

EJERCICIOS DE APLICACIONES. LA INTEGRAL INDEFINIDA.

1.- Calcula las siguientes integrales inmediatas.

- a) $\int -4(3-2x^2)dx$; b) $\int 2x\sqrt{x^2-1} dx$ c) $\int x(1+2x^2)^3 dx$
d) $\int x^2(2-2x^3)^3 dx$ e) $\int \sqrt{7+2x} dx$ f) $\int (x^2+1)^2 dx$
g) $\int \frac{2x}{x^2+3} dx$ h) $\int \frac{2}{x-3} dx$ i) $\int \frac{x}{(3x^2-2)^3} dx$
j) $\int \frac{x}{\sqrt[3]{2-x^2}} dx$ k) $\int (e^{-3x} + e^{x-2}) dx$ l) $\int \frac{3^{5x-1}}{7} dx$
m) $\int \frac{x}{e^{x^2}} dx$ n) $\int \sin(-x) dx$ ñ) $\int \frac{1}{\cos^2(x+1)} dx$
o) $\int \frac{x}{\cos^2(x^2-3)} dx$ p) $\int \frac{1}{\sqrt{1-25x^2}} dx$ q) $\int \frac{1}{\sqrt{1-(2x-3)^2}} dx$
r) $\int \frac{x}{1+9x^4} dx$

Resuelve las siguientes integrales (son por partes)

- a) $\int \ln x dx$ b) $\int x \ln x dx$ c) $\int x^2 \sin x dx$
d) $\int e^x(7+2x) dx$ e) $\int (4x^2-3x)\cos x dx$

Resuelve las siguientes integrales racionales:

- a) $\int \frac{2}{x^2-1} dx$ b) $\int -\frac{3}{x^2+x-2} dx$ c) $\int \frac{2x+1}{x^4-5x^2+4} dx$
d) $\int \frac{7x-2}{x^3-2x^2-x+2} dx$ e) $\int \frac{3x-2}{(2-x)^2} dx$ f) $\int \frac{-2x^2+1}{x^3+6x^2+12x+8} dx$
g) $\int \frac{x-2}{x^4} dx$ h) $\int \frac{4x^2-2x}{(x+2)(x-3)^2} dx$ i) $\int \frac{-x^2+7x}{x^3-x^2-x+1} dx$

SOLUCIONES

$$1. a) \int -4(3-2x^2) dx = \int (-12+8x^2) dx = -12x + 8\frac{x^3}{3} + C$$

$$b) \int 2x\sqrt{x^2-1} dx = \int 2x(x^2-1)^{1/2} dx = \frac{(x^2-1)^{3/2}}{3/2} + C$$

$$c) \int x(1+2x^2)^3 dx = \frac{1}{4} \int 4x(1+2x^2)^3 dx = \frac{1}{4} \frac{(1+2x^2)^4}{4} + C = \\ = \frac{(1+2x^2)^4}{16} + C$$

$$d) \int x^2(2-2x^3)^3 dx = -\frac{1}{6} \int -6x^2(2-2x^3)^3 dx = \\ = -\frac{1}{6} \frac{(2-2x^3)^4}{4} + C = -\frac{(2-2x^3)^4}{24} + C$$

$$e) \int \sqrt{7+2x} dx = \int (7+2x)^{1/2} dx = \frac{1}{2} \int 2(7+2x)^{1/2} dx = \\ = \frac{1}{2} \frac{(7+2x)^{3/2}}{3/2} + C = \frac{(7+2x)^{3/2}}{3} + C$$

$$f) \int (x^2+1)^2 dx = \int (x^4+2x^2+1) dx = \frac{x^5}{5} + \frac{2x^3}{3} + x + C$$

$$g) \int \frac{2x}{x^2+3} dx = \ln|x^2+3| + C$$

$$h) \int \frac{2}{x-3} dx = 2 \int \frac{1}{x-3} dx = 2 \ln|x-3| + C$$

$$\begin{aligned}
 \text{i)} \int \frac{x}{(3x^2-2)^3} dx &= \int x (3x^2-2)^{-3} dx = \frac{1}{6} \int 6x (3x^2-2)^{-3} dx = \\
 &= \frac{1}{6} \frac{(3x^2-2)^{-2}}{-2} + C = \frac{-1}{12(3x^2-2)^2} + C
 \end{aligned}$$

