LIMITES INDETERMINADOS

Los siguientes ejercicios sobre límites están indeterminados, es decir al aplicar el valor en la función se obtiene $\frac{0}{0}$; para levantar la indeterminación se debe descomponer en factores y luego volver a evaluar para el valor dado.

1. $\lim\limits_{x \to 0} \frac{4x^3 - x \quad x}{3x^2 +}$	$\lim\limits_{x \to 0} \frac{4x^3 - 2x \quad x}{3x^2 +} = \frac{4(0)^3 - 2(0) \quad 0}{3(0)^2 + 2(x)} \quad \frac{0}{0}$ Inderminación. $\lim\limits_{x \to 0} \frac{4x^3 - \ x \quad x}{3x^2 +} = \lim\limits_{x\to 0} \frac{x(4x^2 - 2x + 1)}{x(3x + \ )}$ $= \lim\limits_{x\to 0} \frac{4x^2 - \quad + 1}{3x + 2} = \frac{4(0)^2 - 2(0) \quad 1}{3(0) \quad 2} = \frac{1}{2}$
2. $\lim\limits_{x\to 2} \frac{x^2 - 4}{x - 2}$	$\lim\limits_{x\to 2} \frac{x^2 - 4}{x - 2} = \frac{(2)^2 - }{2 \quad 2} \quad \frac{}{0}$ Indeterminación. $\lim\limits_{x \quad 2} \frac{x^2 - 4}{x - 2} = \lim\limits_{\to} \frac{(x+2)(\quad)}{}$ $\lim\limits_{x\to 2} x + 2 = 2 + 2 = 4$
3. $\lim\limits_{x\to 1} \frac{x^3 - 1}{x - 1}$	$\lim\limits_{x\to 1} \frac{x^3 - 1}{x - 1} = \frac{(1)^3 - }{1 \quad 1} \quad \frac{}{0}$ Indeterminación $\lim\limits_{x \quad 1} \frac{x^3 - 1}{x - 1} = \lim\limits_{\to} \frac{(x-1)(x^2 + x + \ )}{}$ $\lim\limits_{x\to 1} (x^2 + \quad + 1) = (1)^2 + 1 + 1 = 3$
4. $\lim\limits_{x\to 2} \frac{x^2 - 5 \quad +}{x^2 - 12 \qquad 20}$	$\lim\limits_{x\to 2} \frac{x^2 - 5 \quad + 6}{x^2 - 12 \quad + 20} = \frac{(2)^2 - 5(2) \quad 6}{(2)^2 - 12(2) \quad 20} = \frac{4 - 10 + 6}{4 - 24 + 20} = \frac{0}{0}$ Indet. $\lim\limits_{x\to 2} \frac{x^2 - 5 \quad +}{x^2 - 12x + 20} = \lim \frac{(x-3)(\qquad)}{(x-10)(x-2)}$ $\lim\limits_{x\to 2} \frac{x-3}{x-10} = \frac{2-3}{2-10} = \frac{-1}{-8} = \frac{1}{8}$

