \\ S_2 = (0,0,-2) \end{cases} \Rightarrow \begin{cases} \begin{cases} \overrightarrow{RS_1} = (3,1,2) - (2,4,0) = (1,-3,2) \\ \overrightarrow{PQ} = (1,-3,2) \end{cases} \Rightarrow \text{Paralelismo } RS \text{ y } PQ \\ \begin{cases} \overrightarrow{QS_2} = (2,-1,1) - (0,0,-2) = (2,-1,3) \\ \overrightarrow{PR} = (2,-1,3) \end{cases} \Rightarrow \text{Paralelismo } QS \text{ y } PR \end{cases}$$

Continuación Problema B.2 de la opción B

Supongamos que **QS** es paralelo a **PR**

$$\begin{cases} \overrightarrow{PR} = (3,1,2)-(1,2,-1)=(2,-1,3) \\ \overrightarrow{QS} = (x,y,z)-(2,-1,1)=(x-2,y+1,z-1) \end{cases} \Rightarrow (x-2,y+1,z-1)=\pm(2,-1,3) \Rightarrow \begin{cases} \begin{cases} x-2=2 \Rightarrow x=4 \\ y+1=-1 \Rightarrow y=-2 \\ z-1=3 \Rightarrow z=4 \end{cases} \\ \begin{cases} x-2=-2 \Rightarrow x=0 \\ y+1=1 \Rightarrow y=0 \\ z-1=-3 \Rightarrow z=-2 \end{cases} \end{cases}$$

$$\begin{cases} S_3 = (4,-2,4) \\ S_4 = S_2 = (0,0,-2) \end{cases}$$

Supongamos que **RS** es paralelo a **PQ**

$$\begin{cases} \overrightarrow{PQ} = (2,-1,1)-(1,2,-1)=(1,-3,2) \\ \overrightarrow{QS} = (x,y,z)-(1,2,-1)=(x-1,y-2,z+1) \end{cases} \Rightarrow (x-1,y-2,z+1)=\pm(1,-3,2) \Rightarrow \begin{cases} \begin{cases} x-1=1 \Rightarrow x=2 \\ y-2=-3 \Rightarrow y=-1 \\ z+1=2 \Rightarrow z=3 \end{cases} \\ \begin{cases} x-1=-1 \Rightarrow x=0 \\ y-2=3 \Rightarrow y=5 \\ z+1=-2 \Rightarrow z=-3 \end{cases} \end{cases}$$

$$\begin{cases} S_5 = (2,-1,3) \\ S_6 = (0,5,-3) \end{cases}$$

B3) Calcula los siguientes límites:

$$\lim_{x\to+\infty} \left(\sqrt{5x^2+4x-1} - \sqrt{5x^2-6x}\right) \qquad \text{(1 punto)}$$

$$\lim_{x\to+\infty} \left(\frac{x^2+2x+1}{x^2+3}\right)^{3x-1} \qquad \text{(1 punto)}$$

$a)$

$$\lim_{x\to\infty}\left(\sqrt{5x^2+4x-1}-\sqrt{5x^2-6x}\right)=\infty-\infty=\lim_{x\to\infty}\frac{\left(\sqrt{5x^2+4x-1}-\sqrt{5x^2-6x}\right)\left(\sqrt{5x^2+4x-1}+\sqrt{5x^2-6x}\right)}{\sqrt{5x^2+4x-1}+\sqrt{5x^2-6x}}=$$

$$\lim_{x\to\infty}\frac{\left(5x^2+4x-1\right)-\left(5x^2-6x\right)}{\sqrt{5x^2+4x-1}+\sqrt{5x^2-6x}}=\lim_{x\to\infty}\frac{5x^2+4x-1-5x^2+6x}{\sqrt{5x^2+4x-1}+\sqrt{5x^2-6x}}=\lim_{x\to\infty}\frac{10x-1}{\sqrt{5x^2+4x-1}+\sqrt{5x^2-6x}}=$$