$$\begin{aligned}
 \text{j)} \int \frac{x}{\sqrt[3]{2-x^2}} dx &= \int x (2-x^2)^{-1/3} dx = -\frac{1}{2} \int -2x (2-x^2)^{-1/3} dx \\
 &= -\frac{1}{2} \frac{(2-x^2)^{2/3}}{2/3} + C = -\frac{3}{4} \sqrt[3]{(2-x^2)^2} + C
 \end{aligned}$$

$$\begin{aligned}
 \text{k)} \int (e^{-3x} + e^{x-2}) dx &= -\frac{1}{3} \int -3e^{-3x} dx + \int e^{x-2} dx = \\
 &= -\frac{1}{3} e^{-3x} + e^{x-2} + C
 \end{aligned}$$

$$\begin{aligned}
 \text{l)} \int \frac{3^{5x-1}}{7} dx &= \frac{1}{7} \int 3^{5x-1} dx = \frac{1}{7} \cdot \frac{1}{5} \int 5 \cdot 3^{5x-1} dx = \\
 &= \frac{1}{35} \frac{3^{5x-1}}{\ln 3} + C
 \end{aligned}$$

$$\begin{aligned}
 \text{m)} \int \frac{x}{e^{x^2}} dx &= \int x \cdot e^{-x^2} dx = -\frac{1}{2} \int -2x e^{-x^2} dx = \\
 &= -\frac{1}{2} e^{-x^2} + C
 \end{aligned}$$

$$\begin{aligned}
 \text{n)} \int \sin(-x) dx &= -1 \int -\sin(-x) dx = -(-\cos(-x)) + C = \\
 &= \cos(-x) + C
 \end{aligned}$$

$$n) \int \frac{1}{\cos^2(x+1)} dx = \tan(x+1) + C$$

$$o) \int \frac{x}{\cos^2(x^2-3)} dx = \frac{1}{2} \int \frac{2x}{\cos^2(x^2-3)} dx = \frac{1}{2} \tan(x^2-3) + C$$

$$p) \int \frac{1}{\sqrt{1-25x^2}} dx = \int \frac{1}{\sqrt{1-(5x)^2}} dx = \frac{1}{5} \int \frac{5}{\sqrt{1-(5x)^2}} dx =$$

$$= \frac{1}{5} \arcsin(5x) + C$$

$$q) \int \frac{1}{\sqrt{1-(2x-3)^2}} dx = \frac{1}{2} \int \frac{2}{\sqrt{1-(2x-3)^2}} dx = \frac{1}{2} \arcsin(2x-3) + C$$

$$r) \int \frac{x}{1+9x^4} dx = \int \frac{x}{1+(3x^2)^2} dx = \frac{1}{6} \int \frac{6x}{1+(3x^2)^2} dx =$$

$$= \frac{1}{6} \arctg(3x^2) + C$$

_____ o _____

$$2. a) \int \ln x \, dx = \begin{cases} u = \ln x \rightarrow du = \frac{1}{x} dx \\ dv = dx \rightarrow v = x \end{cases} =$$

$$= x \cdot \ln x - \int x \cdot \frac{1}{x} dx = x \ln x - \int dx = x \ln x - x + C$$

$$b) \int x \cdot \ln x \cdot dx = \begin{cases} u = \ln x \rightarrow du = \frac{1}{x} dx \\ dv = x dx \rightarrow v = \frac{x^2}{2} \end{cases} =$$

$$= \frac{x^2}{2} \cdot \ln x - \int \frac{x^2}{2} \frac{1}{x} dx = \frac{x^2}{2} \ln x - \frac{1}{2} \int x dx =$$

$$= \frac{x^2}{2} \cdot \ln x - \frac{1}{2} \frac{x^2}{2} + C = \frac{x^2}{2} \ln x - \frac{x^2}{4} + C$$

$$c) \int x^2 \cdot \sin x \, dx = \begin{cases} u = x^2 \rightarrow du = 2x dx \\ dv = \sin x dx \rightarrow v = -\cos x \end{cases}$$