5. $\lim\limits_{x\to2}\frac{x^2+3\quad-10}{3x^2-5\quad-2}$	$\lim\limits_{x\to2}\frac{x^2+3\quad-10}{3x^2-5\quad-2}=\frac{(2)^2+3(2)\quad10}{3(2)^2-5(2)\quad2}=\frac{4+6-10}{12-10-2}=\frac{0}{0}$ Indeterminación $\lim\limits_{x\to2}\frac{x^2+3\quad-10}{3x^2-5x-2}=\lim\frac{(x+5)(\quad)}{(x-2)(3x+1)}$ $\lim\limits_{x\to2}\frac{(x+5)}{(3x+1)}=\frac{2+5}{3(2)\quad1}=\frac{7}{7}=1$
6. $\lim\limits_{y\to-2}\frac{y^3+3y\quad y}{y^2-\quad-6}$	$\lim\limits_{y\to-2}\frac{y^3+3y\quad y}{y^2-y-6}=\lim\limits_{y\to-2}\frac{y(y^2+3y+\quad)}{(y-3)(y+2)}\quad\lim\limits_{y\to-2}\frac{y(y+2)(y\quad)}{(y-3)(y+2)}$ $\lim\limits_{y\to-2}\frac{y(y+1)}{(y-3)}=\frac{-2(-2+1)}{-2-3}=\frac{2}{-5}=-\frac{2}{5}$
7. $\lim\limits_{u\to-2}\frac{u^3+4u\quad u}{(\quad+2)(u\quad)}$	$\lim\limits_{u\to-2}\frac{u^3+4u\quad+4u}{(\quad+2)(u-3)}=\frac{(-2)^3+4(2)\quad+4(-2)}{(-2+2)(-2-3)}=\frac{-8+16-8}{0(-5)}=\frac{0}{0}$ $\lim\limits_{u\to-2}\frac{u^3+4u\quad u}{(u+2)(u-3)}=\lim\limits_{u\to-2}\frac{u(u^2+4u\quad)}{(u+2)(u-3)}\quad\lim\limits_{u\to-2}\frac{u(u+2)^2}{(u+2)(u-3)}$ $\lim\limits_{u\to-2}\frac{u(u+2)}{(u-3)}=\frac{-2(-2+2)}{-2-3}=\frac{0}{-5}=0$
8. $\lim\limits_{x\to2}\frac{x^2-4}{x^2-3\quad+}$	$\lim\limits_{x\to2}\frac{x^2-4}{x^2-3\quad+2}=\frac{(2)^2-}{(2)^2-3(2)\quad2}\quad\frac{}{0}$ Indeterminación. $\lim\limits_{x\quad2}\frac{x^2-4}{x^2-3x+2}=\lim\limits_{\to}\frac{(x+2)(\quad)}{(x-2)(x-1)}$ $\lim\limits_{x\to2}\frac{(x+2)}{(x-1)}=\frac{2+2}{2\quad1}=\frac{4}{1}=4$
9. $\lim\limits_{x\to5}\frac{x^2-7\quad+10}{x^2-25}$	$\lim\limits_{x\to5}\frac{x^2-7\quad+10}{x^2-25}=\frac{(5)^2-7(5)\quad10}{(5)^2-25}=\frac{25-35+10}{25\quad25}=\frac{0}{0}$ Indeterminación. $\lim\limits_{x\to5}\frac{x^2-7\quad+10}{x^2-25}=\lim\frac{(x-5)(\quad)}{(x+5)(x-5)}$ $\lim\limits_{x\to5}\frac{(x-2)}{(x+5)}=\frac{5-}{\quad5}\quad\frac{3}{10}$

10. $\lim\limits_{h\to0}\dfrac{(x+h)^3\quad x^3}{h}$	$\lim\limits_{h\to0}\dfrac{(x+h)^3-x^3}{h}=\dfrac{(x+0)\ \ -x}{0}=\dfrac{x\ \ -x^3}{0}=\dfrac{0}{0}$ Indeterminación. $\lim\limits_{h\to0}\dfrac{(x+h)^3\quad x^3}{h}=\lim\limits_{h\to0}\dfrac{x^3+3x^2h+\ \ xh^2+h^3-x^3}{h}\quad\lim\limits_{h\to0}\dfrac{h(3x^2+\ \ xh\quad h\ )}{h}$ $\lim\limits_{h\to0}(3x^2+3xh+h^2)=3x^2+3x(0)+(\ \ )^2=3x^2$ ■

Los siguientes 8 ejercicios también son indeterminados, la presencia de radicales obliga a levantar la Inderminación mediante la racionalización de los radicales aplicando el producto conjugado.