$$=\lim_{x\to\infty}\frac{10x-1}{\sqrt{5x^2+4x-1}+\sqrt{5x^2-6x}}=\frac{\infty}{\infty}=\lim_{x\to\infty}\frac{10\frac{x}{x}-\frac{1}{x}}{\sqrt{5\frac{x^2}{x^2}+4\frac{x}{x^2}-\frac{1}{x^2}}+\sqrt{5\frac{x^2}{x^2}-6\frac{x}{x^2}}}=$$

$$=\lim_{x\to\infty}\frac{10-\frac{1}{x}}{\sqrt{5+\frac{4}{x}-\frac{1}{x^2}}+\sqrt{5-\frac{6}{x}}}=\frac{10-\frac{1}{\infty}}{\sqrt{5+\frac{4}{\infty}-\frac{1}{\infty}}+\sqrt{5-\frac{6}{\infty}}}=\frac{10-0}{\sqrt{5+0-0}+\sqrt{5-0}}=\frac{10}{2\sqrt{5}}=\frac{5}{\sqrt{5}}=\sqrt{5}$$

Continuación Problema B.3 de la opción B

$b)$

$$\textit{Sabiendo que} \Rightarrow \begin{cases} \lim_{x\to\infty}(3x+1)=\infty \\ \lim_{x\to\infty}\frac{x^2+2x+1}{x^2+3}=\frac{\infty}{\infty}=\lim_{x\to\infty}\frac{\frac{x^2}{x^2}+2\frac{x}{x^2}+\frac{1}{x^2}}{\frac{x^2}{x^2}+\frac{3}{x^2}}=\lim_{x\to\infty}\frac{1+\frac{2}{x}+\frac{1}{x^2}}{1+\frac{3}{x^2}}=\frac{1+\frac{2}{\infty}+\frac{1}{\infty}}{1+\frac{3}{\infty}}=\frac{1+0+0}{1+0}=1 \end{cases}$$

$$\lim_{x\to\infty}\left(\frac{x^2+2x+1}{x^2+3}\right)^{3x+1}=1^{\infty}=\lim_{x\to\infty}\left(\frac{x^2+3+2x+1-3}{x^2+3}\right)^{3x+1}=\lim_{x\to\infty}\left(\frac{x^2+3}{x^2+3}+\frac{2x-2}{x^2+3}\right)^{3x+1}=\lim_{x\to\infty}\left(1+\frac{1}{\frac{x^2+3}{2x-2}}\right)^{3x+1}=$$

$$=\lim_{x\to\infty}\left[\left(1+\frac{1}{\frac{x^2+3}{2x-2}}\right)^{\frac{x^2+3}{2x-2}}\right]^{(3x+1)\cdot\frac{(2x-2)}{x^2+3}}=e^{\lim_{x\to\infty}\frac{6x^2-4x-2}{x^2+3}}=e^6 \quad De\ (1)$$

$$\lim_{x\to\infty}\frac{6x^2-4x-2}{x^2+3}=\frac{\infty}{\infty}=\lim_{x\to\infty}\frac{6\frac{x^2}{x^2}-4\frac{x}{x^2}-\frac{2}{x^2}}{\frac{x^2}{x^2}+\frac{3}{x^2}}=\lim_{x\to\infty}\frac{6-\frac{4}{x}-\frac{2}{x^2}}{1+\frac{3}{x^2}}=\frac{6-\frac{4}{\infty}-\frac{2}{\infty}}{1+\frac{3}{\infty}}=\frac{6-0-0}{1+0}=6 \quad (1)$$

B4) Demuestra que existen $\alpha \in (-1,1)$ y $\beta \in (-1,1)$, $\alpha \neq \beta$, tales que $f'(\alpha) = f'(\beta) = 0$, siendo

$$f(x) = (x^3+1)\, e^{\sqrt[3]{3x+2}} \sqrt[3]{(x-1)\text{sen}\left(\frac{\pi}{2}x\right)}$$

