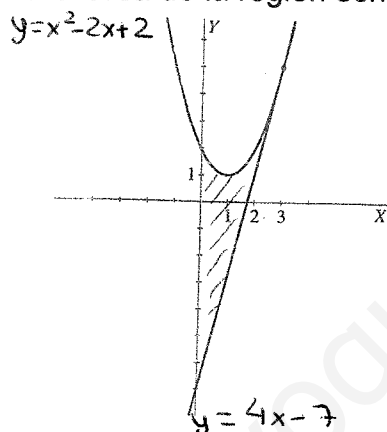


1. Calcular las siguientes integrales por el método que resulte más conveniente:

- $\int \frac{2\sin x \cos x}{1 + \sin^2 x} dx$ (1 punto)
- $\int \frac{1+x}{1+\sqrt{x}} dx$ (1,25 puntos)
- $\int \ln(25+x^2) dx$ (1,25 puntos)
- $\int \frac{x^3+1}{x^2+4} dx$ (1,25 puntos)
- $\int \sqrt{16-x^2} dx$ (1,25 puntos)

2. Calcular el área de la región del plano encerrada por las gráficas de $f(x) = 2x$ y $g(x) = 6 + 3x - x^2$. (1,5 puntos)

3. Calcular el área de la región sombreada. (1 punto)



4. De la función $f: (-1, \infty) \rightarrow \mathbb{R}$ se sabe que $f'(x) = \frac{3}{(x+2)^2}$. (1,5 puntos)

- Determinar la expresión de $f(x)$ si $f(2) = 0$.
- Halla la primitiva de $f'(x)$ cuya gráfica pasa por el punto $(0,1)$.

$$\bullet \int \frac{2\sin x \cos x}{1 + \sin^2 x} dx = \ln |1 + \sin^2 x| + C \quad \left[\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + C \right]$$

$$\bullet \int \frac{1+x}{1+\sqrt{x}} dx = \int \frac{1+t^2}{1+t^2} \cdot 2t dt = \int \frac{(1+t^2) \cdot 2t}{1+t^2} dt = \int \frac{2t^3+2t}{1+t^2} dt =$$

Cambio $\sqrt{x} = t$; $x = t^2$
 $dx = 2t dt$

$$= \int (2t^2 - 2t + 4) dt - \int \frac{4}{t+1} dt = \frac{2t^3}{3} - \frac{2t^2}{2} + 4t - 4 \ln |t+1| + C$$

Des Hacemos el cambio $\boxed{\sqrt{x} = t}$

$$\boxed{I = \frac{2\sqrt{x}^3}{3} - \frac{2(\sqrt{x})^2}{2} + 4\sqrt{x} - 4 \ln |\sqrt{x}+1| + C = \frac{2\sqrt{x}^3}{3} - x + 4\sqrt{x} - 4 \ln |\sqrt{x}+1| + C}$$

$$\bullet \int \ln(25+x^2) dx = x \ln(25+x^2) - \int \frac{x \cdot 2x}{25+x^2} dx$$

Por partes $u = \ln(25+x^2)$; $du = \frac{1}{25+x^2} \cdot 2x dx$

$dv = dx$; $v = x$

$$\textcircled{*} \int \frac{2x^2}{25+x^2} dx = \int \left(2 - \frac{50}{x^2+25} \right) dx =$$

$$= \int 2 dx - 50 \int \frac{dx}{x^2+25} = \int 2 dx - 50 \int \frac{\frac{1}{25} dx}{\left(\frac{x}{5}\right)^2 + 1} =$$

$$= \int 2 dx - \frac{50}{25} \int \frac{1/5 dx}{\left(\frac{x}{5}\right)^2 + 1} = 2x - 10 \operatorname{arctg} \frac{x}{5} + C$$

$$\boxed{\int \ln(25+x^2) dx = x \ln(25+x^2) - 2x + 10 \operatorname{arctg} \frac{x}{5} + C}$$

$$\bullet \int \frac{x^3+1}{x^2+4} dx = \int \left(x + \left(\frac{-4x+1}{x^2+4} \right) \right) dx =$$

$$= \int x dx - \underbrace{\int \frac{4x-1}{x^2+4} dx}_{(*)}$$

$$(*) \int \frac{4x-1}{x^2+4} dx = 4 \int \frac{x - 1/4}{x^2+4} dx =$$

$$= \frac{4}{2} \int \frac{2x - 1/2}{x^2+4} dx = 2 \left[\int \frac{2x}{x^2+4} dx - \frac{1}{2} \int \frac{dx}{x^2+4} \right]$$

$$= 2 \left[\int \frac{2x}{x^2+4} dx - \frac{1}{2} \cdot \int \frac{\frac{dx}{4}}{\left(\frac{x}{2}\right)^2+1} \right] =$$

$$= 2 \left[\int \frac{2x}{x^2+4} dx - \frac{1}{2} \cdot \frac{2}{4} \int \frac{\frac{dx}{2}}{\left(\frac{x}{2}\right)^2+1} \right] =$$

$$= 2 \ln |x^2+4| - \frac{2}{4} \operatorname{arctg} \frac{x}{2} + C.$$

$$\boxed{\int \frac{x^3+1}{x^2+4} dx = \frac{x^2}{2} - 2 \ln |x^2+4| + \frac{1}{2} \operatorname{arctg} \frac{x}{2} + C}$$