11. $\lim\limits_{x\to0}\dfrac{\sqrt{1+x}-}{x}$	$\lim\limits_{x\to0}\dfrac{\sqrt{1+x}-1}{x}\quad\dfrac{\sqrt{1+0}-1}{0}=\dfrac{0}{\ }$ Indeterminado $\lim\limits_{x\to0}\dfrac{\sqrt{1+x}-1}{x}=\lim\dfrac{\sqrt{1+x}-1}{\ }\quad\dfrac{\sqrt{1+x}+1}{\sqrt{1+x}+}$ $\lim\limits_{x\to0}\dfrac{(\sqrt{1+x}-1)(\sqrt{1+x}+1)}{x(\sqrt{1+x}+1)}=\lim\limits_{x\to0}\dfrac{(\sqrt{1+x})^2-1}{x(\sqrt{1+x}+1)}\quad\lim\limits_{x\to0}\dfrac{1+x-}{x(\sqrt{1+x}+1)}$ $\lim\limits_{\to0}\dfrac{x}{x(\sqrt{1+\ \ }+1)}=\lim\limits_{x}\dfrac{1}{\sqrt{1+x}+1}=\dfrac{\ }{\sqrt{1+0}+1}=\dfrac{1}{2}$ ■
12. $\lim\limits_{x\to7}\dfrac{2-\sqrt{x}}{x^2-49}$	$\lim\limits_{x\to7}\dfrac{2-\sqrt{x-3}}{x^2-49}=\dfrac{2-\sqrt{7-3}}{(7)^2-49}=\dfrac{2-\sqrt{4}}{49\quad49}=\dfrac{2-2}{0}=\dfrac{0}{0}$ Indeterminado $\lim\limits_{\to7}\dfrac{2-\sqrt{x-3}}{\ ^2-49}=\lim\limits_{x}\dfrac{2-\sqrt{x-3}}{x\quad49}\quad\dfrac{2+\sqrt{x-3}}{2+\sqrt{x}}$ $\lim\limits_{\to7}\dfrac{(2-\sqrt{x-3})(2+\sqrt{x-3})}{(x^2-49)(2+\sqrt{x-3})}=\lim\limits_{x}\dfrac{(2)^2-(\sqrt{x-3})^2}{(x+7)(x-7)(2+\sqrt{x-3})}$ $\lim\limits_{x\to7}\dfrac{4-(x-\ \ )}{(x+7)(x-7)(2+\sqrt{x-3})}=\lim\dfrac{4-x+}{(x+7)(x-7)(2+\sqrt{x-3})}$ $\lim\limits_{x\to7}\dfrac{7-x}{(x+7)(x-7)(2+\sqrt{x-3})}=\lim\dfrac{-(x-7)}{(x+7)(x-7)(2+\sqrt{x-3})}$ $\lim\limits_{x\to7}\dfrac{-1}{(x+7)(2+\sqrt{\ \ -3})}=\dfrac{-1}{(7+7)(2+\sqrt{7-3})}=\dfrac{-1}{14(2+\sqrt{4})}=\dfrac{-1}{56}$ ■

13. $\lim\limits_{x\to 0}\frac{\sqrt{1+x+x^2}-}{x}$

$$\lim_{x\to 0}\frac{\sqrt{1+x+x^2}-1}{x} \quad \frac{\sqrt{1+0+(0)^2}-1}{0}=\frac{0}{\ }$$ Indeterminado

$$\lim_{x\to 0}\frac{\sqrt{1+x+x^2}-1}{x}=\lim\frac{\sqrt{1+x+x^2}-1}{\ } \quad \frac{\sqrt{1+x+x^2}+1}{\sqrt{1+x+x^2}+}$$

$$\lim_{x\to 0}\frac{\left(\sqrt{1+x+x^2}-1\right)\left(\sqrt{1+x+x^2}+1\right)}{x\left(\sqrt{1+x+x^2}+1\right)}=\lim_{x\to 0}\frac{\left(\sqrt{1+x+x^2}\right)^2-1}{x\left(\sqrt{1+x+x^2}+1\right)} \quad \lim_{x\to 0}\frac{1+x+x^2-}{x\left(\sqrt{1+x+x^2}+1\right)}$$

$$\lim_{x\to 0}\frac{x+x^2}{x\left(\sqrt{1+x+x^2}+1\right)}=\lim\frac{x(1+\ )}{x\left(\sqrt{1+x+x\ }+1\right)}$$

$$\lim_{x\to 0}\frac{1+x}{\sqrt{1+x+x\ }+}=\frac{1\quad 0}{\sqrt{1+\ +(0)^2}+1}=\frac{1}{\sqrt{1}+}=\frac{1}{2}$$ ■