(3 puntos)

Teorema del valor medio o de Lagrange

Si **f(x)** es continua en **[a , b]** y derivable en **(a , b)**, entonces existe, al menos, un punto $c \in (a\,,\,b)$ tal que: $f(b) - f(a) = f'(c)\,(b-a) \Rightarrow f'(c) = \dfrac{f(b)-f(a)}{b-a}$

La función derivada es continua en **[-1 , 0]** y derivada en **(-1 , 0)**, entonces al menos un punto $\alpha \in (-1\,,\,0)$ tal que: $f(0) - f(-1) = f'(\alpha)\,[0-(-1)] \Rightarrow f'(\alpha) = \dfrac{f(0)-f(-1)}{0+1}$

$$\begin{cases} f(-1) = \left[(-1)^3+1\right] e^{\sqrt[3]{3\cdot(1)+2}} \sqrt[3]{[(-1)-1]\cdot sen\left[\frac{\pi}{2}(-1)\right]} = (-1+1) e^{\sqrt[3]{-1}} \sqrt[3]{(-2)\cdot sen\left(-\frac{\pi}{2}\right)} = 0\cdot e^{-1}\sqrt[3]{(-2)\cdot 0} \\ f(0) = \left[0^3+1\right] e^{\sqrt[3]{3\cdot 0+2}} \sqrt[3]{[0-1]\cdot sen\left(\frac{\pi}{2}\cdot 0\right)} = e^{\sqrt[3]{2}} \sqrt[3]{(-1)\cdot sen\,0} = e^{\sqrt[3]{2}} \sqrt[3]{(-1)\cdot 0} = 0 \Rightarrow \end{cases}$$

$$f'(\alpha) = \frac{f(0)-f(-1)}{1} = \frac{0-0}{1} = \frac{0}{1} = 0$$

La función derivada es continua en **[0 , 1]** y derivada en **(0 , 1)**, entonces al menos un punto $\beta \in (0\,,\,1)$ tal que: $f(1) - f(0) = f'(\beta)\,[1-0] \Rightarrow f'(\beta) = \dfrac{f(0)-f(-1)}{1-0}$

$$\begin{cases} f(1) = \left[1^3+1\right] e^{\sqrt[3]{3\cdot 1+2}} \sqrt[3]{[1-1]\cdot sen\left(\frac{\pi}{2}\cdot 1\right)} = (1+1) e^{\sqrt[3]{5}} \sqrt[3]{0\cdot sen\left(\frac{\pi}{2}\right)} = 2\cdot e^{\sqrt[3]{5}}\sqrt[3]{0\cdot 0} = 0 \\ f(0) = \left[0^3+1\right] e^{\sqrt[3]{3\cdot 0+2}} \sqrt[3]{[0-1]\cdot sen\left(\frac{\pi}{2}\cdot 0\right)} = e^{\sqrt[3]{2}} \sqrt[3]{(-1)\cdot sen\,0} = e^{\sqrt[3]{2}} \sqrt[3]{(-1)\cdot 0} = 0 \Rightarrow \end{cases}$$

$$f'(\beta) = \frac{f(1)-f(0)}{1} = \frac{0-0}{1} = \frac{0}{1} = 0\ con\ \alpha \neq \beta$$

Opción A

A1) Estudia el siguiente sistema de ecuaciones lineales dependiente del parámetro real a y resuélvelo en los casos en que es compatible:

$$\begin{cases} ax - y = 0 \\ -2ax + a^2y + az = -2a \\ -ax + (a^2-1)y + (a+1)z = -a-2 \end{cases}$$

