

CALCULA LOS SIGUIENTES LÍMITES:

$$1) \lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = \frac{0}{0} \text{ (I)} = \lim_{x \rightarrow 1} \frac{(x-1)(x+1)}{(x-1)} = \lim_{x \rightarrow 1} (x+1) = 2$$

$$2) \lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1} = \frac{0}{0} \text{ (I)} = \lim_{x \rightarrow 1} \frac{(x-1)(x^2 + x + 1)}{(x-1)(x+1)} = \lim_{x \rightarrow 1} \frac{x^2 + x + 1}{x+1} = \frac{(1)^2 + (1) + 1}{(1) + 1} = \frac{3}{2}$$

$$\begin{array}{r|rrrr} & 1 & 0 & 0 & -1 \\ 1 & & +1 & +1 & +1 \\ \hline & 1 & +1 & +1 & \boxed{0} \end{array} \Rightarrow x^3 - 1 = (x-1)(x^2 + x + 1)$$

$$3) \lim_{x \rightarrow -1} \frac{2x^2 + 5x + 3}{x^3 + 2x^2 + 2x + 1} = \frac{0}{0} \text{ (I)} = \lim_{x \rightarrow -1} \frac{(x+1)(2x+3)}{(x+1)(x^2 + x + 1)} = \lim_{x \rightarrow -1} \frac{(2x+3)}{(x^2 + x + 1)} = \frac{1}{1} = 1$$

$$\begin{array}{r|rrr} & 2 & 5 & 3 \\ -1 & & -2 & -3 \\ \hline & 2 & +3 & \boxed{0} \end{array} \Rightarrow 2x^2 + 5x + 3 = (x+1)(2x+3)$$

$$\begin{array}{r|rrrr} & 1 & +2 & +2 & +1 \\ -1 & & -1 & -1 & -1 \\ \hline & 1 & +1 & +1 & \boxed{0} \end{array} \Rightarrow x^3 + 2x^2 + 2x + 1 = (x+1)(x^2 + x + 1)$$

$$4) \lim_{x \rightarrow 3} \frac{x^2 - 6x + 9}{x - 3} = \frac{0}{0} \text{ (I)} = \lim_{x \rightarrow 3} \frac{(x-3)^2}{(x-3)} = \lim_{x \rightarrow 3} (x-3) = 0$$

$$5) \lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x^2 - 4x + 4} = \frac{0}{0} \text{ (I)} = \lim_{x \rightarrow 2} \frac{(x-2)(x-3)}{(x-2)^2} = \lim_{x \rightarrow 2} \frac{x-3}{x-2} = \frac{-1}{0} = \begin{cases} \lim_{x \rightarrow 2^-} \frac{x-3}{x-2} = \frac{-1}{0^-} = +\infty \\ \lim_{x \rightarrow 2^+} \frac{x-3}{x-2} = \frac{-1}{0^+} = -\infty \end{cases} \Rightarrow x = 2 \text{ es A.V.}$$

$$6) \lim_{x \rightarrow 0} \frac{2x^3 + 6x^2 - 3x}{2x^2 + 5x} = \frac{0}{0} \text{ (I)} = \lim_{x \rightarrow 0} \frac{x(2x^2 + 6x - 3)}{x(2x + 5)} = \lim_{x \rightarrow 0} \frac{2x^2 + 6x - 3}{2x + 5} = \frac{2 \cdot (0)^2 + 6 \cdot (0) - 3}{2 \cdot (0) + 5} = -\frac{3}{5}$$

$$\lim_{x \rightarrow +\infty} \frac{2x^3 + 6x^2 - 3x}{2x^2 + 5x} = \frac{+\infty}{+\infty} \text{ (I)} = \lim_{x \rightarrow +\infty} \frac{\frac{2x^3}{x^3} + \frac{6x^2}{x^3} - \frac{3x}{x^3}}{\frac{2x^2}{x^3} + \frac{5x}{x^3}} = \lim_{x \rightarrow +\infty} \frac{2 + \frac{6}{x} - \frac{3}{x^2}}{\frac{2}{x} + \frac{5}{x^2}} = \frac{2 + 0 - 0}{0 + 0} = +\infty$$

