Resolver el sistema:

$$\begin{cases}\mathbf{2X+Y=A}\\ \mathbf{4X-3Y=B}\end{cases}$$

sabiendo que X e Y son matrices de dimensión 3x4 y que:

$$\mathbf{A}=\begin{pmatrix}-1 & 8 & 7\\ -3 & 6 & 12\end{pmatrix} \qquad \mathbf{B}=\begin{pmatrix}13 & -4 & -21\\ -11 & 12 & 14\end{pmatrix}$$

$$\begin{cases}2X+Y=A\\ 4X-3Y=B\end{cases} \Rightarrow \begin{cases}-4X-2Y=-2A\\ 4X-3Y=B\end{cases} \Rightarrow -5Y=B-2A$$

$$\Rightarrow \quad 5Y=2A-B \quad \Rightarrow \quad Y=\frac{1}{5}(2A-B)$$

$$\Rightarrow \quad 2X+\frac{1}{5}(2A-B)=A \quad \Rightarrow \quad 2X=\frac{3}{5}A+\frac{1}{5}B \quad \Rightarrow \quad X=\frac{1}{10}(3A+B)$$

- $X=\frac{1}{10}\cdot\left[\begin{pmatrix}-3 & 24 & 21\\ -9 & 18 & 36\end{pmatrix}+\begin{pmatrix}13 & -4 & -21\\ -11 & 12 & 14\end{pmatrix}\right]=$

$$=\frac{1}{10}\cdot\begin{pmatrix}10 & 20 & 0\\ -20 & 30 & 50\end{pmatrix}=\begin{pmatrix}1 & 2 & 0\\ -2 & 3 & 5\end{pmatrix}$$

- $Y=\frac{1}{5}\cdot\left[\begin{pmatrix}-2 & 16 & 14\\ -6 & 12 & 24\end{pmatrix}-\begin{pmatrix}13 & -4 & -21\\ -11 & 12 & 14\end{pmatrix}\right]=$

$$=\frac{1}{5}\cdot\begin{pmatrix}-15 & 20 & 35\\ 5 & 0 & 10\end{pmatrix}=\begin{pmatrix}-3 & 4 & 7\\ 1 & 0 & 2\end{pmatrix}$$

Dadas las matrices:

$$\mathbf{M}=\begin{pmatrix}1 & 5\\ -1 & 3\end{pmatrix} \qquad \mathbf{N}=\begin{pmatrix}1 & 0\\ 3 & 0\end{pmatrix}$$

halla dos matrices X e Y que verifiquen:

$$\begin{cases}\mathbf{X-2M=3N}\\ \mathbf{M+N-Y=I}\end{cases}$$

- $X=3N+2M=3\cdot\begin{pmatrix}1 & 0\\ 3 & 0\end{pmatrix}+2\cdot\begin{pmatrix}1 & 5\\ -1 & 3\end{pmatrix}=$

$$=\begin{pmatrix}3 & 0\\ 9 & 0\end{pmatrix}+\begin{pmatrix}2 & 10\\ -2 & 6\end{pmatrix}=\begin{pmatrix}5 & 10\\ 7 & 6\end{pmatrix}$$

- $Y=M+N-I=\begin{pmatrix}1 & 5\\ -1 & 3\end{pmatrix}+\begin{pmatrix}1 & 0\\ 3 & 0\end{pmatrix}-\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=\begin{pmatrix}1 & 5\\ 2 & 2\end{pmatrix}$