(3 puntos)

$$|A| = \begin{vmatrix} a & -1 & 0 \\ -2a & a^2 & a \\ -a & a^2-1 & a+1 \end{vmatrix} = \begin{vmatrix} a & -1 & 0 \\ 0 & a^2-2 & a \\ 0 & a^2-2 & a+1 \end{vmatrix} = a \cdot \begin{vmatrix} a^2-2 & a \\ a^2-2 & a+1 \end{vmatrix} = a \cdot \left[(a^2-2)(a+1) - (a^2-2)a \right]$$

$$|A| = a \cdot \left[(a^2-2)(a+1-a) \right] = a \cdot (a^2-2) \Rightarrow Si\ |A| = 0 \Rightarrow a \cdot (a^2-2) = 0 \Rightarrow a \cdot (a-\sqrt{2}) \cdot (a+\sqrt{2}) = 0 \Rightarrow$$

$$\begin{cases} a = 0 \\ a - \sqrt{2} = 0 \Rightarrow a = \sqrt{2} \\ a + \sqrt{2} = 0 \Rightarrow a = -\sqrt{2} \end{cases} \Rightarrow$$

$$\forall a \in \Re - \left\{ -\sqrt{2}, 0, \sqrt{2} \right\} \Rightarrow |A| \neq 0 \Rightarrow rang\ (A) = 3 = Numero\ de\ incógnitas \Rightarrow Sist.Compatible\ Determ.$$

$Si\ a = -\sqrt{2}$

$$\left(\begin{array}{ccc|c} -\sqrt{2} & -1 & 0 & 0 \\ 2\sqrt{2} & 2 & -\sqrt{2} & 2\sqrt{2} \\ \sqrt{2} & 1 & -\sqrt{2}+1 & \sqrt{2}-2 \end{array} \right) \equiv \left(\begin{array}{ccc|c} -\sqrt{2} & -1 & 0 & 0 \\ 0 & 0 & -\sqrt{2} & 2\sqrt{2} \\ 0 & 0 & -\sqrt{2}+1 & \sqrt{2}-2 \end{array} \right) \equiv \left(\begin{array}{ccc|c} -\sqrt{2} & -1 & 0 & 0 \\ 0 & 0 & -\sqrt{2} & 2\sqrt{2} \\ 0 & 0 & 1 & -\sqrt{2}-2 \end{array} \right) \equiv$$

$$\left(\begin{array}{ccc|c} -\sqrt{2} & -1 & 0 & 0 \\ 0 & 0 & 0 & 2\sqrt{2}-2-2\sqrt{2} \\ 0 & 0 & 1 & -\sqrt{2}-2 \end{array} \right) \equiv \left(\begin{array}{ccc|c} -\sqrt{2} & -1 & 0 & 0 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 1 & -\sqrt{2}-2 \end{array} \right) \Rightarrow rang\ (A) = 2 \neq rang\ (A/B) = 3$$

Sistema Incompatible

$Si\ a = 0$

$$\left(\begin{array}{ccc|c} 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & -2 \end{array} \right) \Rightarrow rang\ (A) = rang\ (A/B) = 2 \Rightarrow Sist.Compatible\ Indeterminado$$

Continuación Problema A.1 de la opción A

$Si\ a=\sqrt{2}$

$$\left(\begin{array}{ccc|c} \sqrt{2} & -1 & 0 & 0 \\ -2\sqrt{2} & 2 & \sqrt{2} & -2\sqrt{2} \\ -\sqrt{2} & 1 & \sqrt{2}+1 & -\sqrt{2}-2 \end{array}\right) \equiv \left(\begin{array}{ccc|c} \sqrt{2} & -1 & 0 & 0 \\ 0 & 0 & \sqrt{2} & -2\sqrt{2} \\ 0 & 0 & \sqrt{2}+1 & -\sqrt{2}-2 \end{array}\right) \equiv \left(\begin{array}{ccc|c} \sqrt{2} & -1 & 0 & 0 \\ 0 & 0 & \sqrt{2} & -2\sqrt{2} \\ 0 & 0 & 1 & \sqrt{2}-2 \end{array}\right) \equiv$$

$$\equiv \left(\begin{array}{ccc|c} \sqrt{2} & -1 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & \sqrt{2}-2 \end{array}\right) \Rightarrow rang\ (A)=2 \neq rang\ (A/B)=3 \Rightarrow Sistema\ Incompatible$$