14. $\lim\limits_{x\to 0}\frac{\sqrt{1+x}-\sqrt{\ -}}{x}$

$$\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-}}{x}=\frac{\sqrt{1+0}-\sqrt{1-0}}{0}=\frac{\sqrt{1}-\sqrt{1}}{0}=\frac{0}{0}$$ Indeterminado.

$$\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}=\lim\frac{\sqrt{1+x}-\sqrt{1-x}}{\ } \quad \frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{\ -}}$$

$$\lim_{x\to 0}\frac{\left(\sqrt{1+x}-\sqrt{1-x}\right)\left(\sqrt{1+x}+\sqrt{1-x}\right)}{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}=\lim\frac{\left(\sqrt{1+x}\right)^2-\left(\sqrt{1-x}\right)}{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}$$

$$\lim_{x\to 0}\frac{\left(\sqrt{1+x}\right)^2-\left(\sqrt{\ -}\right)}{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}=\lim\frac{1+\ \ -(\ \ -x)}{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}$$

$$\lim_{x\to 0}\frac{1+x-1+x}{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}=\lim\frac{2x}{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}$$

$$\lim_{x\to 0}\frac{2}{\sqrt{1+x}+\sqrt{1-}}=\frac{2}{\sqrt{1+0}+\sqrt{1-0}}=\frac{2}{2\sqrt{1}}=1$$ ■

15. $\lim_{x\to3}\dfrac{\sqrt{x^2-2x+6}-\sqrt{x\ \ +2x-6}}{\ ^2-4x+}$

$$\lim_{x\to3}\frac{\sqrt{x^2-2x+6}-\sqrt{x\ \ +2x-6}}{x^2-4\ \ +3}=\frac{\sqrt{(3)^2-2(3)+6}-\sqrt{(3)\ \ +2(3)-6}}{(3)^2-4(3)\ \ 3}=\frac{\sqrt{9-6+6}-\sqrt{9+6-6}}{9-12+3}=\frac{0}{0}$$

$$\lim_{x\to3}\frac{\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}}{x^2-4\ \ +}\times\frac{\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}}{\sqrt{x^2-2x+6}+\sqrt{x\ \ +2x-6}}$$

$$\lim_{x\to3}\frac{\left(\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}\right)\left(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}\right)}{\left(x^2-4x+3\right)\left(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}\right)}$$

$$\lim_{x\to3}\frac{\left(\sqrt{x^2-2x+6}\right)^2-\left(\sqrt{x\ \ +2x-6}\right)}{\left(x^2-4x+3\right)\left(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}\right)}=\frac{x^2-2x+6-\left(x\ \ +2x-6\right)}{\left(x^2-4x+3\right)\left(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}\right)}$$

$$\lim_{x\to3}\frac{x^2-2x+6-x\ \ -2x+6}{(x-3)(x-1)\left(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}\right)}=\lim\frac{-4x+12}{(x-3)(x-1)\left(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}\right)}$$

$$\lim_{x\to3}\frac{-4(x\ \ \ )}{(x-3)(x-1)\left(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}\right)}=\lim_{x}\frac{-4}{(x-1)\left(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6}\right)}$$

$$=\frac{-4}{(3-1)\left(\sqrt{(3)^2-2(3)+6}+\sqrt{(3)\ \ \ 2(3)-6}\right)}=\frac{-4}{2\left(\sqrt{9}+\sqrt{9}\right)}=\frac{-4}{2(\ \ )}=-\frac{1}{3}$$

16. $\lim_{h\to0}\dfrac{\sqrt{x+h}\ \ \ \sqrt{x}}{h}$

$$\lim_{h\to0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\qquad\frac{\sqrt{x+0}-\sqrt{x}}{0}=-\ \text{Indeterminado}$$

$$\lim_{h\to0}\frac{\sqrt{x+h}-\sqrt{x}}{h}=\lim\frac{\sqrt{x+h}-\sqrt{x}}{}\qquad\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}\ \ \ \sqrt{x}}$$

$$\lim_{h\to0}\frac{\left(\sqrt{x+h}-\sqrt{x}\right)\left(\sqrt{x+h}+\sqrt{x}\right)}{h\left(\sqrt{x+h}+\sqrt{x}\right)}=\lim\frac{\left(\sqrt{x+h}\right)^2-\left(\sqrt{x}\right)}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$\lim_{h\to0}\frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}=\lim\frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}\qquad\lim\frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$\lim_{h\to0}\frac{1}{\sqrt{x+h}+\sqrt{x}}=\frac{1}{\sqrt{x+0}+\sqrt{x}}\qquad\frac{1}{2\sqrt{x}}$$