$$\bullet \int \sqrt{16-x^2} dx = \int \sqrt{16-(4\text{sen}t)^2} \cdot 4\text{Cost} dt =$$

$$\text{Cambio } x = 4\text{sen}t \quad dx = 4\text{Cost} dt$$

$$= \int \sqrt{16-16\text{sen}^2t} \cdot 4\text{Cost} dt = \int 4 \cdot \sqrt{1-\text{sen}^2t} \cdot 4\text{Cost} dt =$$

$$= \int 16 \cdot \text{Cost} \cdot \text{Cost} dt = 16 \int \text{Cos}^2t dt =$$

$$= 16 \int \frac{1+\text{Cos}2t}{2} dt = \frac{16}{2} \cdot \int (1+\text{Cos}2t) dt =$$

$$= 8 \cdot \left[t + \frac{\text{Sen}2t}{2} + C \right] = 8t + 4\text{Sen}2t + C.$$

Desfazemos el cambio: $x = 4\text{sen}t$

$$\left\{ \begin{array}{l} \frac{x}{4} = \text{sen}t ; t = \arcsen \frac{x}{4} \end{array} \right.$$

$$I = 8 \arcsen \frac{x}{4} + 4 \text{Sen} 2 \left(\arcsen \frac{x}{4} \right) + C$$

$$(2^\circ) f(x) = 2x ; g(x) = 6+3x-x^2$$

$$(1) \text{ Corte } f(x) \text{ y } g(x) \rightarrow 2x = 6+3x-x^2 ; x^2 + 2x - 3x - 6 = 0$$

$$x^2 - x - 6 = 0 ; x = \frac{1 \pm \sqrt{1-4(-6)}}{2} = \frac{1 \pm \sqrt{25}}{2} \left\{ \begin{array}{l} \frac{1+5}{2} = \frac{6}{2} = 3 \\ \frac{1-5}{2} = \frac{-4}{2} = -2 \end{array} \right.$$

(2) En $(-2, 3)$ vemos qué función queda por encima:

$$\left. \begin{array}{l} f(0) = 0 \\ g(0) = 6 \end{array} \right\} g(x) > f(x) \text{ en } (-2, 3)$$

$$(3) A = \int_{-2}^3 |g(x) - f(x)| dx = \int_{-2}^3 |6+3x-x^2-2x| dx$$

$$= \left| \int_{-2}^3 -x^2 + x + 6 dx \right| = \left| \left[-\frac{x^3}{3} + \frac{x^2}{2} + 6x \right]_{-2}^3 \right| =$$

$$\left| -\frac{3^3}{3} + \frac{3^2}{2} + 6 \cdot 3 - \left[-\frac{(-2)^3}{3} + \frac{(-2)^2}{2} + 6 \cdot (-2) \right] \right| =$$

$$\left| -9 + \frac{9}{2} + 18 - \left[\frac{8}{3} + 2 - 12 \right] \right| = \left| 9 + \frac{9}{2} - \left[\frac{8}{3} - 10 \right] \right|$$

$$= \left| \frac{27}{2} - \left[-\frac{22}{3} \right] \right| = \left| \frac{27}{2} + \frac{22}{3} \right| = \left| \frac{81+44}{6} \right| = \boxed{\frac{125}{6} u}$$

$$\textcircled{3} \quad A = \int_0^3 |x^2 - 2x + 2 - (4x - 7)| dx =$$

$$= \int_0^3 |x^2 - 2x + 2 - 4x + 7| dx = \left| \int_0^3 x^2 - 6x + 9 dx \right| =$$

$$= \left| \left[\frac{x^3}{3} - \frac{6x^2}{2} + 9x \right]_0^3 \right| = \left| \frac{3^3}{3} - \frac{6 \cdot 3^2}{2} + 9 \cdot 3 \right| = |9 - 27 + 27| =$$

$$= \boxed{9 u^2}$$

$$\textcircled{4} \quad f'(x) = \frac{3}{(x+2)^2}$$

$$a) \quad F(x) = \int f'(x) dx = \int 3(x+2)^{-2} dx = 3 \int (x+2)^{-2} dx$$

$$= 3 \frac{(x+2)^{-2+1}}{-1} + C = \frac{3(x+2)^{-1}}{-1} + C = \boxed{\frac{-3}{(x+2)^1} + C = F(x)}$$

$F(x)$ son todas las primitivas de $f'(x)$

$$\text{Como } f(2) = 0 \Rightarrow 0 = \frac{-3}{(2+2)^1} + C; \quad 0 = \frac{-3}{4} + C;$$

$$C = \frac{3}{4}$$

Así $f(x) = \frac{-3}{(x+2)^4} + \frac{3}{24}$ Solución del apartado a

b) Haciendo ahora $F(0)=1$ obtenemos la función primitiva de $f(x)$ que verifica la 2ª condición.

$$\frac{-3}{(0+2)^4} + C = 1 ; \quad -\frac{3}{2} + C = 1 ; \quad C = 1 + \frac{3}{2}$$

$$C = \frac{5}{2}$$

$f(x) = \frac{-3}{(x+2)^4} + \frac{5}{2}$ Solución del apartado b