

1.- a) (1,5 puntos) Resolver por el Método de Gauss el sistema:
$$\begin{cases} 2x - y + 3z = 2 \\ 5x - y + z = 6 \\ x + y + 2z = 2 \end{cases}$$

b) (1 punto) Calcular el término 19 del desarrollo por Newton de $(a^3 b - a^2)^{20}$

2.- (2,5 puntos) Resolver las ecuaciones:

a) $\log x^2 - \log \frac{10x + 11}{10} = 1$

b) $2 \cdot \binom{2x}{x} = 7 \cdot \binom{2x-2}{x-1}$

3.- (2,5 puntos)

a) Calcular $\sec 280^\circ$ si sabemos que $\cotg 10^\circ = h$

b) Demostrar la identidad trigonométrica: $\frac{\cos^2 x - \cos 2x}{1 - \cos x} = 1 + \cos x$

4.- (2,5 puntos)

a) Si β es un ángulo del segundo cuadrante y $\cos \beta = -3/4$, calcular las razones trigonométricas (solo sen y cos) de $\beta/2$ y 2β

b) Resolver la ecuación: $2 \cdot \operatorname{sen} x + \sqrt{3} \cdot \operatorname{tg} x = 0$

$$\textcircled{1} \begin{cases} 2x - y + 3z = 2 \\ 5x - y + z = 6 \\ x + y + 2z = 2 \end{cases} \xrightarrow{E_3 \rightarrow E_1} \begin{cases} x + y + 2z = 2 \\ 2x - y + 3z = 2 \\ 5x - y + z = 6 \end{cases} \xrightarrow{\begin{matrix} E_2 - 2E_1 \\ E_3 - 5E_1 \end{matrix}} \begin{cases} x + y + 2z = 2 \\ -3y - z = -2 \\ -6y - 9z = -4 \end{cases}$$

$$\xrightarrow{E_3 - 2E_2} \begin{cases} x + y + 2z = 2 \\ -3y - z = -2 \\ -7z = 0 \end{cases} \rightarrow \begin{matrix} x = 2 - \frac{2}{3} - 2 \cdot 0 = \frac{4}{3} \\ -3y - 0 = -2 \Rightarrow 3y = 2 \Rightarrow y = \frac{2}{3} \\ \Rightarrow z = 0 \end{matrix}$$

$$\boxed{S_0\left(\frac{4}{3}, \frac{2}{3}, 0\right)}$$

$$b) T_k = (-1)^{k-1} \binom{n}{k-1} a^{n-(k-1)} \cdot b^{k-1} \quad \text{con } (a^3b - a^2)^{20} \text{ ¿} T_{19}?$$

$$T_{19} = (-1)^{19-1} \binom{20}{19-1} (a^3b)^{20-(19-1)} \cdot (a^2)^{19-1} =$$

$$= (-1)^{18} \binom{20}{18} (a^3b)^2 \cdot (a^2)^{18} = 1 \cdot 190 \cdot a^6b^2 \cdot a^{36} = \boxed{190 a^{42} b^2}$$

$$\textcircled{2} a) \log x^2 - \log \frac{10x+11}{10} = 1 \Rightarrow \log \frac{x^2}{\frac{10x+11}{10}} = \log 10 \Rightarrow$$

$$\Rightarrow \frac{10x^2}{10x+11} = 10 \Rightarrow 10x^2 = 100x + 110 \Rightarrow 10x^2 - 100x - 110 = 0$$

$$\Rightarrow \text{Simplificando: } x^2 - 10x - 11 = 0 \quad \begin{cases} \boxed{x_1 = 11} \\ \boxed{x_2 = -1} \end{cases}$$

Comprobación:

$$x_1 = 11 \Rightarrow \log 121 - \log \frac{121}{10} = \log 121 - (\log 121 - \log 10) = \log 121 - \log 121 + \log 10 = 1$$

$$+ \log 10 = \underline{\underline{1}}$$

$$x_2 = -1 \Rightarrow \log 1 - \log \frac{-10+11}{10} = 0 - \log \frac{1}{10} = 0 - (-1) = \underline{\underline{1}}$$

② b) $2 \binom{2x}{x} = 7 \binom{2x-2}{x-1}$ $x \neq 0, 1$ pues no tendría sentido

$$\frac{2 \cdot (2x)!}{x! (2x-x)!} = \frac{7 (2x-2)!}{(x-1)! (2x-2-x+1)!} \rightarrow \frac{2 \cdot (2x)!}{x! \cdot x!} = \frac{7 (2x-2)!}{(x-1)! \cdot (x-1)!} \rightarrow$$

$$\rightarrow 2 \cdot (2x)! \cdot (x-1)! \cdot (x-1)! = 7 (2x-2)! \cdot x! \cdot x! \rightarrow$$

$$\rightarrow 2 \cdot 2x \cdot (2x-1) \cdot \cancel{(2x-2)!} \cdot \cancel{(x-1)!} \cdot \cancel{(x-1)!} = 7 \cdot \cancel{(2x-2)!} \cdot x \cdot \cancel{(x-1)!} \cdot x \cdot \cancel{(x-1)!} \rightarrow$$