Halla las matrices X e Y que verifican el sistema:

$$\begin{cases}\mathbf{2X+Y}=\begin{pmatrix}\mathbf{1} & \mathbf{4}\\ \mathbf{2} & \mathbf{0}\end{pmatrix}\\ \mathbf{X-Y}=\begin{pmatrix}\mathbf{1} & \mathbf{-1}\\ \mathbf{1} & \mathbf{0}\end{pmatrix}\end{cases}$$

$$\begin{cases}2X+Y=\begin{pmatrix}1 & 4\\ 2 & 0\end{pmatrix}\\ X-Y=\begin{pmatrix}1 & -1\\ 1 & 0\end{pmatrix}\end{cases} \Rightarrow \quad 3X=\begin{pmatrix}2 & 3\\ 3 & 0\end{pmatrix} \quad \Rightarrow \quad X=\begin{pmatrix}\frac{2}{3} & 1\\ 1 & 0\end{pmatrix}$$

$$\Rightarrow \quad X-Y=\begin{pmatrix}1 & -1\\ 1 & 0\end{pmatrix} \quad \Rightarrow \quad -Y=\begin{pmatrix}1 & -1\\ 1 & 0\end{pmatrix}-\begin{pmatrix}\frac{2}{3} & 1\\ 1 & 0\end{pmatrix}=\begin{pmatrix}\frac{1}{3} & -2\\ 0 & 0\end{pmatrix}$$

$$\Rightarrow \quad Y=\begin{pmatrix}\frac{-1}{3} & 2\\ 0 & 0\end{pmatrix}$$

Siendo A y B dos matrices cuadradas de orden 2, resolver el siguiente sistema matricial:

$$\begin{cases} \mathbf{3A - 5B} = \begin{pmatrix} \mathbf{1} & \mathbf{-2} \\ \mathbf{8} & \mathbf{1} \end{pmatrix} \\ \mathbf{-A + 3B} = \begin{pmatrix} \mathbf{2} & \mathbf{4} \\ \mathbf{3} & \mathbf{0} \end{pmatrix} \end{cases}$$

$$\begin{cases} 3A - 5B = \begin{pmatrix} 1 & -2 \\ 8 & 1 \end{pmatrix} \\ -A + 3B = \begin{pmatrix} 2 & 4 \\ 3 & 0 \end{pmatrix} \end{cases} \Rightarrow \begin{cases} 3A - 5B = \begin{pmatrix} 1 & -2 \\ 8 & 1 \end{pmatrix} \\ -3A + 9B = \begin{pmatrix} 6 & 12 \\ 9 & 0 \end{pmatrix} \end{cases}$$

$$\Rightarrow \quad 3A - 5B + (-3A + 9B) = \begin{pmatrix} 1 & -2 \\ 8 & 1 \end{pmatrix} + \begin{pmatrix} 6 & 12 \\ 9 & 0 \end{pmatrix} \Rightarrow$$

$$4B = \begin{pmatrix} 7 & 10 \\ 17 & 1 \end{pmatrix} \quad \Rightarrow \quad B = \begin{pmatrix} \frac{7}{4} & \frac{10}{4} \\ \frac{17}{4} & \frac{1}{4} \end{pmatrix}$$

$$-A + 3B = \begin{pmatrix} 2 & 4 \\ 3 & 0 \end{pmatrix} \quad \Rightarrow \quad A = 3B - \begin{pmatrix} 2 & 4 \\ 3 & 0 \end{pmatrix} \Rightarrow$$

$$\Rightarrow A = 3 \cdot \begin{pmatrix} \frac{7}{4} & \frac{10}{4} \\ \frac{17}{4} & \frac{1}{4} \end{pmatrix} - \begin{pmatrix} 2 & 4 \\ 3 & 0 \end{pmatrix}$$

$$\Rightarrow \quad A = \begin{pmatrix} \frac{21}{4} & \frac{30}{4} \\ \frac{51}{4} & \frac{3}{4} \end{pmatrix} - \begin{pmatrix} 2 & 4 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} \frac{13}{4} & \frac{14}{4} \\ \frac{39}{4} & \frac{3}{4} \end{pmatrix}$$