17. $\lim_{x\to 4}\frac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{\ }}$

$$\lim_{x\to 4}\frac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}}=\frac{\sqrt{2(4)+1}-3}{\sqrt{4-2}-\sqrt{2}}=\frac{\sqrt{9}-3}{\sqrt{2}-\sqrt{2}}=\frac{0}{0}\text{ Indeterminado}$$

La presencia de radicales en el numerador y en el denominador obliga a la formación de dos productos conjugados:

$$\lim_{x\to 4}\frac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}}=\lim\frac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}}\times\frac{\sqrt{2x+1}+3}{\sqrt{2x+1}+3}\times\frac{\sqrt{x-2}+\sqrt{2}}{\sqrt{x-2}+\sqrt{2}}$$

$$\lim_{x\to 4}\frac{\left(\sqrt{2x+1}-3\right)\left(\sqrt{2x+1}+3\right)\left(\sqrt{x-2}+\sqrt{2}\right)}{\left(\sqrt{x-2}-\sqrt{2}\right)\left(\sqrt{x-2}+\sqrt{2}\right)\left(\sqrt{2x+1}+3\right)}=\lim\frac{\left[\left(\sqrt{2x+1}\right)^2-9\right.\ \left(\sqrt{\ }-2+\sqrt{2}\right)}{\left[\left(\sqrt{\ }-2\right)^2-\left(\sqrt{2}\right)\right.\ \left(\sqrt{2x+1}+3\right)}$$

$$\lim_{x\to 4}\frac{[2x+1-9]\left(\sqrt{x-2}+\sqrt{2}\right)}{[x-2-2]\left(\sqrt{2x+1}+3\right)}=\lim_{x\to 4}\frac{[2x-8]\left(\sqrt{x-2}+\sqrt{2}\right)}{[x-4]\left(\sqrt{2x+1}+3\right)}\ \lim_{x\to 4}\frac{2(x-4)\left(\sqrt{x-2}+\sqrt{2}\right)}{(x-4)\left(\sqrt{2x+1}+3\right)}$$

$$\lim_{x\to 4}\frac{2\left(\sqrt{x-2}+\sqrt{2}\right)}{\left(\sqrt{2x+1}+\ \right)}=\frac{2\left(\sqrt{4-2}+\sqrt{2}\right)}{\sqrt{2(4)+1}+3}=\frac{2\left(\sqrt{2}+\sqrt{2}\right)}{\sqrt{9}+\ }=\frac{2\left(2\sqrt{2}\right)}{3+3}=\frac{4\sqrt{\ }}{6}=\frac{2\sqrt{\ }}{3}\ \blacksquare$$

18. $\lim_{x\to 0}\frac{\sqrt{x^2+p}\quad p}{\sqrt{x^2+q}\quad q}$

$$\lim_{x\to 0}\frac{\sqrt{x^2+p}\quad p}{\sqrt{x^2\quad q^2}-q}=\frac{\sqrt{(0)^2+p^2}}{\sqrt{(0)^2+q^2}}=\frac{p-p}{q-q}=\frac{0}{0}\text{ Indeterminado}$$

$$\lim_{x\to 0}\frac{\sqrt{x^2+p^2}-p}{\sqrt{x^2+q^2}-q}=\lim\frac{\sqrt{x^2+p^2}-p}{\sqrt{x^2+q^2}-q}\times\frac{\sqrt{x^2+p^2}+p}{\sqrt{x^2+p^2}+p}\times\frac{\sqrt{x^2+q^2}+q}{\sqrt{x^2+q^2}+q}$$