$Si\ a=0 \Rightarrow Sistema\ Compatible\ In\det er\min ado$

$$\left(\begin{array}{ccc|c} 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & -2 \end{array}\right) \Rightarrow -y=0 \Rightarrow y=0 \Rightarrow -0+z=-2 \Rightarrow Solución \Rightarrow (x\ ,\ y\ ,\ z)=(\lambda\ ,\ 0\ ,\ -2)$$

$\forall a \in \Re - \{-\sqrt{2}\ ,\ 0\ ,\ \sqrt{2}\} \Rightarrow Sistema\ Compatible\ Deter\min ado$

$$\left(\begin{array}{ccc|c} a & -1 & 0 & 0 \\ 0 & a^2-2 & a & -2a \\ 0 & a^2-2 & a+1 & -a-2 \end{array}\right) \equiv \left(\begin{array}{ccc|c} a & -1 & 0 & 0 \\ 0 & a^2-2 & a & -2a \\ 0 & 0 & 1 & a-2 \end{array}\right) \Rightarrow z=a-2 \Rightarrow \left(a^2-2\right)y+a\cdot(a-2)=-2a \Rightarrow$$

$$\left(a^2-2\right)y=-2a-a^2+2a \Rightarrow y=-\frac{a^2}{a^2-2} \Rightarrow ax+\frac{a^2}{a^2-2}=0 \Rightarrow ax=-\frac{a^2}{a^2-2} \Rightarrow x=-\frac{a^2}{a\left(a^2-2\right)}$$

$$\Rightarrow x=-\frac{a}{a^2-2} \Rightarrow Solución \Rightarrow (x\ ,\ y\ ,\ z)=\left(a-2\ ,\ -\frac{a^2}{a^2-2}\ ,\ -\frac{a}{a^2-2}\right)$$

A2) Encuentra la ecuación continua de la recta que pasa por el punto $P \equiv (1,-2,3)$ y corta perpendicularmente a la recta

$$r \equiv \begin{cases} x+y+z-4=0 \\ 3x+y-3z-2=0 \end{cases}$$

(2 puntos)

Hallaremos un plano π que conteniendo al punto **P** sea perpendicular a la recta **r**, debido a ello el vector director del plano es el de la recta. El producto escalar del vector director del plano y del vector, perpendicular a él, **PG**, donde G es el punto genérico del plano, es nulo y la ecuación pedida del plano Una vez hallado el plano calcularemos el punto **Q** intersección de recta y plano. Los dos puntos, P y Q, determinan la recta **s** pedida.

$$\begin{cases} x+y+z-4=0 \\ -3x-y+3z+2=0 \end{cases} \Rightarrow -2x+4z-2=0 \Rightarrow 2x=4z-2 \Rightarrow x=-1+2z \Rightarrow -1+2z+y+z-4=0 \Rightarrow$$

$$y=5-3z \Rightarrow r \equiv \begin{cases} x=-1+2\lambda \\ y=5-3\lambda \\ z=\lambda \end{cases} \Rightarrow \begin{cases} \overrightarrow{v_\pi}=\overrightarrow{v_r}=(2\,,-3\,,1) \\ \overrightarrow{PG}=(x\,,\,y\,,\,z)-(1\,,-2\,,3)=(x-1\,,\,y+2\,,\,z-3) \end{cases} \Rightarrow \overrightarrow{v_\pi} \perp \overrightarrow{PG} \Rightarrow$$

$$\overrightarrow{v_\pi}\cdot\overrightarrow{PG}=0 \Rightarrow (2\,,-3\,,1)\cdot(x-1\,,\,y+2\,,\,z-3)=0 \Rightarrow 2\,(x-1)-3\,(y+2)+(z-3)=0 \Rightarrow$$