$$= -x^2 \cos x - \int -\cos x \cdot 2x dx = -x^2 \cos x + 2 \int x \cos x dx =$$

$$= \begin{cases} u = x \rightarrow du = dx \\ dv = \cos x dx \rightarrow v = \sin x \end{cases} = -x^2 \cos x + 2 \left[x \sin x - \int \sin x dx \right] =$$

$$= -x^2 \cos x + 2x \sin x - 2(-\cos x) + C =$$

$$= -x^2 \cos x + 2x \sin x + 2 \cos x + C$$

$$d) \int e^x (7+2x) dx = \begin{cases} u=7+2x \rightarrow du=2dx \\ dv=e^x dx \rightarrow v=e^x \end{cases}$$

$$= (7+2x)e^x - \int e^x \cdot 2 dx = (7+2x)e^x - 2e^x + C$$

$$e) \int (4x^2-3x) \cos x dx = \begin{cases} u=4x^2-3x \rightarrow du=(8x-3)dx \\ dv=\cos x dx \rightarrow v=\sin x \end{cases}$$

$$= (4x^2-3x) \sin x - \int \sin x (8x-3) dx = \begin{cases} u=8x-3 \rightarrow du=8dx \\ dv=\sin x dx \rightarrow v=-\cos x \end{cases}$$

$$= (4x^2-3x) \sin x - \left[(8x-3)(-\cos x) - \int -\cos x \cdot 8 dx \right] =$$

$$= (4x^2-3x) \sin x + (8x-3) \cos x - 8 \int \cos x dx =$$

$$= (4x^2-3x) \sin x + (8x-3) \cos x - 8 \sin x + C =$$

$$= (4x^2-3x-8) \sin x + (8x-3) \cos x + C$$

————— 0 —————

INTEGRALES RACIONALES

$$a) I = \int \frac{2}{x^2-1} dx = \int \frac{2}{(x+1)(x-1)} dx = \int \left(\frac{A}{x+1} + \frac{B}{x-1} \right) dx$$

$$\frac{2}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1} = \frac{A(x-1) + B(x+1)}{(x+1)(x-1)} \rightarrow$$

$$2 = A(x-1) + B(x+1)$$

$$\text{Si } x=1 \quad 2 = B \cdot 2 \rightarrow B=1$$

$$\text{Si } x=-1 \quad 2 = -2A \rightarrow A=-1$$

$$\begin{aligned} \Rightarrow I &= \int \left(\frac{-1}{x+1} + \frac{1}{x-1} \right) dx = -\ln|x+1| + \ln|x-1| + C = \\ &= \ln \left| \frac{x-1}{x+1} \right| + C \end{aligned}$$

$$b) I = \int \frac{-3}{x^2+x-2} dx = \int \frac{-3}{(x-1)(x+2)} dx = \int \left(\frac{A}{x-1} + \frac{B}{x+2} \right) dx$$

$$x^2+x-2=0$$

$$x = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2} < \frac{1}{-2}$$

$$\frac{-3}{x^2+x-2} = \frac{A}{x-1} + \frac{B}{x+2} = \frac{A(x+2) + B(x-1)}{x^2+x-2}$$

$$-3 = A(x+2) + B(x-1)$$

$$\text{Si } x=-2 \rightarrow -3 = -3B \rightarrow B=1$$

$$\text{Si } x=1 \rightarrow -3 = 3A \rightarrow A=-1$$

$$\Rightarrow I = \int \left[\frac{-1}{x-1} + \frac{1}{x+2} \right] dx = -\ln|x-1| + \ln|x+2| + C = \ln \left| \frac{x+2}{x-1} \right| + C$$

$$c) \int \frac{2x+1}{x^4-5x^2+4} dx = \int \left[\frac{A}{(x+2)} + \frac{B}{(x-2)} + \frac{C}{(x+1)} + \frac{D}{(x-1)} \right] dx$$

$$x^4-5x^2+4=0$$

$$x^2 = \frac{5 \pm \sqrt{25-16}}{2} = \frac{5 \pm 3}{2} < 4 \rightarrow x = \pm 2, x = \pm 1$$