$$\lim_{x \rightarrow -\infty} \frac{2x^3 + 6x^2 - 3x}{2x^2 + 5x} = \frac{-\infty}{+\infty} \text{ (I)} = \lim_{x \rightarrow -\infty} \frac{\frac{2x^3}{x^3} + \frac{6x^2}{x^3} - \frac{3x}{x^3}}{\frac{2x^2}{x^3} + \frac{5x}{x^3}} = \lim_{x \rightarrow -\infty} \frac{2 + \frac{6}{x} - \frac{3}{x^2}}{\frac{2}{x} + \frac{5}{x^2}} = \frac{2 + 0 - 0}{0 + 0} = -\infty$$

$$7) \lim_{x \rightarrow 3^+} \left(\frac{4x-2}{x-3} \right)^{\frac{1}{x}} = \left(\frac{12}{0^+} \right)^{\frac{1}{3}} = (+\infty)^{\frac{1}{3}} = +\infty$$

$$8) \lim_{x \rightarrow +\infty} \frac{1+2x-3x^3}{x^2-x^3-4} = \frac{-\infty}{-\infty} \text{ (I)} = \lim_{x \rightarrow +\infty} \frac{\frac{1}{x^3} + \frac{2x}{x^3} - \frac{3x^3}{x^3}}{\frac{x^2}{x^3} - \frac{x^3}{x^3} - \frac{4}{x^3}} = \lim_{x \rightarrow +\infty} \frac{\frac{1}{x^3} + \frac{2}{x^2} - 3}{\frac{1}{x} - 1 - \frac{4}{x^3}} = \frac{0+0-3}{0-1-0} = \frac{-3}{-1} = 3$$

$$9) \lim_{x \rightarrow -\infty} \frac{5x^2-3x+1}{x^2-x^3-4} = \frac{+\infty}{+\infty} \text{ (I)} = \lim_{x \rightarrow -\infty} \frac{\frac{5x^2}{x^3} - \frac{3x}{x^3} + \frac{1}{x^3}}{\frac{x^2}{x^3} - \frac{x^3}{x^3} - \frac{4}{x^3}} = \lim_{x \rightarrow -\infty} \frac{\frac{5}{x} - \frac{3}{x^2} + \frac{1}{x^3}}{\frac{1}{x} - 1 - \frac{4}{x^3}} = \frac{0+0+0}{0-1-0} = \frac{0}{-1} = 0$$

$$10) \lim_{x \rightarrow -\infty} \frac{x^4-3x+1}{2x-3x^3-4} = \frac{+\infty}{+\infty} \text{ (I)} = \lim_{x \rightarrow -\infty} \frac{\frac{x^4}{x^4} - \frac{3x}{x^4} + \frac{1}{x^4}}{\frac{2x}{x^4} - \frac{3x^3}{x^4} - \frac{4}{x^4}} = \lim_{x \rightarrow -\infty} \frac{1 - \frac{3}{x^3} + \frac{1}{x^4}}{\frac{2}{x^3} - \frac{3}{x} - \frac{4}{x^4}} = \frac{1-0+0}{0-0-0} = \frac{1}{0} = +\infty$$

$$11) \lim_{x \rightarrow -\infty} \frac{-2x^4-3x+1}{x^3-3x^4-4} = \frac{-\infty}{-\infty} \text{ (I)} = \lim_{x \rightarrow -\infty} \frac{\frac{-2x^4}{x^4} - \frac{3x}{x^4} + \frac{1}{x^4}}{\frac{x^3}{x^4} - \frac{3x^4}{x^4} - \frac{4}{x^4}} = \lim_{x \rightarrow -\infty} \frac{-2 - \frac{3}{x^3} + \frac{1}{x^4}}{\frac{1}{x} - 3 - \frac{4}{x^4}} = \frac{-2-0+0}{0-3-0} = \frac{2}{3}$$

$$12) \lim_{x \rightarrow 1} \frac{2x^3-4x^2+2x}{x^3-x^2-x+1} = \frac{0}{0} \text{ (I)} = \lim_{x \rightarrow 1} \frac{(x-1)^2 \cdot 2x}{(x-1)^2 \cdot (x+1)} = \lim_{x \rightarrow 1} \frac{2x}{x+1} = \frac{2}{2} = 1$$

