

EXAMEN DE TRIGONOMETRÍA  
1º BACHILLERATO

1. Sabiendo que  $\operatorname{tag} \alpha = \frac{-3}{4}$  y que  $90^\circ \leq \alpha \leq 180^\circ$ , calcular:

- a)  $\cos (\alpha + 60^\circ)$
- b)  $\operatorname{cotag} 2\alpha$
- c) ¿En qué cuadrante se encuentra  $2\alpha$ ?
- d)  $\operatorname{sen} \frac{\alpha}{2}$

2. Demostrar:

a) 
$$\frac{\operatorname{tag} \alpha \cdot \operatorname{cotag} \alpha}{\operatorname{cosec} \alpha} = \operatorname{sen} \alpha$$

b) 
$$\operatorname{tag}^2 \alpha - \operatorname{sen}^2 \alpha = \operatorname{tag}^2 \alpha \cdot \operatorname{sen}^2 \alpha$$

3. Resuelve las siguientes ecuaciones:

a)  $\cos x - \operatorname{sen} 2x = 0$

b)  $4 \operatorname{sen}^2 x + 4 \cos x = 5$

4. Dos trenes salen de una estación a la misma hora con direcciones distintas que forman  $78^\circ$ . Al cabo de una hora se encuentran a 234 Km de distancia. Si uno va a 150 Km/h, calcula la velocidad del otro.

5. Entre Pedradas y Ocenilla hay 10 Km. Desde ambos se observa en el mismo instante un globo situado en la vertical que une ambos pueblos. Desde Pedradas el ángulo es de  $70^\circ$  y desde Ocenilla  $80^\circ$ . Calcula la altura del globo y la distancia a los pueblos.

SOLUCIONES

$$1. \operatorname{tag} \alpha = \frac{-3}{4} \rightarrow 1 + \operatorname{tag}^2 \alpha = \frac{1}{\cos^2 \alpha} \rightarrow \cos \alpha = \frac{-4}{5} \rightarrow \operatorname{sen}^2 \alpha + \cos^2 \alpha = 1 \rightarrow \operatorname{sen} \alpha = \frac{3}{5}$$

$$a) \cos(\alpha + 60^\circ) = \cos \alpha \cdot \cos 60^\circ - \operatorname{sen} \alpha \cdot \operatorname{sen} 60^\circ = \frac{-4}{5} \cdot \frac{1}{2} - \frac{3}{5} \cdot \frac{\sqrt{3}}{2} = \frac{-4 - 3\sqrt{3}}{10}$$

$$b) \cot \operatorname{ag} 2\alpha = \frac{1}{\operatorname{tag} 2\alpha} = \frac{\cos 2\alpha}{\operatorname{sen} 2\alpha} = \frac{\cos^2 \alpha - \operatorname{sen}^2 \alpha}{2 \cdot \operatorname{sen} \alpha \cdot \cos \alpha} = \frac{\left(\frac{-4}{5}\right)^2 - \left(\frac{3}{5}\right)^2}{2 \cdot \frac{3}{5} \cdot \frac{-4}{5}} = \frac{\frac{16}{25} - \frac{9}{25}}{\frac{-24}{25}} = \frac{-7}{24}$$

$$c) \alpha \in 2^\circ \text{ cuadrante} \rightarrow 2\alpha \in 4^\circ \text{ cuadrante}$$

$$d) \operatorname{sen} \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}} = + \sqrt{\frac{1 - \left(\frac{-4}{5}\right)}{2}} = + \sqrt{\frac{9}{10}} = \frac{3\sqrt{10}}{10}$$

$$2. a) \frac{\operatorname{tag} \alpha \cdot \cot \operatorname{ag} \alpha}{\cos \operatorname{ag} \alpha} = \frac{\frac{\operatorname{sen} \alpha \cdot \cos \alpha}{\cos \alpha} \cdot \frac{\cos \alpha}{\operatorname{sen} \alpha}}{\frac{1}{\operatorname{sen} \alpha}} = \frac{1}{\frac{1}{\operatorname{sen} \alpha}} = \operatorname{sen} \alpha$$

$$b) \operatorname{tag}^2 \alpha - \operatorname{sen}^2 \alpha = \operatorname{tag}^2 \alpha \cdot \operatorname{sen}^2 \alpha$$

$$\frac{\operatorname{sen}^2 \alpha}{\cos^2 \alpha} - \operatorname{sen}^2 \alpha = \frac{\operatorname{sen}^2 \alpha}{\cos^2 \alpha} \cdot \operatorname{sen}^2 \alpha \rightarrow \frac{\operatorname{sen}^2 \alpha - \operatorname{sen}^2 \alpha \cdot \cos^2 \alpha}{\cos^2 \alpha} = \frac{\operatorname{sen}^4 \alpha}{\cos^2 \alpha} \rightarrow$$

$$\operatorname{sen}^2 \alpha - \operatorname{sen}^2 \alpha \cdot \cos^2 \alpha = \operatorname{sen}^4 \alpha \rightarrow \operatorname{sen}^2 \alpha (1 - \cos^2 \alpha) = \operatorname{sen}^4 \alpha \rightarrow \operatorname{sen}^2 \alpha \cdot \operatorname{sen}^2 \alpha = \operatorname{sen}^4 \alpha \rightarrow \operatorname{sen}^4 \alpha = \operatorname{sen}^4 \alpha$$

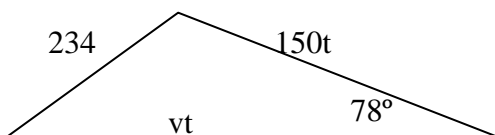
$$3. a) \cos x - \operatorname{sen} 2x = 0 \rightarrow \cos x - 2 \cdot \operatorname{sen} x \cdot \cos x = 0 \rightarrow \cos x (1 - 2 \cdot \operatorname{sen} x) = 0 \rightarrow$$

$$\begin{cases} \cos x = 0 \rightarrow x = 90^\circ + k360^\circ, x = 270^\circ + k360^\circ, \\ (1 - 2\operatorname{sen} x) = 0 \rightarrow \operatorname{sen} x = \frac{1}{2} \rightarrow x = 30^\circ + k360^\circ, x = 150^\circ + k360^\circ \end{cases}$$

$$b) 4 \operatorname{sen}^2 x + 4 \cos x = 5 \rightarrow 4(1 - \cos^2 x) + 4 \cos x = 5 \rightarrow 4 \cos^2 x - 4 \cos x + 1 = 0$$

$$\cos x = \frac{4 \pm \sqrt{16 - 16}}{8} = \frac{1}{2} \rightarrow x = 60^\circ + k360^\circ, x = 300^\circ + k360^\circ$$

4.

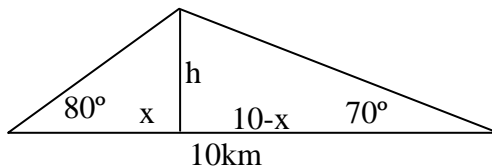


aplicando el teorema del coseno

$$234^2 = (150t)^2 + (vt)^2 - 2 \cdot (150t) \cdot vt \cdot \cos 78^\circ$$

$$V = 213.47 \text{ Km/h.}$$

5.



$$\operatorname{tag} 80^\circ = \frac{h}{x} \left. \vphantom{\operatorname{tag} 80^\circ} \right\} \rightarrow h = 18.51 \text{ Km}$$

$$\operatorname{tag} 70^\circ = \frac{h}{10-x} \left. \vphantom{\operatorname{tag} 70^\circ} \right\} d_1 = 19.7 \text{ Km} \quad d_2 = 18.79 \text{ Km}$$