

ACTIVIDADES FINALES

Fracciones algebraicas

□ 58. Realiza las siguientes sumas y restas formadas por fracciones algebraicas:

- a) $\frac{1}{x} - \frac{x}{x+1}$
 b) $\frac{2x}{x+2} + \frac{x^2-1}{x^2}$
 c) $\frac{x}{2x-1} + \frac{6x}{4x^2-4x+1}$
 d) $\frac{1-x}{x+1} - \frac{x+1}{x-1}$
 e) $\frac{2x-1}{2-x} - \frac{4x+2}{x+2} - \frac{x^2-x}{4-x^2}$
 f) $\frac{5x-1}{x^3} - \frac{2x-1}{x} + x$

□ 59. Realiza las siguientes operaciones formadas por fracciones algebraicas:

- a) $\frac{2}{x} + \frac{3}{x^2} - \frac{4}{x^3}$
 b) $\frac{x-1}{2x} + \frac{2x+3}{2x^2} - \frac{2(x^3-x^2)}{4x^3}$

■ 60. Realiza las siguientes operaciones formadas por fracciones algebraicas:

- a) $\frac{x}{x+2} - \frac{3x-1}{x^2+2x} - \frac{1}{x}$
 b) $\frac{x+4}{2x-1} - \frac{3x+1}{2x+1}$
 c) $\frac{2x}{-x+3} - \frac{x-3}{x+3} + \frac{x-5}{x^2-9}$
 d) $\frac{2}{x-3} - \frac{x+3}{x} + \frac{3x^2-1}{x^2-9}$

■ 61. Realiza las siguientes operaciones formadas por fracciones algebraicas:

- a) $\frac{2}{x^2-1} + \frac{3x-2}{x^3-x} - \frac{5x}{x+1}$
 b) $\frac{3x-1}{4x^2+4x+1} - \frac{2x^2-5x}{6x-3}$
 c) $\frac{2x+3}{2x-3} - \frac{2x-3}{2x+3}$
 d) $\frac{x-2}{x^2+2x} - \frac{x+2}{x-2} + \frac{x^2+4}{x^2-4}$
 e) $\frac{5x}{2x^2-12x+18} - \frac{x+5}{2x-6} + \frac{3x+1}{x+3}$
 f) $\frac{2x-1}{9x^2-6x+1} - \frac{3x-2}{3x^2+x} + \frac{x}{6x-2}$

■ 62. Opera y simplifica si es posible:

- a) $\frac{x-1}{x} \cdot \frac{x+1}{x^2-2x+1}$
 b) $\frac{x-1}{x+1} \cdot \frac{x-2}{x^2-x}$
 c) $\frac{x^2-1}{2x} : \frac{x+1}{3x^2-x}$
 d) $\frac{2x-1}{x^2-3x} : \frac{4x^2+4x+1}{x^2-6x+9}$
 e) $\frac{x-2}{2x^4+4x^2} : \frac{2x-4}{5x^3-3x^2}$
 f) $\frac{3a-b}{b-a} \cdot \frac{a^2-b^2}{6a-2b}$
 g) $\frac{a^2-b^2}{2ab^2+b^3} : \frac{a^2-ab}{-4ab^3-2b^4}$

■ 63. Opera y simplifica:

- a) $\left(x - \frac{1}{x}\right) : \frac{x-1}{x^2+x}$
 b) $\left(\frac{2}{x} - \frac{x}{3}\right) : \frac{x-6}{x^2-4x}$
 c) $\frac{x^3}{x^2-1} \cdot \frac{x^2-4x+3}{x^4-5x^3}$
 d) $\left(\frac{x+1}{x^2+x-2} - \frac{x-1}{x^2-x-2}\right) : \frac{x}{x^2-4}$

■ 64. Opera y simplifica:

$$\frac{2x^3+6x^2+4x}{x^2-4x+4} \cdot \frac{x^2-5x+6}{x^6+5x^5+7x^4+3x^3}$$

■ 65. Opera y simplifica:

$$\left(\frac{5x}{1-2x} - \frac{3x}{1+2x}\right) : \frac{3x-1}{4x^2-4x+1}$$

■ 66. Opera y simplifica:

