

6. a) Siendo $z = 1 - 3i$ y $w = \sqrt{8}_{45^\circ}$, calcular: $z + w, z \cdot w, 2z, \frac{z^3}{w}$
- b) Calcula: $\frac{i^{35} - i^5}{2i^{10}}$
7. a) Opera: $\frac{(1-2i)(-2+i)}{3i(1-i)}$
- b) Resuelve la siguiente ecuación y expresa el resultado en forma binómica:
 $\frac{3 - zi + 2i}{2} = z + i$
8. Siendo $z = 4\sqrt{3} - 4i$, se pide:
- a) z^5
- b) Las raíces cúbicas de z
9. Halla el valor de x para que el cociente $(1 + 3xi) : (3 - 4i)$
- a) Sea un número imaginario puro.
- b) Sea un número real.
- c) Tenga módulo 1
10. Encontrar a y b para que $\frac{a - 6i}{3 + bi} = \sqrt{2}_{315}$

NÚMEROS COMPLEJOS

⑥ a) $z = 1 - 3i$, $w = \sqrt{8} \angle 45^\circ = 2 + 2i$

↓

$$z = \sqrt{10} \angle 288.4^\circ$$

$$z + w = (1 - 3i) + (2 + 2i) = 3 - i$$

$$z \cdot w = (1 - 3i) \cdot (2 + 2i) = 8 - 4i$$

$$2z = 2 - 6i$$

$$\frac{z^3}{w} = \frac{(1 - 3i)^3}{2 + 2i} = \frac{-26 + 18i}{2 + 2i} = -2 + 11i$$

b) $\frac{i^{35} - i^5}{2 \cdot i^{10}} = \frac{(-i) - i}{2 \cdot (-1)} = \frac{-2i}{-2} = i$

$$\begin{array}{r} 35 \cancel{14} \\ 3 \quad 8 \end{array} \quad \begin{array}{r} 5 \cancel{14} \\ 1 \quad 1 \end{array} \quad \begin{array}{r} 10 \cancel{14} \\ 2 \quad 2 \end{array}$$

$i^1 = i$	R
$i^2 = -1$	1
$i^3 = -i$	2
$i^4 = 1$	3
	0

⑦ a) $\frac{(1 - 2i)(1 - 2i)}{3i(1 - i)} = \frac{5i}{3 + 3i} = \frac{5}{6} + \frac{5}{6}i$

b) $\frac{3 - 2i + 2i}{2} = z + i \rightarrow 3 - 2i + 2i = 2z + 2i \rightarrow 3 = 2z + 2i$

$$3 = (2 + i)z \rightarrow z = \frac{3}{2 + i} \cdot \frac{2 - i}{2 - i} = \frac{6 - 3i}{5} = \frac{6}{5} - \frac{3}{5}i$$

⑧ $z = 4\sqrt{3} - 4i = 8 \angle 330^\circ$

a) $z^5 = (8 \angle 330^\circ)^5 = 8^5 \angle (330 \cdot 5) = 32768 \angle 1650^\circ = 32768 \angle 210^\circ$

b) $\sqrt[3]{z} = \sqrt[3]{8 \angle 330^\circ} = \sqrt[3]{8} \angle \frac{330 + k \cdot 360}{3} = 2 \begin{cases} 110^\circ \\ 230^\circ \\ 350^\circ \end{cases}$

$$(9) \frac{1+3xi}{3-4i} \cdot \frac{3+4i}{3+4i} = \frac{(3-12x) + (9x+4i)}{25}$$

$$a) \frac{3-12x}{25} = 0 \quad x = \frac{1}{4}$$

$$b) \frac{9x+4}{25} = 0 \quad x = -\frac{4}{9}$$

$$c) \sqrt{\left(\frac{3-12x}{25}\right)^2 + \left(\frac{9x+4}{25}\right)^2} = 1 \rightarrow x = \pm \sqrt{\frac{8}{3}}$$

$$(10) \frac{a-6i}{3+bi} = \sqrt{2} \angle 315^\circ$$

$$\frac{a-6i}{3+bi} \cdot \frac{3-bi}{3-bi} = \frac{3a-18i-abi+6bi^2}{9+b^2} = \frac{(3a-6b)}{9+b^2} + \frac{(-18-ab)i}{9+b^2}$$

$$\sqrt{2} \angle 315^\circ = \sqrt{2} \cos 315^\circ + i \sqrt{2} \sin 315^\circ = \sqrt{2} \cdot \frac{\sqrt{2}}{2} - \sqrt{2} \cdot \frac{\sqrt{2}}{2} i = 1 - i$$

$$\Rightarrow \frac{3a-6b}{9+b^2} = 1 \quad \left\{ \begin{array}{l} 3a-6b=9+b^2 \\ -18-ab=-9-b^2 \end{array} \right. \quad \left\{ \begin{array}{l} a = \frac{9+b^2+6b}{3} \\ -18 - \left(\frac{9+b^2+6b}{3}\right)b = -9-b^2 \end{array} \right.$$

$$54 + 9b + b^3 + 6b^2 = -27 - 3b^2 \rightarrow b^3 + 9b^2 + 9b + 81 = 0$$

$$\begin{array}{r|rrrr} 1 & 9 & 9 & 81 \\ -9 & & & \\ \hline & 1 & 0 & 9 & 0 \end{array}$$

$$\begin{array}{l} b = -9 \rightarrow a = 12 \\ b = 3i \rightarrow a = 6i \\ b = -3i \rightarrow a = -6i \end{array}$$

$$x^2 + 9 = 0$$

$$x^2 = -9$$

$$x = \pm \sqrt{-9} = \pm 3i$$

(2)