$$\pi \equiv 2x-3y+z-11=0$$

Intersección

$$2\,(-1+2\lambda)-3\,(5-3\lambda)+\lambda-11=0 \Rightarrow -2+4\lambda-15+9\lambda+\lambda-11=0 \Rightarrow 14\lambda-28=0 \Rightarrow 14\lambda=28 \Rightarrow$$

$$\lambda=2 \Rightarrow Q \begin{cases} x=-1+2\cdot 2 \\ y=5-3\cdot 2 \\ z=2 \end{cases} \Rightarrow Q(3\,,-1\,,2) \Rightarrow \overrightarrow{v_s}=\overrightarrow{PQ}=(3\,,-1\,,2)-(1\,,-2\,,3)=(2\,,1\,,-1) \Rightarrow$$

$$s \equiv \begin{cases} x=1+2\gamma \\ y=-2+\gamma \\ z=3-\gamma \end{cases}$$

A3) Halla las asíntotas de la función

$$f(x)=\frac{2x^2-1}{x-2}$$

(2 puntos)

$$x-2=0 \Rightarrow f(2)=\frac{2\cdot 2^2-1}{2-2}=\frac{7}{0} \Rightarrow \textit{Sin solución} \Rightarrow$$

Asíntota vertical $\Rightarrow x=2$

Asíntotas horizontales

$$y=\lim_{x\to\infty} f(x)=\lim_{x\to\infty}\frac{2x^2-1}{x-2}=\frac{\infty}{\infty}=\lim_{x\to\infty}\frac{2\frac{x^2}{x^2}-\frac{1}{x^2}}{\frac{x}{x^2}-\frac{2}{x^2}}=\lim_{x\to\infty}\frac{2-\frac{1}{x^2}}{\frac{1}{x}-\frac{2}{x^2}}=\frac{2-\frac{1}{\infty}}{\frac{1}{\infty}-\frac{2}{\infty}}=\frac{2-0}{0-0}=\frac{2}{0} \Rightarrow \textit{No existe}$$

No existe asíntota horizontal cuando $x \to \infty$

Continuación Problema A.3 de la opción A

Asíntotas horizontales (Continuación)

$$y=\lim_{x\to-\infty} f(x)=\lim_{x\to-\infty}\frac{2x^2-1}{x-2}=\frac{\infty}{-\infty}=\xrightarrow{\text{Utilixando L'Hopital}}=\lim_{x\to-\infty}\frac{4x}{1}=\lim_{x\to-\infty}4x=4\cdot(-\infty)=-\infty$$

No existe asíntota horizontal cuando $x\to-\infty$

Asíntotas oblicuas

$$m=\lim_{x\to\infty}\frac{f(x)}{x}=\lim_{x\to\infty}\frac{\frac{2x^2-1}{x-2}}{x}=\lim_{x\to\infty}\frac{2x^2-1}{x^2-2x}=\frac{\infty}{\infty}=\lim_{x\to\infty}\frac{2\frac{x^2}{x^2}-\frac{1}{x^2}}{\frac{x^2}{x^2}-2\frac{x}{x^2}}=\lim_{x\to\infty}\frac{2-\frac{1}{x^2}}{1-\frac{2}{x}}=\frac{2-\frac{1}{\infty}}{1-\frac{2}{\infty}}=\frac{2-0}{1-0}=2$$

$$n=\lim_{x\to\infty}[f(x)-mx]=\lim_{x\to\infty}\left[\frac{2x^2-1}{x-2}-2\cdot x\right]=\lim_{x\to\infty}\frac{2x^2-1-2x^2+4x}{x-2}=\lim_{x\to\infty}\frac{4x-1}{x-2}=\frac{\infty}{\infty}=\lim_{x\to\infty}\frac{4\frac{x}{x}-\frac{1}{x}}{\frac{x}{x}-\frac{2}{x}}=$$