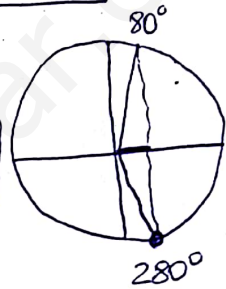
$$\rightarrow 4x \cdot (2x-1) = 7x^2 \Rightarrow 8x^2 - 4x = 7x^2 \Rightarrow 8x^2 - 7x^2 - 4x = 0$$

$$\Rightarrow x^2 - 4x = 0 \Rightarrow x(x-4) = 0 \Rightarrow \begin{cases} x=0 \text{ NO VALE } \parallel \text{ (por enunciado)} \\ x-4=0 \Rightarrow \boxed{x=4} \end{cases}$$

③ a) ¿ $\sec 280^\circ$? siendo $\cotg 10^\circ = h$

$$\sec 280^\circ = \frac{1}{\cos 280^\circ} = \frac{1}{\cos 80^\circ} \stackrel{(*)}{=} \frac{1}{\sin 10^\circ} \stackrel{(*)}{=} \sqrt{1+h^2}$$

(complementarios)



$$(*) \quad 1 + \cotg^2 10^\circ = \operatorname{cosec}^2 10^\circ \Rightarrow 1 + h^2 = \frac{1}{\sin^2 10^\circ}$$

b) $\frac{\cos^2 x - \cos 2x}{1 - \cos x} = 1 + \cos x$

$$\frac{\cos^2 x - \cos 2x}{1 - \cos x} = \frac{\cos^2 x - (\cos^2 x - \sin^2 x)}{1 - \cos x} = \frac{\cancel{\cos^2 x} - \cancel{\cos^2 x} + \sin^2 x}{1 - \cos x} =$$

$$= \frac{1 - \cos^2 x}{1 - \cos x} = \frac{(1 - \cos x)(1 + \cos x)}{(1 - \cos x)} = \underline{\underline{1 + \cos x}}$$

④ a) $\beta \in \text{II} \quad \cos \beta = -\frac{3}{4}$

$$\sin \frac{\beta}{2} = \pm \sqrt{\frac{1 - \cos \beta}{2}} \Rightarrow \beta \in \text{II} \Rightarrow \frac{\beta}{2} \in \text{I} \Rightarrow \sin \frac{\beta}{2} = + \sqrt{\frac{1 - (-\frac{3}{4})}{2}} = \sqrt{\frac{1 + \frac{3}{4}}{2}} =$$

$$= \sqrt{\frac{7/4}{2}} = \sqrt{\frac{7}{8}} = \frac{1}{2} \sqrt{\frac{7}{2}} = \frac{1}{2} \frac{\sqrt{7}}{\sqrt{2}} = \frac{1}{2} \frac{\sqrt{7} \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \boxed{\frac{\sqrt{14}}{4}}$$

$$\cos \frac{\beta}{2} = \pm \sqrt{\frac{1 + \cos \beta}{2}} \Rightarrow \cos \frac{\beta}{2} = + \sqrt{\frac{1 + (-\frac{3}{4})}{2}} = \sqrt{\frac{1/4}{2}} = \sqrt{\frac{1}{8}} = \frac{1}{2} \cdot \sqrt{\frac{1}{2}} =$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{2}}{4}}$$

$$\operatorname{sen} 2\beta = 2 \operatorname{sen} \beta \cos \beta = 2 \cdot \frac{\sqrt{7}}{4} \cdot \left(-\frac{3}{4}\right) = -\frac{6\sqrt{7}}{16} = \boxed{-\frac{3\sqrt{7}}{8}}$$

$$\text{Como } \cos \beta = -\frac{3}{4} \Rightarrow \operatorname{sen}^2 \beta + \cos^2 \beta = 1 \Rightarrow \operatorname{sen}^2 \beta = 1 - \cos^2 \beta = 1 - \left(-\frac{3}{4}\right)^2 = 1 - \frac{9}{16} = \frac{7}{16} \Rightarrow \operatorname{sen} \beta = \pm \sqrt{\frac{7}{16}} = \pm \frac{\sqrt{7}}{4}$$

$\beta \in \text{II}$

$$\begin{aligned} \cos 2\beta &= \cos^2 \beta - \operatorname{sen}^2 \beta = \left(-\frac{3}{4}\right)^2 - \left(\frac{\sqrt{7}}{4}\right)^2 = \\ &= \frac{9}{16} - \frac{7}{16} = \frac{2}{16} = \boxed{\frac{1}{8}} \end{aligned}$$

④ b) $2 \operatorname{sen} x + \sqrt{3} \operatorname{tg} x = 0 \Rightarrow 2 \operatorname{sen} x + \sqrt{3} \frac{\operatorname{sen} x}{\cos x} = 0 \Rightarrow$

$$\Rightarrow \frac{2 \operatorname{sen} x \cos x + \sqrt{3} \operatorname{sen} x}{\cos x} = 0 \Rightarrow \operatorname{sen} x (2 \cos x + \sqrt{3}) = 0 \Rightarrow$$

$$\Rightarrow \begin{cases} \operatorname{sen} x = 0 \Rightarrow x = \operatorname{arc} \operatorname{sen} 0 \Rightarrow x = \begin{cases} 0^\circ + 2\pi K \\ 180^\circ + 2\pi K \end{cases} \Rightarrow \boxed{0^\circ + \pi K} \quad K \in \mathbb{Z} \\ 2 \cos x = -\sqrt{3} \Rightarrow \cos x = -\frac{\sqrt{3}}{2} \Rightarrow x = \operatorname{arc} \cos -\frac{\sqrt{3}}{2} = \boxed{\begin{cases} x = 150^\circ + 2\pi K \\ x = 210^\circ + 2\pi K \end{cases}} \quad K \in \mathbb{Z} \end{cases}$$