$$\lim_{x\to 0}\frac{\left(\sqrt{x^2+p^2}-p\right)\left(\sqrt{x^2+p^2}+p\right)\left(\sqrt{x^2+q^2}+q\right)}{\left(\sqrt{x^2+q^2}-q\right)\left(\sqrt{x^2+q^2}+q\right)\left(\sqrt{x^2+p^2}+p\right)}$$

$$\lim_{\to 0}\frac{\left|\left(\sqrt{x^2+p^2}\right)^2-p^2\right]\left(\sqrt{x^2+q^2}+q\right)}{\left[\left(\sqrt{x^2+q^2}\right)^2-q^2\ \left(\sqrt{x^2+p^2}+p\right)\right.}=\lim_{x}\frac{\left[x^2+p^2-p^2\ \left(\sqrt{x^2+q^2}+q\right)\right.}{\left[x^2+q^2-q^2\ \left(\sqrt{x^2+p^2}+p\right)\right.}$$

$$\lim_{x\to 0}\frac{x^2\left(\sqrt{x^2\quad q^2}+q\right)}{x^2\left(\sqrt{x^2+p^2}\quad\right)}=\lim\frac{\sqrt{x^2+q^2}+q}{\sqrt{x^2+p^2}+p}=\frac{\sqrt{(0)^2+q^2}}{\sqrt{(0)^2+p^2}\quad p}=\frac{\sqrt{q^2}+q}{\sqrt{p^2}+p}=\frac{2q}{2p}=\frac{q}{p}\ \blacksquare$$

Los 2 siguientes ejercicios están indeterminados en primera instancia, la presencia de radicales cúbicos obliga a buscar un factor racionalizante que permita levantar dicha indeterminación; recuerde que:

$$a^3+b^3=(a+b)\left(a^2-ab+b^2\right)$$
$$a^3-b^3=(a-b)\left(a^2+ab+b^2\right)$$

19. $\lim\limits_{x\to 8}\dfrac{x-8}{\sqrt[3]{x}-2}$

$$\lim_{x\to 8}\frac{x-8}{\sqrt[3]{x}-2}=\frac{8\quad 8}{\sqrt[3]{8}\quad 2}\quad\frac{0}{0}$$ Está indeterminado.

$$\lim_{x\to 8}\frac{x-8}{\sqrt[3]{x}-2}=\lim_{x\to 8}\frac{\left(x^{\cdot}\right)^3-(2)^3}{x^{\cdot}-2}\qquad\lim_{x\to 8}\frac{\left(x^{\cdot}-2\right)\left(x^{\cdot}+2x^{\cdot}+4\right)}{\left(x^{\cdot}-2\right)}$$

$$\lim_{x\to 8}\left(x^{\cdot}+2x^{\cdot}+4\right)=\lim\left(\sqrt[3]{x^2}+2\sqrt[3]{x}+4\right)=\sqrt[3]{64}+2\left(\sqrt[3]{8}\right)+4=4+4+4=12$$

20. $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x}-1}{\sqrt{x}-1}$

$$\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt{x}-1}=\frac{\sqrt[3]{1}\quad 1}{\sqrt{1}\quad 1}\quad\frac{0}{0}$$ Está indeterminado.

$$\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt{x}-1}=\lim\frac{x^{\cdot}-1}{\sqrt{x}-1}\times\frac{x^{\cdot}+x^{\cdot}+1}{x^{\cdot}+x^{\cdot}\quad 1}\times\frac{\sqrt{x}+1}{\sqrt{x}+1}$$

Reordenando los factores se tiene:

$$\lim_{\to 1}\frac{\left(x^{\cdot}-1\right)\left(x^{\cdot}+x^{\cdot}+1\right)\left(\sqrt{x}+1\right)}{\left(x^{\cdot}+x^{\cdot}+1\right)\left(\sqrt{x}-1\right)\left(\sqrt{x}+1\right)}=\lim_{x}\frac{(x-1)\left(\sqrt{x}+1\right)}{\left(x^{\cdot}+x^{\cdot}+1\right)(x-\ )}$$

$$\lim_{x\to 1}\frac{\sqrt{x}+1}{x^{\cdot}+x^{\cdot}+1}=\frac{\sqrt{1}\quad 1}{\ +1+1}\quad\frac{2}{3}$$