Halla dos matrices A y B tales que:

$$\begin{cases} \mathbf{2A + 3B} = \begin{pmatrix} \mathbf{8} & \mathbf{4} & \mathbf{7} \\ \mathbf{18} & \mathbf{11} & \mathbf{-6} \\ \mathbf{8} & \mathbf{3} & \mathbf{13} \end{pmatrix} \\ \mathbf{-A + 5B} = \begin{pmatrix} \mathbf{9} & \mathbf{-2} & \mathbf{16} \\ \mathbf{17} & \mathbf{1} & \mathbf{-10} \\ \mathbf{9} & \mathbf{5} & \mathbf{13} \end{pmatrix} \end{cases}$$

$$\begin{cases} 2A + 3B = \begin{pmatrix} 8 & 4 & 7 \\ 18 & 11 & -6 \\ 8 & 3 & 13 \end{pmatrix} \\ -A + 5B = \begin{pmatrix} 9 & -2 & 16 \\ 17 & 1 & -10 \\ 9 & 5 & 13 \end{pmatrix} \end{cases} \Rightarrow \begin{cases} 2A + 3B = \begin{pmatrix} 8 & 4 & 7 \\ 18 & 11 & -6 \\ 8 & 3 & 13 \end{pmatrix} \\ -2A + 10B = \begin{pmatrix} 18 & -4 & 32 \\ 34 & 2 & -20 \\ 18 & 10 & 26 \end{pmatrix} \end{cases}$$

$$\Rightarrow \quad 13B = \begin{pmatrix} 8 & 4 & 7 \\ 18 & 11 & -6 \\ 8 & 3 & 13 \end{pmatrix} + \begin{pmatrix} 18 & -4 & 32 \\ 34 & 2 & -20 \\ 18 & 10 & 26 \end{pmatrix} = \begin{pmatrix} 26 & 0 & 39 \\ 52 & 13 & -26 \\ 26 & 13 & 39 \end{pmatrix}$$

$$\Rightarrow \quad B = \begin{pmatrix} 2 & 0 & 3 \\ 4 & 1 & -2 \\ 2 & 1 & 3 \end{pmatrix}$$

$$\Rightarrow \quad A = 5B - \begin{pmatrix} 9 & -2 & 16 \\ 17 & 1 & -10 \\ 9 & 5 & 13 \end{pmatrix} = 5 \cdot \begin{pmatrix} 2 & 0 & 3 \\ 4 & 1 & -2 \\ 2 & 1 & 3 \end{pmatrix} - \begin{pmatrix} 9 & -2 & 16 \\ 17 & 1 & -10 \\ 9 & 5 & 13 \end{pmatrix} =$$

$$= \begin{pmatrix} 10 & 0 & 15 \\ 20 & 5 & -10 \\ 10 & 5 & 15 \end{pmatrix} - \begin{pmatrix} 9 & -2 & 16 \\ 17 & 1 & -10 \\ 9 & 5 & 13 \end{pmatrix} = \begin{pmatrix} 1 & 2 & -1 \\ 3 & 4 & 0 \\ 1 & 0 & 2 \end{pmatrix}$$

Resolver el sistema matricial:

$$\begin{cases} 2X + Y = A \\ 3X - 2Y = B \end{cases}$$

siendo:

$$A = \begin{pmatrix} 3 & 1 & 6 \\ 2 & 9 & 3 \\ 12 & 4 & 15 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & -2 & 2 \\ -4 & 3 & -6 \\ 4 & -8 & 5 \end{pmatrix}$$

$$\begin{cases} 2X + Y = A \\ 3X - 2Y = B \end{cases} \Rightarrow \begin{cases} 4X + 2Y = 2A \\ 3X - 2Y = B \end{cases} \Rightarrow 7X = 2A + B$$

$$\Rightarrow \quad 7X = 2A + B \quad \Rightarrow \quad X = \frac{1}{7}\cdot(2A + B)$$

$$\Rightarrow \quad 2\cdot\frac{1}{7}(2A+B) + Y = A \quad \Rightarrow \quad Y = A - \frac{4}{7}A - \frac{2}{7}B \quad \Rightarrow \quad Y = \frac{1}{7}\cdot(3A - 2B)$$

