Resuelve las siguientes integrales:

a) $\displaystyle\int \frac{2x-4}{(x-1)^2(x+3)}\,dx$ b) $\displaystyle\int \frac{2x+3}{(x-2)(x+5)}\,dx$

c) $\displaystyle\int \frac{1}{(x-1)(x+3)^2}\,dx$ d) $\displaystyle\int \frac{3x-2}{x^2-4}\,dx$

a) $\displaystyle\int \frac{2x-4}{(x-1)^2(x+3)}\,dx$

Descomponemos en fracciones simples:

$$\frac{2x-4}{(x-1)^2(x+3)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+3}$$

$$\frac{2x-4}{(x-1)^2(x+3)} = \frac{A(x-1)(x+3) + B(x+3) + C(x-1)^2}{(x-1)^2(x+3)}$$

$$2x-4 = A(x-1)(x+3) + B(x+3) + C(x-1)^2$$

Hallamos A, B y C:

$$\left.\begin{array}{llll} x = 1 & \rightarrow & -2 = 4B & \rightarrow\ B = -1/2 \\ x = -3 & \rightarrow & -10 = 16C & \rightarrow\ C = -5/8 \\ x = 0 & \rightarrow & -4 = -3A + 3B + C & \rightarrow\ A = 5/8 \end{array}\right\}$$

Por tanto:

$$\int \frac{2x-4}{(x-1)^2(x+3)}\,dx = \int \frac{5/8}{x-1}\,dx + \int \frac{-1/2}{(x-1)^2}\,dx + \int \frac{-5/8}{x+3}\,dx =$$

$$= \frac{5}{8}\,ln|x-1| + \frac{1}{2}\cdot\frac{1}{(x-1)} - \frac{5}{8}\,ln|x+3| + k = \frac{5}{8}\,ln\left|\frac{x-1}{x+3}\right| + \frac{1}{2x-2} + k$$

b) $\displaystyle\int \frac{2x+3}{(x-2)(x+5)}\,dx$

Descomponemos en fracciones simples:

$$\frac{2x+3}{(x-2)(x+5)} = \frac{A}{x-2} + \frac{B}{x+5} = \frac{A(x+5) + B(x-2)}{(x-2)(x+5)}$$

$$2x+3 = A(x+5) + B(x-2)$$

Hallamos A y B:

$$\left.\begin{array}{llll} x = 2 & \rightarrow & 7 = 7A & \rightarrow\ A = 1 \\ x = -5 & \rightarrow & -7 = -7B & \rightarrow\ B = 1 \end{array}\right\}$$

Por tanto:

$$\int \frac{2x+3}{(x-2)(x+5)}\,dx = \int \frac{1}{x-2}\,dx + \int \frac{1}{x+5}\,dx =$$

$$= ln|x-2| + ln|x+5| + k = ln|(x-2)(x+5)| + k$$

c) $\displaystyle\int \frac{1}{(x-1)(x+3)^2}\,dx$

Descomponemos en fracciones simples:

$$\frac{1}{(x-1)(x+3)^2} = \frac{A}{x-1} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$$

$$\frac{1}{(x-1)(x+3)^2} = \frac{A(x+3)^2 + B(x-1)(x+3) + C(x-1)}{(x-1)(x+3)^2}$$

$$1 = A(x+3)^2 + B(x-1)(x+3) + C(x-1)$$

Hallamos A, B y C:

$$\left.\begin{array}{llll} x=1 & \to\ 1 = 16A & \to\ A = 1/16 \\ x=-3 & \to\ 1 = -4C & \to\ C = -1/4 \\ x=0 & \to\ 1 = 9A - 3B - C & \to\ B = -1/16 \end{array}\right\}$$

Por tanto:

$$\int \frac{1}{(x-1)(x+3)^2}\,dx = \int \frac{1/16}{x-1}\,dx + \int \frac{-1/16}{x+3}\,dx + \int \frac{-1/4}{(x+3)^2}\,dx =$$

$$= \frac{1}{16}\ln|x-1| - \frac{1}{16}\ln|x+3| + \frac{1}{4}\cdot\frac{1}{(x+3)} + k =$$

$$= \frac{1}{16}\ln\left|\frac{x-1}{x+3}\right| + \frac{1}{4(x+3)} + k$$

d) $\displaystyle\int \frac{3x-2}{x^2-4}\,dx = \int \frac{3x-2}{(x-2)(x+2)}\,dx$

Descomponemos en fracciones simples:

$$\frac{3x-2}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2} = \frac{A(x+2) + B(x-2)}{(x-2)(x+2)}$$

$$3x - 2 = A(x+2) + B(x-2)$$

Hallamos A y B:

$$\left.\begin{array}{llll} x=2 & \to\ 4 = 4A & \to\ A = 1 \\ x=-2 & \to\ -8 = -4B & \to\ B = 2 \end{array}\right\}$$

Por tanto:

$$\int \frac{3x-2}{x^2-4}\,dx = \int \frac{1}{x-2}\,dx + \int \frac{2}{x+2}\,dx =$$

$$= \ln|x-2| + 2\ln|x+2| + k = \ln\left[|x-2|(x+2)^2\right] + k$$