- a) $\frac{x-2}{x^3-4x} - \frac{5x}{x+2} : \frac{x^2-4}{3x+1}$
 b) $\frac{x+3}{x-2} - \frac{x+1}{x^2-4} + \frac{2x+1}{x+2}$
 c) $\frac{x-1}{x-2} + \frac{x+3}{2x^2-5x+2} - \frac{2x}{x^2-4x+4}$

■ 67. Opera y simplifica:

$$\frac{\frac{x}{x-1} + \frac{1}{x^2-1}}{x + \frac{1}{x} \cdot \frac{x^2-4x}{x+1}}$$

SOLUCIONES

Fraciones algebraicas.

58.

$$a) \frac{1}{x} - \frac{x}{x+1} = \frac{x+1}{x \cdot (x+1)} - \frac{x^2}{x \cdot (x+1)} = \frac{-x^2 + x + 1}{x \cdot (x+1)}$$

$$b) \frac{2x}{x+2} - \frac{x^2-1}{x^2} = \frac{2x^3}{x^2 \cdot (x+2)} - \frac{(x^2-1) \cdot (x+2)}{x^2 \cdot (x+2)} = \frac{2x^3}{x^2 \cdot (x+2)} - \frac{x^3 + 2x^2 - x - 2}{x^2 \cdot (x+2)} = \frac{x^3 - 2x^2 + x + 2}{x^2 \cdot (x+2)}$$

$$c) \frac{x}{2x-1} - \frac{6x}{4x^2-4x+1} = \frac{x}{2x-1} - \frac{6x}{(2x-1)^2} = \frac{x \cdot (2x-1)}{(2x-1)^2} - \frac{6x}{(2x-1)^2} = \frac{2x^2 - 7x}{(2x-1)^2}$$

$$d) \frac{1-x}{x+1} - \frac{x+1}{x-1} = \frac{-(x-1) \cdot (x-1)}{(x-1) \cdot (x+1)} - \frac{(x+1)^2}{(x-1) \cdot (x+1)} = \frac{-(x^2 - 2x + 1)}{x^2 - 1} - \frac{x^2 + 2x + 1}{x^2 - 1} = \frac{-2x^2 - 2}{x \cdot (x+1)}$$

$$e) \frac{2x-1}{2-x} - \frac{4x^2+2}{x+2} - \frac{x^2-x}{4-x^2} = \frac{(2x-1) \cdot (2+x)}{4-x^2} - \frac{(4x^2+2) \cdot (2-x)}{4-x^2} - \frac{x^2-x}{4-x^2} = \frac{2x^2+2x}{4-x^2} - \frac{-4x^3+8x^2-2x+4}{4-x^2} - \frac{x^2-x}{4-x^2} = \frac{-4x^3-7x^2+5x-4}{4-x^2}$$

$$f) \frac{5x-1}{x^3} - \frac{2x-1}{x} + x = \frac{5x-1}{x^3} - \frac{2x^3-x^2}{x^3} + \frac{x^4}{x^3} = \frac{x^4 - 2x^3 + x^2 + 5x - 1}{x^3}$$

59.

$$a) \frac{2}{x} + \frac{3}{x^2} - \frac{4}{x^3} = \frac{2x^2}{x^3} + \frac{3x}{x^3} - \frac{4}{x^3} = \frac{2x^2 + 3x - 4}{x^3}$$

$$b) \frac{x-1}{2x} + \frac{2x+3}{2x^2} - \frac{2 \cdot (x^3-x^2)}{4x^3} = \frac{x-1}{2x} + \frac{2x+3}{2x^2} - \frac{x^3-x^2}{2x^3} = \frac{x^3-x^2}{2x^3} + \frac{2x^2+3x}{2x^3} - \frac{x^3-x^2}{2x^3} = \frac{2x+3}{2x^2}$$

60.