$$\frac{2x+1}{x^4-5x^2+4} = \frac{A(x-2)(x+1)(x-1) + B(x+2)(x+1)(x-1) + C(x+2)(x-2)(x-1) + D(x+2)(x-2)(x+1)}{x^4-5x^2+4}$$

$$\text{Si } x=2 \rightarrow 5=12B \rightarrow B=5/12$$

$$\text{Si } x=-2 \rightarrow -3=-40A \rightarrow A=3/40$$

$$\text{Si } x=1 \rightarrow 3=-6D \rightarrow D=-1/2$$

$$\text{Si } x=-1 \rightarrow -1=6C \rightarrow C=-1/6$$

$$\Rightarrow I = \int \left[\frac{3/40}{x+2} + \frac{5/12}{x-2} + \frac{-1/6}{x+1} + \frac{-1/2}{x-1} \right] dx =$$

$$= \frac{3}{40} \ln|x+2| + \frac{5}{12} \ln|x-2| - \frac{1}{6} \ln|x+1| - \frac{1}{2} \ln|x-1| + C.$$

$$d) \int \frac{7x-2}{x^3-2x^2-x+2} dx = \int \left[\frac{A}{x-2} + \frac{B}{x-1} + \frac{C}{x+1} \right] dx$$

$$\begin{array}{r|rrrr} & 1 & -2 & -1 & 2 \\ 2 & & 2 & 0 & -2 \\ \hline & 1 & 0 & -1 & 0 \\ 1 & & 1 & 1 & \\ \hline & 1 & 1 & 0 & \\ -1 & & -1 & & \\ \hline & 1 & 0 & & \end{array}$$

$$7x-2 = A(x-1)(x+1) + B(x-2)(x+1) + C(x-2)(x-1)$$

$$\text{Si } x=1 \rightarrow 5 = -2B \rightarrow B = -5/2$$

$$\text{Si } x=-1 \rightarrow -9 = 6C \rightarrow C = -3/2$$

$$\text{Si } x=2 \rightarrow 12 = 3A \rightarrow A = 4$$

$$I = \int \left[\frac{4}{x-2} + \frac{-5/2}{x-1} + \frac{-3/2}{x+1} \right] dx =$$

$$= 4 \cdot \ln|x-2| - \frac{5}{2} \ln|x-1| - \frac{3}{2} \ln|x+1| + C$$

$$e) \int \frac{3x-2}{(2-x)^2} dx = \int \left(\frac{A}{2-x} + \frac{B}{(2-x)^2} \right) dx$$

$$\frac{3x-2}{(2-x)^2} = \frac{A(2-x) + B}{(2-x)^2}$$

$$\left. \begin{array}{l} \text{Si } x=2 \rightarrow 4 = B \\ \text{Si } x=0 \rightarrow 1 = A+B \end{array} \right\} \rightarrow A = 1-4 = -3$$

$$I = \int \left[\frac{-3}{2-x} + \frac{4}{(2-x)^2} \right] dx = -3 \ln|2-x| + 4 \int (2-x)^{-2} dx$$

$$= -3 \ln|2-x| + 4(-1) \int (-1)(2-x)^{-2} dx = -3 \ln|2-x| - 4 \frac{(2-x)^{-1}}{-1} + C =$$

$$= -3 \ln|2-x| + \frac{4}{2-x} + C$$

$$f) \int \frac{-2x^2+1}{x^3+6x^2+12x+8} dx = \int \frac{-2x^2+1}{(x+2)^3} dx =$$

$$= \int \left[\frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{(x+2)^3} \right] dx$$