- $X = \frac{1}{7}\cdot(2A+B) = \frac{1}{7}\cdot\left[\begin{pmatrix} 6 & 2 & 12 \\ 4 & 18 & 6 \\ 24 & 8 & 30 \end{pmatrix} + \begin{pmatrix} 1 & -2 & 2 \\ -4 & 3 & -6 \\ 4 & -8 & 5 \end{pmatrix}\right] =$

$$= \frac{1}{7}\cdot\begin{pmatrix} 7 & 0 & 14 \\ 0 & 21 & 0 \\ 28 & 0 & 35 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 3 & 0 \\ 4 & 0 & 5 \end{pmatrix}$$

- $Y = \frac{1}{7}\cdot(3A-2B) = \frac{1}{7}\cdot\left[\begin{pmatrix} 9 & 3 & 18 \\ 6 & 27 & 9 \\ 36 & 12 & 45 \end{pmatrix} - \begin{pmatrix} 2 & -4 & 4 \\ -8 & 6 & -12 \\ 8 & -16 & 10 \end{pmatrix}\right] =$

$$= \frac{1}{7}\cdot\begin{pmatrix} 7 & 7 & 14 \\ 14 & 21 & 21 \\ 28 & 28 & 35 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 3 & 3 \\ 4 & 4 & 5 \end{pmatrix}$$

Halla la matriz $X^2 + Y$, donde X e Y son dos matrices del sistema:

$$\begin{cases} 5X + 3Y = \begin{pmatrix} 2 & 0 \\ -4 & 15 \end{pmatrix} \\ 3X + 2Y = \begin{pmatrix} 1 & -1 \\ -2 & 9 \end{pmatrix} \end{cases}$$

Multiplicamos la primera ecuación por 2 y la segunda ecuación por 3.

$$\begin{cases} 5X + 3Y = \begin{pmatrix} 2 & 0 \\ -4 & 15 \end{pmatrix} \\ 3X + 2Y = \begin{pmatrix} 1 & -1 \\ -2 & 9 \end{pmatrix} \end{cases} \Rightarrow \begin{cases} 10X + 6Y = \begin{pmatrix} 4 & 0 \\ -8 & 30 \end{pmatrix} \\ 9X + 6Y = \begin{pmatrix} 3 & -3 \\ -6 & 27 \end{pmatrix} \end{cases}$$

Si restamos la segunda ecuación a la primera obtenemos:

$$\Rightarrow \quad X = \begin{pmatrix} 1 & 3 \\ -2 & 3 \end{pmatrix}$$

Multiplicamos la primera ecuación por 3 y la segunda ecuación por 5.

$$\begin{cases} 5X + 3Y = \begin{pmatrix} 2 & 0 \\ -4 & 15 \end{pmatrix} \\ 3X + 2Y = \begin{pmatrix} 1 & -1 \\ -2 & 9 \end{pmatrix} \end{cases} \Rightarrow \begin{cases} 15X + 9Y = \begin{pmatrix} 6 & 0 \\ -12 & 45 \end{pmatrix} \\ 15X + 10Y = \begin{pmatrix} 5 & -5 \\ -10 & 45 \end{pmatrix} \end{cases}$$

Si restamos la primera ecuación a la segunda obtenemos:

$$\Rightarrow \quad Y = \begin{pmatrix} -1 & -5 \\ 2 & 0 \end{pmatrix}$$

Una vez calculadas las matrices X e Y podemos resolver el problema:

$$\Rightarrow \quad X^2 + Y = \begin{pmatrix} 1 & 3 \\ -2 & 3 \end{pmatrix}^2 + \begin{pmatrix} -1 & -5 \\ 2 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 3 \\ -2 & 3 \end{pmatrix}\cdot\begin{pmatrix} 1 & 3 \\ -2 & 3 \end{pmatrix} + \begin{pmatrix} -1 & -5 \\ 2 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} -5 & 12 \\ -8 & 3 \end{pmatrix} + \begin{pmatrix} -1 & -5 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} -6 & 7 \\ -6 & 3 \end{pmatrix}$$