$$=\lim_{x\to\infty}\frac{4-\frac{1}{x}}{1-\frac{2}{x}}=\frac{4-\frac{1}{\infty}}{1-\frac{2}{\infty}}=\frac{4-0}{1-0}=4\Rightarrow \textit{Existe asíntota oblicua, } y=2x+4 \textit{ cuando } x\to\infty$$

$$m=\lim_{x\to-\infty}\frac{f(x)}{x}=\lim_{x\to-\infty}\frac{\frac{2x^2-1}{x-2}}{x}=\lim_{x\to-\infty}\frac{2x^2-1}{x^2-2x}=\frac{\infty}{\infty}=\lim_{x\to-\infty}\frac{2\frac{x^2}{x^2}-\frac{1}{x^2}}{\frac{x^2}{x^2}-2\frac{x}{x^2}}=\lim_{x\to-\infty}\frac{2-\frac{1}{x^2}}{1-\frac{2}{x}}=\frac{2-\frac{1}{\infty}}{1-\frac{2}{-\infty}}=\frac{2-0}{1-0}=2$$

$$n=\lim_{x\to-\infty}[f(x)-mx]=\lim_{x\to-\infty}\left[\frac{2x^2-1}{x-2}-2\cdot x\right]=\lim_{x\to-\infty}\frac{2x^2-1-2x^2+4x}{x-2}=\lim_{x\to-\infty}\frac{4x-1}{x-2}=\frac{\infty}{\infty}=\lim_{x\to-\infty}\frac{4\frac{x}{x}-\frac{1}{x}}{\frac{x}{x}-\frac{2}{x}}=$$

$$=\lim_{x\to-\infty}\frac{4-\frac{1}{x}}{1-\frac{2}{x}}=\frac{4-\frac{1}{-\infty}}{1-\frac{2}{-\infty}}=\frac{4-0}{1-0}=4\Rightarrow \textit{Existe asíntota oblicua, } y=2x+4 \textit{ cuando } x\to-\infty$$

A4) Dadas las funciones $f(x) = \text{sen}(\frac{\pi}{2}x)\cos(\frac{\pi}{2}x)$ y $g(x) = 4 - 4x^2$, encuentra los dos puntos en que se cortan. Calcula el área de la región del plano encerrada entre ambas curvas.

(3 puntos)

$$\begin{cases} sen\left(\frac{\pi}{2}x\right)\cdot\cos\left(\frac{\pi}{2}x\right)=0 \Rightarrow \begin{cases} sen\left(\frac{\pi}{2}x\right)=0 \Rightarrow \frac{\pi}{2}x = 0+\pi k \Rightarrow x = 0+2k\ ,\ k\in Z \\ \cos\left(\frac{\pi}{2}x\right)=0 \Rightarrow \frac{\pi}{2}x=\frac{\pi}{2}+\pi k \Rightarrow x = 1+2k\ ,\ k\in Z \end{cases} \\ 4-4x^2=0 \Rightarrow 4x^2=4 \Rightarrow x^2=\frac{4}{4}=1 \Rightarrow x=\pm\sqrt{1} \Rightarrow \begin{cases} x=-1 \\ x=1 \end{cases} \end{cases}$$

$$x=1 \Rightarrow \begin{cases} 1=0+2k \Rightarrow k=\frac{1}{2}\notin Z \\ 1=1+2k \Rightarrow k=0\in Z \end{cases} \Rightarrow \begin{cases} sen\left(\frac{\pi}{2}\right)\cdot\cos\left(\frac{\pi}{2}\right)=0\cdot 1=0 \\ 4-4\cdot 1^2=0 \end{cases} \Rightarrow \textit{Punto de corte}$$

$$x=-1 \Rightarrow \begin{cases} -1=0+2k \Rightarrow k=-\frac{1}{2}\notin Z \\ -1=1+2k \Rightarrow k=-1\in Z \end{cases} \Rightarrow \begin{cases} sen\left(-\frac{\pi}{2}\right)\cdot\cos\left(-\frac{\pi}{2}\right)=0\cdot 1=0 \\ 4-4\cdot(-1)^2=0 \end{cases} \Rightarrow \textit{Punto de corte}$$