$$a) \frac{x}{x+2} - \frac{3x-1}{x^2+2x} - \frac{1}{x} = \frac{x^2}{x^2+2x} - \frac{3x-1}{x^2+2x} - \frac{x+2}{x^2+2x} = \frac{x^2-4x-1}{x^2+2x}$$

$$b) \frac{x+4}{2x-1} - \frac{3x+1}{2x+1} = \frac{(x+4) \cdot (2x-1)}{(2x-1) \cdot (2x+1)} - \frac{(2x-1) \cdot (3x+1)}{(2x-1) \cdot (2x+1)} = \frac{2x^2+7x-4-(6x^2-x-1)}{(2x-1) \cdot (2x+1)} = \frac{-4x^2+8x-3}{4x^2-1}$$

$$c) \frac{2x}{-x+3} - \frac{x-3}{x+3} - \frac{x-5}{x^2-9} = \frac{2x}{3-x} - \frac{x-3}{3+x} + \frac{x-5}{9-x^2} = \frac{2x^2+6x}{9-x^2} + \frac{(x-3)^2}{9-x^2} + \frac{x-5}{x^2-9} = \frac{2x^2+6x}{9-x^2} + \frac{x^2-6x+9}{9-x^2} + \frac{x-5}{x^2-9} = \frac{3x^2+x+4}{9-x^2}$$

$$d) \frac{2}{x-3} - \frac{x+3}{x} + \frac{3x^2-1}{x^2-9} = \frac{2x \cdot (x+3)}{x^3-9x} - \frac{x \cdot (x+3) \cdot (x-3)}{x^3-9x} + \frac{3x^3-x}{x^3-9x} = \frac{2x^2+6x}{x^3-9x} - \frac{x^3-9x}{x^3-9x} + \frac{3x^3-x}{x^3-9x} = \frac{2x^3+2x^2+14x}{x^3-9x}$$

61.

a)

$$\frac{2}{x^2-1} + \frac{3x-2}{x^3-x} - \frac{5x}{x+1} = \frac{2x}{x \cdot (x-1) \cdot (x+1)} + \frac{3x-2}{x \cdot (x-1) \cdot (x+1)} - \frac{5x \cdot x \cdot (x-1)}{x \cdot (x-1) \cdot (x+1)} = \frac{2x}{x^3-x} + \frac{3x-2}{x^3-x} - \frac{5x^3-5x^2}{x^3-x} = \frac{-5x^3+5x^2+5x-2}{x^3-x}$$

b)

$$\frac{3x-1}{4x^2+4x+1} - \frac{2x^2-5x}{6x-3} = \frac{3x-1}{(2x+1)^2} - \frac{2x^2-5x}{3 \cdot (2x-1)} = \frac{18x^2-15x+3}{24x^3+20x^2+2x-1} - \frac{8x^4-12x^3-18x^2-5x}{24x^3+20x^2+2x-1} = \frac{-8x^4-12x^3+36x^2-10x+3}{24x^3+20x^2+2x-1}$$

c)

$$\frac{2x+3}{2x-3} - \frac{2x-3}{2x+3} = \frac{(2x+3)^2}{(2x-3) \cdot (2x+3)} - \frac{(2x-3)^2}{(2x-3) \cdot (2x+3)} = \frac{24x}{4x^2-9}$$

d)

$$\frac{x-2}{x^2+2x} - \frac{x+2}{x-2} + \frac{x^2+4}{x^2-4} = \frac{(x-2)^2}{x \cdot (x^2-4)} - \frac{x \cdot (x+2)^2}{x \cdot (x^2-4)} + \frac{x^3+4x}{x \cdot (x^2-4)} = \frac{x^2-4x+4-x^3-4x^2-4x+x^3+4x}{x \cdot (x^2-4)} = \frac{-3x^2-4x+4}{x \cdot (x^2-4)}$$

e)

$$\frac{5x}{2x^2-12x+18} - \frac{x+5}{2x-6} + \frac{3x+1}{x+3} = \frac{5x \cdot (x+3) - (x+5) \cdot (x-3) \cdot (x+3) + 2 \cdot (x-3)^2 \cdot (3x+1)}{2 \cdot (x-3)^2 \cdot (x+3)} = \frac{5x^3-34x^2+66x+27}{2x^3-6x^2-18x+54}$$

