

4 Sean $A = \begin{pmatrix} 3 & 0 \\ 5 & -1 \end{pmatrix}$ y $B = \begin{pmatrix} 0 & 6 \\ 1 & -3 \end{pmatrix}$.

Encuentra X que cumpla: $3 \cdot X - 2 \cdot A = 5 \cdot B \rightarrow 3X = 5B + 2A \rightarrow X = \frac{1}{3}(5B + 2A)$

$$5B = \begin{pmatrix} 0 & 30 \\ 5 & -15 \end{pmatrix}$$

$$2A = \begin{pmatrix} 6 & 0 \\ 10 & -2 \end{pmatrix}$$

$$5B + 2A = \begin{pmatrix} 6 & 30 \\ 15 & -17 \end{pmatrix}$$

$$X = \frac{1}{3} \begin{pmatrix} 6 & 30 \\ 15 & -17 \end{pmatrix} = \begin{pmatrix} 2 & 10 \\ 5 & -17/3 \end{pmatrix}$$

5 Encuentra dos matrices, A y B , de dimensión 2×2 que cumplan:

$$2A + B = \begin{pmatrix} 1 & 4 \\ 2 & 0 \end{pmatrix}$$

$$A - B = \begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 4 \\ 2 & 0 \end{pmatrix}$$

$$D = \begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix}$$

$$\left. \begin{array}{l} 2A + B = C \\ A - B = D \end{array} \right\}$$

$$A = \frac{1}{3}(C + D)$$

$$C + D = \begin{pmatrix} 0 & 6 \\ 3 & 0 \end{pmatrix}$$

$$A = \frac{1}{3} \begin{pmatrix} 0 & 6 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix} = A$$

$$3A = C + D$$

$$A - B = D \Rightarrow A - D = B$$

$$B = \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = B$$

1 Calcula el rango de las siguientes matrices:

$$A = \begin{pmatrix} 1 & 4 & -1 \\ -1 & 3 & 2 \\ 2 & 2 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & -2 & 0 & -3 \\ -1 & 3 & 1 & 4 \\ 2 & 1 & 5 & -1 \end{pmatrix}$$

Triangulación por Gauss

$$A = \begin{pmatrix} 1 & 4 & -1 \\ -1 & 3 & 2 \\ 2 & 2 & 0 \end{pmatrix} \xrightarrow{\substack{-2 \cdot 1F + 3F \\ 1F + 2F}} \begin{pmatrix} 1 & 4 & -1 \\ 0 & 7 & 1 \\ 0 & -5 & 2 \end{pmatrix} \xrightarrow{\substack{5 \cdot 2F + 1 \cdot 3F \\ 5 - 14 = -9}} \begin{pmatrix} 1 & 4 & -1 \\ 0 & 7 & 1 \\ 0 & -9 & 17 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 4 & -1 \\ 0 & 7 & 1 \\ 0 & 0 & -9 \end{pmatrix}$$

$$\text{rang}(A) = 3$$

$$1F + 2F$$

$$-2 \cdot 1F + 3F$$

$$-5 \cdot 2F + 3F$$

$$C = \begin{pmatrix} 1 & -2 & 0 & -3 \\ -1 & 3 & 1 & 4 \\ 2 & 1 & 5 & -1 \end{pmatrix} \xrightarrow{1F + 2F} \begin{pmatrix} 1 & -2 & 0 & -3 \\ 0 & 1 & 1 & 1 \\ 0 & 5 & 5 & 5 \end{pmatrix} \xrightarrow{\substack{-2 \cdot 1F + 3F \\ -5 \cdot 2F + 3F}} \begin{pmatrix} 1 & -2 & 0 & -3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{rang}(C) = 2$$

1 Expresa en forma matricial y resuelve los siguientes sistemas de ecuaciones:

$$b) \begin{cases} 2x - y = 7 \\ x - 2y = 11 \end{cases}$$

$$\begin{pmatrix} 2 & -1 \\ 1 & -2 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 11 \end{pmatrix}$$

A X B

$$AX = B$$

$$\underline{A^{-1} \cdot A} \cdot X = A^{-1} \cdot B$$

I

$$X = A^{-1} \cdot B$$

$$A^{-1} = \frac{[\text{adj}(A)]^T}{|A|}$$

$$\text{Adj}(A) = \begin{pmatrix} -2 & -1 \\ +1 & 2 \end{pmatrix}$$

$$[\text{Adj}(A)]^T = \begin{pmatrix} -2 & 1 \\ -1 & 2 \end{pmatrix}$$

$$|A| = -4 + 1 = -3$$

$$A^{-1} = \begin{pmatrix} 2/3 & -1/3 \\ 1/3 & -2/3 \end{pmatrix}$$

$$X = A^{-1} \cdot B = \begin{pmatrix} 2/3 & -1/3 \\ 1/3 & -2/3 \end{pmatrix} \begin{pmatrix} 7 \\ 11 \end{pmatrix} = \begin{pmatrix} 14/3 - 11/3 \\ 7/3 - 22/3 \end{pmatrix} = \begin{pmatrix} 1 \\ -5 \end{pmatrix}$$

$$\boxed{\begin{matrix} x = 1 \\ y = -5 \end{matrix}}$$

1 Escribe las matrices traspuestas de:

$$A = \begin{pmatrix} 3 & 1 \\ 2 & 5 \\ 7 & 6 \end{pmatrix}$$

$$B = \begin{pmatrix} 2 & 5 & 7 \\ 4 & 1 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 3 & 5 & -1 \\ 0 & 2 & 4 & 1 \\ 6 & 1 & 0 & 3 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 3 & 2 & 7 \\ 1 & 5 & 6 \end{pmatrix}$$

$$B^T = \begin{pmatrix} 2 & 4 \\ 5 & 1 \\ 7 & 0 \end{pmatrix}$$

$$C^T = \begin{pmatrix} 1 & 0 & 6 \\ 3 & 2 & 1 \\ 5 & 4 & 0 \\ -1 & 1 & 3 \end{pmatrix}$$