$$\begin{array}{r|rrrr} & 2 & -4 & +2 & 0 \\ +1 & & +2 & -2 & 0 \\ \hline & 2 & -2 & 0 & 0 \\ +1 & & +2 & 0 & \\ \hline & 2 & 0 & 0 & \end{array} \Rightarrow 2x^3 - 4x^2 + 2x = (x-1)^2 \cdot 2x$$

$$\begin{array}{r|rrrr} & 1 & -1 & -1 & +1 \\ +1 & & +1 & 0 & 0 \\ \hline & 1 & 0 & -1 & 0 \\ +1 & & +1 & +1 & \\ \hline & 1 & +1 & 0 & \end{array} \Rightarrow x^3 - x^2 - x + 1 = (x-1)^2 \cdot (x+1)$$

$$13) \lim_{x \rightarrow -2} \frac{2x^2 + 7x + 6}{x^3 + 3x^2 + 3x + 2} = \frac{0}{0} \text{ (I)} = \lim_{x \rightarrow -2} \frac{(x+2)(2x+3)}{(x+2)(x^2+x+1)} = \lim_{x \rightarrow -2} \frac{2x+3}{x^2+x+1} = \frac{2 \cdot (-2) + 3}{(-2)^2 + (-2) + 1} = -\frac{1}{3}$$

$$\begin{array}{r|rrrr} & 2 & +7 & +6 & \\ -2 & & -3 & -6 & \\ \hline & 2 & +3 & \boxed{0} & \end{array} \Rightarrow 2x^2 + 7x + 6 = (x+2)(2x+3)$$

$$\begin{array}{r|rrrrr} & 1 & +3 & +3 & +2 & \\ -2 & & -2 & -2 & -2 & \\ \hline & 1 & +1 & +1 & \boxed{0} & \end{array} \Rightarrow x^3 + 3x^2 + 3x + 2 = (x+2)(x^2+x+1)$$

$$14) \lim_{x \rightarrow 1} \frac{x^4 - x^3 + x^2 - 2x + 1}{x^3 - x^2 + x - 1} = \frac{0}{0} \text{ (I)} = \lim_{x \rightarrow 1} \frac{(x-1)(x^3+x-1)}{(x-1)(x^2+1)} = \lim_{x \rightarrow 1} \frac{x^3+x-1}{x^2+1} = \frac{1}{2}$$

$$\begin{array}{r|rrrrr} & 1 & -1 & +1 & -2 & +1 \\ 1 & & +1 & 0 & +1 & -1 \\ \hline & 1 & 0 & +1 & -1 & \boxed{0} \end{array} \Rightarrow x^4 - x^3 + x^2 - 2x + 1 = (x-1)(x^3+x-1)$$

$$\begin{array}{r|rrrr} & 1 & -1 & +1 & -1 \\ 1 & & +1 & 0 & +1 \\ \hline & 1 & 0 & +1 & \boxed{0} \end{array} \Rightarrow x^3 - x^2 + x - 1 = (x-1)(x^2+1)$$

$$15) \lim_{x \rightarrow \frac{1}{2}} \frac{8x^3 + 4x^2 - 10x + 3}{4x^4 - 15x^2 + 13x - 3} = \frac{0}{0} \text{ (I)} = \lim_{x \rightarrow \frac{1}{2}} \frac{\left(x - \frac{1}{2}\right)^2 (8x+12)}{\left(x - \frac{1}{2}\right)^2 (4x^2 + 4x - 12)} = \lim_{x \rightarrow \frac{1}{2}} \frac{(8x+12)}{(4x^2 + 4x - 12)} = \frac{4+12}{1+2-12} = -\frac{16}{9}$$

$$\begin{array}{r|rrrr} \frac{1}{2} & 8 & +4 & -10 & +3 \\ \hline & 8 & +8 & -6 & \boxed{0} \\ \frac{1}{2} & & +4 & +6 & \\ \hline & 8 & +12 & \boxed{0} & \end{array} \Rightarrow 8x^3 + 4x^2 - 10x + 3 = \left(x - \frac{1}{2}\right)^2 \cdot (8x+12)$$