f)

$$\frac{2x-1}{9x^2-6x+1} - \frac{3x-2}{3x^2+x} + \frac{x}{6x-2} = \frac{2 \cdot (2x-1) \cdot (3x+1) - 2 \cdot (3x-1)^2 \cdot (3x-2) + x \cdot (3x-1) \cdot (3x+1)}{2 \cdot (3x-1)^2 \cdot (3x+1)} = \frac{-45x^3-60x^2+27x-6}{54x^2-24x+2}$$

62.

a)

$$\frac{x-1}{x} \cdot \frac{x+1}{x^2-2x+1} = \frac{(x-1) \cdot (x+1)}{x \cdot (x-1)^2} = \frac{x+1}{x \cdot (x-1)}$$

b)

$$\frac{x-1}{x+1} \cdot \frac{x-2}{x^2-x} = \frac{(x-1) \cdot (x-2)}{x^2 \cdot (x+1) \cdot (x-1)} = \frac{x-2}{x^2 \cdot (x+1)}$$

c)

$$\frac{x^2-1}{2x} \cdot \frac{x+1}{3x^2-x} = \frac{x \cdot (x-1) \cdot (x+1) \cdot (3x-1)}{2x \cdot (x+1)} = \frac{(x-1) \cdot (3x-1)}{2}$$

d)

$$\frac{2x-1}{x^2-3x} \cdot \frac{4x^2+4x+1}{x^2-6x+9} = \frac{(2x-1) \cdot (x-3)^2}{x \cdot (x-3) \cdot (2x+1)^2} = \frac{(2x-1) \cdot (x-3)}{x \cdot (2x+1)^2}$$

e)

$$\frac{x-2}{2x^4+4x^3} \cdot \frac{2x-4}{5x^3-3x^2} = \frac{x^2 \cdot (x-2) \cdot (5x-3)}{4x^3 \cdot (x+2) \cdot (x-2)} = \frac{5x-3}{4x \cdot (x+2)}$$

f)

$$\frac{3a-b}{b-a} \cdot \frac{a^2-b^2}{6a-2b} = \frac{(3a-b) \cdot (a+b) \cdot (a-b)}{-(a-b) \cdot 2 \cdot (3a-b)} = -\frac{a+b}{2}$$

g)

$$\frac{a^2-b^2}{2ab^2+b^3} \cdot \frac{a^2-ab}{-4ab^3-2b^4} = \frac{-2 \cdot b^3 \cdot (a-b) \cdot (a+b) \cdot (2a+b)}{b^2 \cdot (2a+b) \cdot a \cdot (a-b)} = \frac{-2b \cdot (a+b)}{a}$$

63.

a)

$$\left(x - \frac{1}{x}\right) : \frac{x-1}{x^2+x} = \frac{x^2-1}{x} : \frac{x-1}{x^2+x} = \frac{(x-1) \cdot (x+1)^2 \cdot x}{x \cdot (x-1)} = (x+1)^2$$

b)

$$\left(\frac{2}{x} - \frac{x}{3}\right) : \frac{x-6}{x^2-4x} = \frac{6-x^2}{3x} : \frac{x-6}{x^2-4x} = \frac{(6-x^2) \cdot (x-4) \cdot x}{3x \cdot (x-6)} = \frac{(6-x^2) \cdot (x-4)}{3(x-6)}$$

c)

$$\frac{x^3}{x^2-1} \cdot \frac{x^2-4x+3}{x^4-5x^3} = \frac{x^3 \cdot (x-1) \cdot (x-3)}{(x-1) \cdot (x+1) \cdot x^3 \cdot (x-5)} = \frac{(x-3)}{(x+1) \cdot (x-5)}$$

d)

$$\begin{aligned} \left(\frac{x+1}{x^2+x-2} - \frac{x-1}{x^2-x-2}\right) : \frac{x}{x^2-4} &= \frac{(x+1) \cdot (x^2-x-2) - (x-1) \cdot (x^2+x-2)}{(x^2+x-2) \cdot (x^2-x-2)} : \frac{x}{x^2-4} = \\ &= \frac{-4}{(x-1) \cdot (x+2) \cdot (x-2) \cdot (x+1)} : \frac{x}{(x+2) \cdot (x-2)} = \frac{-4 \cdot (x+2) \cdot (x-2)}{x \cdot (x-1) \cdot (x+2) \cdot (x-2) \cdot (x+1)} = \frac{-4}{x \cdot (x-1) \cdot (x+1)} \end{aligned}$$