1 Dadas las siguientes matrices:

$$A = \begin{pmatrix} 1 & 0 & -2 \\ 4 & 1 & -3 \end{pmatrix}$$

$$B = \begin{pmatrix} -1 & 0 & 1 \\ -4 & 1 & 3 \end{pmatrix}$$

$$C = \begin{pmatrix} 7 & 1 & -1 \\ 8 & -10 & 0 \end{pmatrix}$$

$$D = \begin{pmatrix} -3 & 1 & 5 \\ 6 & 2 & 4 \end{pmatrix}$$

calcula $E = 2A - 3B + C - 2D$.

$$2A = \begin{pmatrix} 2 & 0 & -4 \\ 8 & 2 & -6 \end{pmatrix}$$

$$3B = \begin{pmatrix} -3 & 0 & 3 \\ -12 & 3 & 9 \end{pmatrix}$$

$$2D = \begin{pmatrix} -6 & 2 & 10 \\ 12 & 4 & 8 \end{pmatrix}$$

$$E = \begin{pmatrix} 2 & 0 & -4 \\ 8 & 2 & -6 \end{pmatrix} + \begin{pmatrix} 3 & 0 & -3 \\ 12 & -3 & -9 \end{pmatrix} + \begin{pmatrix} 7 & 1 & -1 \\ 8 & -10 & 0 \end{pmatrix} + \begin{pmatrix} 6 & -2 & -10 \\ -12 & -4 & -8 \end{pmatrix} = \begin{pmatrix} 18 & -1 & -18 \\ 16 & -15 & -23 \end{pmatrix}$$

1 Calcula, utilizando el método de Gauss, la inversa de cada una de las siguientes matrices en el supuesto de que la tengan:

a) $\Delta = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

b) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

$B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

a) $\Delta = \begin{pmatrix} 1 & 1 & | & 1 & 0 \\ 0 & 1 & | & 0 & 1 \end{pmatrix} \xrightarrow{-1 \cdot 2F + 1F} \begin{pmatrix} 1 & 0 & | & 1 & -1 \\ 0 & 1 & | & 0 & 1 \end{pmatrix} \Rightarrow \Delta^{-1} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$

b) $B = \begin{pmatrix} 1 & 2 & | & 1 & 0 \\ 3 & 4 & | & 0 & 1 \end{pmatrix} \xrightarrow{-3 \cdot 1F + 2F} \begin{pmatrix} 1 & 2 & | & 1 & 0 \\ 0 & -2 & | & -3 & 1 \end{pmatrix} \xrightarrow{-1/2 \cdot 2F} \begin{pmatrix} 1 & 2 & | & 1 & 0 \\ 0 & 1 & | & 3/2 & -1/2 \end{pmatrix} \xrightarrow{-2 \cdot 2F + 1F} \begin{pmatrix} 1 & 0 & | & -2 & 1 \\ 0 & 1 & | & 3/2 & -1/2 \end{pmatrix}$

$B^{-1} = \begin{pmatrix} -2 & 1 \\ 3/2 & -1/2 \end{pmatrix}$

2 Calcula la inversa de cada una de las siguientes matrices o averigua que no la tiene:

a) $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$

b) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 1 & 2 & 4 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 2 & 1 \\ 2 & 0 & 0 \end{pmatrix}$

a) $|\Delta| = 1 \cdot 5 \cdot 9 + 2 \cdot 6 \cdot 7 + 3 \cdot 4 \cdot 8 - 7 \cdot 5 \cdot 3 - 8 \cdot 6 \cdot 1 - 9 \cdot 4 \cdot 2 = 0$

$|\Delta| = 0$ No tiene inversa

b) $\Delta = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 1 & 2 & 4 \end{pmatrix} \Rightarrow |\Delta| = 1$

$\text{Adj}(\Delta) = \begin{pmatrix} 0 & 2 & -1 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{pmatrix}$

$[\text{Adj}(\Delta)]^T = \begin{pmatrix} 0 & -2 & 1 \\ 2 & 1 & -2 \\ -1 & 0 & 1 \end{pmatrix}$

$\Delta^{-1} = \frac{[\text{adj}(\Delta)]^T}{|\Delta|} = \begin{pmatrix} 0 & -2 & 1 \\ 2 & 1 & -2 \\ -1 & 0 & 1 \end{pmatrix}$

c) $\Delta = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 2 & 1 \\ 2 & 0 & 0 \end{pmatrix}$

$\text{Adj}(\Delta) = \begin{pmatrix} 0 & 2 & -4 \\ 0 & -6 & 2 \\ -5 & 2 & 1 \end{pmatrix}$

$[\text{Adj}(\Delta)]^T = \begin{pmatrix} 0 & 0 & -5 \\ 2 & -6 & 2 \\ -4 & 2 & 1 \end{pmatrix}$

$|\Delta| = -10$

$\Delta^{-1} = \frac{[\text{adj}(\Delta)]^T}{|\Delta|} =$

$\begin{pmatrix} 0 & 0 & 1/2 \\ -1/5 & 3/5 & -1/5 \\ 2/5 & -1/5 & -1/10 \end{pmatrix}$