$$\begin{array}{r|rrrrr} \frac{1}{2} & 4 & 0 & -15 & +13 & -3 \\ \hline & 4 & +2 & -14 & +6 & \boxed{0} \\ \frac{1}{2} & & +2 & +2 & -6 & \\ \hline & 4 & +4 & -12 & \boxed{0} & \end{array} \Rightarrow 4x^4 - 15x^2 + 13x - 3 = \left(x - \frac{1}{2}\right)^2 \cdot (4x^2 + 4x - 12)$$

$$16) \lim_{x \rightarrow a} \frac{x^6 - a^6}{x^3 - a^3} = \frac{0}{0} \text{ (I)} = \lim_{x \rightarrow a} \frac{(x^3 - a^3)(x^3 + a^3)}{(x^3 - a^3)} = \lim_{x \rightarrow a} (x^3 + a^3) = a^3 + a^3 = 2a^3$$

$$17) \lim_{x \rightarrow a} \frac{x^2 - (a+1)x + a}{x^2 - a^2} = \lim_{x \rightarrow a} \frac{(x-a)(x-1)}{(x-a)(x+a)} = \lim_{x \rightarrow a} \frac{x-1}{x+a} = \frac{a-1}{2a}$$

	1	-(a+1)	+a	
a		+a	-a	
	1	-1	0	$\Rightarrow x^2 - (a+1)x + a = (x-a)(x-1)$

$$18) \lim_{x \rightarrow a} \frac{x^3 - ax^2 - a^2x + a^3}{x^3 - 3ax^2 + 3a^2x - a^3} = \frac{0}{0} \text{ (I)} = \lim_{x \rightarrow a} \frac{(x-a)^2(x+a)}{(x-a)^3} = \lim_{x \rightarrow a} \frac{x+a}{x-a}$$

$$\left\{ \begin{array}{l} \bullet \text{ Si } a > 0 \Rightarrow \begin{cases} \lim_{x \rightarrow a^-} \frac{x+a}{x-a} = \frac{2a}{0^-} = -\infty \\ \lim_{x \rightarrow a^+} \frac{x+a}{x-a} = \frac{2a}{0^+} = +\infty \end{cases} \\ \bullet \text{ Si } a < 0 \Rightarrow \begin{cases} \lim_{x \rightarrow a^-} \frac{x+a}{x-a} = \frac{2a}{0^-} = +\infty \\ \lim_{x \rightarrow a^+} \frac{x+a}{x-a} = \frac{2a}{0^+} = -\infty \end{cases} \\ \bullet \text{ Si } a = 0 \Rightarrow \lim_{x \rightarrow a} \frac{x}{x} = \lim_{x \rightarrow a} 1 = 1 \end{array} \right.$$

	1	-a	-a^2	+a^3	
a		+a	0	-a^3	
	1	0	-a^2	0	
a		+a	+a^2		
	1	+a	0	$\Rightarrow x^3 - ax^2 - a^2x + a^3 = (x-a)^2 \cdot (x+a)$	

	1	-3a	+3a^2	-a^3	
a		+a	-2a^2	+a^3	
	1	-2a	+a^2	0	
a		+a	-a^2		
	1	-a	0	$\Rightarrow x^3 - 3ax^2 + 3a^2x - a^3 = (x-a)^2 \cdot (x-a) = (x-a)^3$	

$$19) \lim_{x \rightarrow 2} \left(\frac{3}{x^2 - 5x + 6} - \frac{4}{x - 2} \right) = \infty - \infty \text{ (I)} = \lim_{x \rightarrow 2} \left(\frac{3}{(x-2)(x-3)} - \frac{4}{x-2} \right) = \lim_{x \rightarrow 2} \left(\frac{3 - 4(x-3)}{(x-2)(x-3)} \right) =$$

$$= \lim_{x \rightarrow 2} \left(\frac{3 - 4x + 12}{(x-2)(x-3)} \right) = \lim_{x \rightarrow 2} \frac{15 - 4x}{(x-2)(x-3)} = \frac{7}{0} = \begin{cases} \lim_{x \rightarrow 2^-} \frac{15 - 4x}{(x-2)(x-3)} = \frac{7}{(0^-)(-1)} = \frac{7}{0^+} = +\infty \\ \lim_{x \rightarrow 2^+} \frac{15 - 4x}{(x-2)(x-3)} = \frac{7}{(0^+)(-1)} = \frac{7}{0^-} = -\infty \end{cases}$$