64.

$$\frac{2x^3+6x^2+4x}{x^2-4x+4} \cdot \frac{x^2-5x+6}{x^6+5x^5+7x^4+3x^3} = \frac{2x \cdot (x+1) \cdot (x+2) \cdot (x-2) \cdot (x-3)}{(x-2)^2 \cdot x^3 \cdot (x+1)^2 \cdot (x+3)} = \frac{2(x+2) \cdot (x-3)}{x^2 \cdot (x-2) \cdot (x+1) \cdot (x+3)}$$

65.

$$\left(\frac{5x}{1-2x} - \frac{3x}{1+2x}\right) : \frac{3x-1}{4x^2-4x+1} = \frac{5x \cdot (1+2x) - 3x \cdot (1-2x)}{(1-2x) \cdot (1+2x)} : \frac{3x-1}{(1-2x)^2} = \frac{2x \cdot (8x+1) \cdot (1-2x)^2}{(1-2x) \cdot (1+2x) \cdot (3x-1)} = \frac{2x \cdot (8x+1) \cdot (1-2x)}{(1+2x) \cdot (3x-1)}$$

66.

a)

$$\frac{x-2}{x^3-4x} - \frac{5x}{x+2} : \frac{x^2-4}{3x+1} = \frac{x-2}{x^3-4x} - \frac{5x \cdot (3x+1)}{(x+2)^2 \cdot (x-2)} = \frac{(x+2) \cdot (x-2)}{x \cdot (x+2)^2 \cdot (x-2)} - \frac{5x^2 \cdot (3x+1)}{x \cdot (x+2)^2 \cdot (x-2)} = \frac{-15x^3 - 4x^2 - 4}{x^4 + 2x^3 - 4x^2 - 8x}$$

b)

$$\frac{x+3}{x-2} - \frac{x+1}{x^2-4} + \frac{2x+1}{x+2} = \frac{(x+3) \cdot (x+2) - (x+1) + (2x+1) \cdot (x-2)}{x^2-4} = \frac{3x^2+3x+3}{x^2-4}$$

c)

$$\begin{aligned} \frac{x-3}{x-2} + \frac{x+3}{2x^2-5x+2} - \frac{2x}{x^2-4x+4} &= \frac{x-3}{x-2} + \frac{x+3}{(x-1) \cdot (4x-1)} - \frac{2x}{(x-2)^2} = \\ &= \frac{(x-2) \cdot (4x-1) \cdot (x-1)^2 + (x+3) \cdot (x-2)^2 - 2x \cdot (x-1) \cdot (4x-1)}{(x-1) \cdot (4x-1) \cdot (x-2)^2} = \frac{4x^4 - 32x^3 + 31x^2 - 14x + 12}{4x^4 - 21x^3 + 37x^2 - 24x + 4} \end{aligned}$$

67.

$$\frac{\frac{x}{x-1} + \frac{1}{x^2-1}}{x + \frac{1}{x}} \cdot \frac{x \cdot (x+1) + 1}{x+1} = \frac{\frac{x^2+x+1}{x^2-1}}{x + \frac{x-4}{x+1}} = \frac{\frac{x^2+x+1}{x^2-1}}{\frac{x^2+2x-4}{x+1}} = \frac{(x^2+x+1) \cdot (x+1)}{(x^2+2x-4) \cdot (x+1) \cdot (x-1)} = \frac{(x^2+x+1)}{(x^2+2x-4) \cdot (x-1)}$$