$$x=-\frac{1}{2}\in(-1\ ,\ 0) \Rightarrow \begin{cases} f\left(-\frac{1}{2}\right)=sen\left[\frac{\pi}{2}\left(-\frac{1}{2}\right)\right]\cdot\cos\left[\frac{\pi}{2}\left(-\frac{1}{2}\right)\right]=sen\left(-\frac{\pi}{4}\right)\cdot\cos\left(-\frac{\pi}{4}\right)=-\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}=-\frac{1}{2}<0 \\ g\left(-\frac{1}{2}\right)=4-4\cdot\left(-\frac{1}{2}\right)^2=4-4\cdot\frac{1}{4}=3>0 \end{cases}$$

$$x=\frac{1}{2}\in(0\ ,\ 1) \Rightarrow \begin{cases} f\left(\frac{1}{2}\right)=sen\left[\frac{\pi}{2}\left(\frac{1}{2}\right)\right]\cdot\cos\left[\frac{\pi}{2}\left(\frac{1}{2}\right)\right]=sen\left(\frac{\pi}{4}\right)\cdot\cos\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}=\frac{1}{2}>0 \\ g\left(\frac{1}{2}\right)=4-4\cdot\left(\frac{1}{2}\right)^2=4-4\cdot\frac{1}{4}=3>0 \end{cases}$$

$$A=\int_{-1}^{0}\left(4-4x^2\right)dx+\left|\int_{-1}^{0}sen\left(\frac{\pi}{2}x\right)\cdot\cos\left(\frac{\pi}{2}x\right)dx\right|+\int_{0}^{1}\left(4-4x^2\right)dx-\int_{0}^{1}sen\left(\frac{\pi}{2}x\right)\cdot\cos\left(\frac{\pi}{2}x\right)dx$$

$$A=\int_{-1}^{0}\left(4-4x^2\right)dx-\int_{-1}^{0}sen\left(\frac{\pi}{2}x\right)\cdot\cos\left(\frac{\pi}{2}x\right)dx+\int_{0}^{1}\left(4-4x^2\right)dx-\int_{0}^{1}sen\left(\frac{\pi}{2}x\right)\cdot\cos\left(\frac{\pi}{2}x\right)dx$$

$$A=\int_{-1}^{1}\left(4-4x^2\right)dx-\int_{-1}^{1}sen\left(\frac{\pi}{2}x\right)\cdot\cos\left(\frac{\pi}{2}x\right)dx=4\cdot[x]_{-1}^{1}-4\cdot\frac{1}{3}\cdot\left[x^3\right]_{-1}^{1}-\int_{-1}^{1}t\cdot\frac{2}{\pi}\,dt$$

$$sen\left(\frac{\pi}{2}x\right)=t \Rightarrow \frac{\pi}{2}\cos\left(\frac{\pi}{2}x\right)=dt \Rightarrow \cos\left(\frac{\pi}{2}x\right)=\frac{2}{\pi}dt \Rightarrow \begin{cases} x=1 \Rightarrow t=sen\left(\frac{\pi}{2}\right)=1 \\ x=-1 \Rightarrow t=sen\left(-\frac{\pi}{2}\right)=-1 \end{cases}$$

$$A=4\cdot[1-(-1)]-\frac{4}{3}\cdot\left[1^3-(-1)^3\right]-\int_{-1}^{1}t\cdot\frac{2}{\pi}\,dt-\frac{2}{\pi}\cdot\frac{1}{2}\cdot\left[t^2\right]_{-1}^{1}=4\cdot(1+1)-\frac{4}{3}\cdot(1+1)-\frac{1}{\pi}\left[1^2-(-1)^2\right]=8-\frac{4}{3}-\frac{1}{\pi}\cdot 0$$

$$A=\frac{16}{3}\,u^2$$