1	6	12	8
-2	-2	-8	-8
1	4	4	0
-2	-2	-4	
1	2	0	
-2	-2		
1	0		

$$-2x^2+1 = A(x+2)^2 + B(x+2) + C$$

$$\begin{aligned} \text{Si } x=-2 &\rightarrow -7=C \\ \text{Si } x=0 &\rightarrow 1=4A+2B+C \\ \text{Si } x=-1 &\rightarrow -1=A+B+C \end{aligned} \quad \left\{ \begin{array}{l} 4A+2B=8 \rightarrow 2A+B=4 \\ A+B=6 \end{array} \right\}$$

$$\begin{array}{r} 2A+B=4 \\ A+B=6 \\ \hline A=-2 \end{array} \rightarrow B=8$$

$$\Rightarrow I = \int \left[\frac{-2}{x+2} + \frac{8}{(x+2)^2} + \frac{-7}{(x+2)^3} \right] dx =$$

$$= -2 \ln|x+2| + 8 \int (x+2)^{-2} dx - 7 \int (x+2)^{-3} dx =$$

$$= -2 \ln|x+2| + 8 \frac{(x+2)^{-1}}{-1} - 7 \frac{(x+2)^{-2}}{-2} + C =$$

$$= -2 \ln|x+2| - \frac{8}{x+2} + \frac{7}{2(x+2)^2} + C$$

$$g) \int \frac{x-2}{x^4} dx = \int \left(\frac{x}{x^4} - \frac{2}{x^4} \right) dx = \int \left(\frac{1}{x^3} - \frac{2}{x^4} \right) dx$$

Ab es rational

$$= \int (x^{-3} - 2x^{-4}) dx = \frac{x^{-2}}{-2} - 2 \frac{x^{-3}}{-3} + C = -\frac{1}{2x^2} + \frac{2}{3x^3} + C$$

$$h) \int \frac{4x^2 - 2x}{(x+2)(x-3)^2} dx = \int \left(\frac{A}{x+2} + \frac{B}{x-3} + \frac{C}{(x-3)^2} \right) dx$$

$$\frac{4x^2 - 2x}{(x+2)(x-3)^2} = \frac{A(x-3)^2 + B(x+2)(x-3) + C(x+2)}{(x+2)(x-3)^2}$$

$$\text{Si } x=2 \rightarrow 20 = 25A \rightarrow \boxed{A = \frac{20}{25} = \frac{4}{5}}$$

$$\text{Si } x=3 \rightarrow 30 = 5C \rightarrow \boxed{C=6}$$

$$\text{Si } x=0 \rightarrow 0 = 9A - 6B + 2C \rightarrow 0 = 9 \cdot \frac{4}{5} - 6B + 2 \cdot 6 \rightarrow \boxed{B = \frac{16}{5}}$$

$$I = \int \frac{4/5}{x+2} dx + \int \frac{16/5}{x-3} dx + \int \frac{6}{(x-3)^2} dx =$$

$$= \frac{4}{5} \ln|x+2| + \frac{16}{5} \ln|x-3| + 6 \int (x-3)^{-2} dx =$$

$$= \frac{4}{5} \ln|x+2| + \frac{16}{5} \ln|x-3| - \frac{6}{x-3} + C$$

$$c) \int \frac{-x^2+7x}{x^3-x^2-x+1} dx = \int \frac{-x^2+7x}{(x-1)^2(x+1)} dx =$$

$$\begin{array}{r|rrrr} 1 & 1 & -1 & -1 & 1 \\ & & 1 & 0 & -1 \\ \hline 1 & 1 & 0 & -1 & 0 \\ & & 1 & 1 & \\ \hline 1 & 1 & 1 & 0 & \\ -1 & & -1 & & \\ \hline 1 & 0 & & & \end{array} = \int \left(\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1} \right) dx$$

$$\frac{-x^2+7x}{(x-1)^2(x+1)} = \frac{A(x-1)(x+1) + B(x+1) + C(x-1)^2}{(x-1)^2(x+1)}$$

$$\text{Si } x=1 \rightarrow 6=2B \rightarrow \boxed{B=3}$$

$$\text{Si } x=-1 \rightarrow -8=4C \rightarrow \boxed{C=-2}$$

$$\text{Si } x=0 \rightarrow 0=-A+B+C \rightarrow A=B+C=1 \rightarrow \boxed{A=1}$$

$$\Rightarrow I = \int \left(\frac{1}{x-1} + \frac{3}{(x-1)^2} + \frac{-2}{x+1} \right) dx =$$

$$= \ln|x-1| - \frac{3}{x-1} - 2\ln|x+1| + C$$