$$20) \lim_{x \rightarrow +\infty} \left(\frac{4x^2 - x + 3}{3x^2 + x - 3} \right)^{\frac{x}{1-x}} = \left[\lim_{x \rightarrow +\infty} \left(\frac{4x^2 - x + 3}{3x^2 + x - 3} \right) \right]^{\lim_{x \rightarrow +\infty} \left(\frac{x}{1-x} \right)} = \left(\frac{4}{3} \right)^{-1} = \frac{3}{4}$$

$$\blacksquare \lim_{x \rightarrow +\infty} \left(\frac{4x^2 - x + 3}{3x^2 + x - 3} \right) = \frac{+\infty}{+\infty} \text{ (I)} = \lim_{x \rightarrow +\infty} \frac{4x^2 - x + 3}{3x^2 + x - 3} = \lim_{x \rightarrow +\infty} \frac{4 - \frac{1}{x} + \frac{3}{x^2}}{3 + \frac{1}{x} - \frac{3}{x^2}} = \frac{4 - 0 + 0}{3 + 0 - 0} = \frac{4}{3}$$

$$\blacksquare \lim_{x \rightarrow +\infty} \left(\frac{x}{1-x} \right) = \frac{+\infty}{-\infty} \text{ (I)} = \lim_{x \rightarrow +\infty} \frac{\frac{x}{x}}{\frac{1}{x} - \frac{x}{x}} = \lim_{x \rightarrow +\infty} \frac{1}{\frac{1}{x} - 1} = \frac{1}{0 - 1} = -1$$

$$21) \lim_{x \rightarrow +\infty} \left(\frac{x}{3x^2 + 2} \right)^{\frac{x^2}{1+x}} = \left[\lim_{x \rightarrow +\infty} \left(\frac{x}{3x^2 + 2} \right) \right]^{\lim_{x \rightarrow +\infty} \left(\frac{x^2}{1+x} \right)} = 0^{+\infty} = 0$$

$$\blacksquare \lim_{x \rightarrow +\infty} \left(\frac{x}{3x^2 + 2} \right) = \frac{+\infty}{+\infty} \text{ (I)} = \lim_{x \rightarrow +\infty} \frac{\frac{x}{x^2}}{\frac{3x^2}{x^2} + \frac{2}{x^2}} = \lim_{x \rightarrow +\infty} \frac{\frac{1}{x}}{3 + \frac{2}{x^2}} = \frac{0}{3 + 0} = \frac{0}{3} = 0$$

$$\blacksquare \lim_{x \rightarrow +\infty} \left(\frac{x^2}{1+x} \right) = \frac{+\infty}{+\infty} \text{ (I)} = \lim_{x \rightarrow +\infty} \frac{\frac{x^2}{x^2}}{\frac{1}{x^2} + \frac{x}{x^2}} = \lim_{x \rightarrow +\infty} \frac{1}{\frac{1}{x^2} + \frac{1}{x}} = \frac{1}{0} = +\infty$$

$$22) \lim_{x \rightarrow +\infty} \left(\frac{x^2 + 3}{3x^2 - 5} \right)^{\frac{3x^2}{x-2}} = \left[\lim_{x \rightarrow +\infty} \left(\frac{x^2 + 3}{3x^2 - 5} \right) \right]^{\lim_{x \rightarrow +\infty} \left(\frac{3x^2}{x-2} \right)} = \left(\frac{1}{3} \right)^{+\infty} = 0$$

$$\blacksquare \lim_{x \rightarrow +\infty} \left(\frac{x^2 + 3}{3x^2 - 5} \right) = \frac{+\infty}{+\infty} \text{ (I)} = \lim_{x \rightarrow +\infty} \frac{\frac{x^2}{x^2} + \frac{3}{x^2}}{\frac{3x^2}{x^2} - \frac{5}{x^2}} = \lim_{x \rightarrow +\infty} \frac{1 + \frac{3}{x^2}}{3 - \frac{5}{x^2}} = \frac{1 + 0}{3 - 0} = \frac{1}{3}$$

$$\blacksquare \lim_{x \rightarrow +\infty} \left(\frac{3x^2}{x-2} \right) = \frac{+\infty}{+\infty} \text{ (I)} = \lim_{x \rightarrow +\infty} \frac{\frac{3x^2}{x^2}}{\frac{x}{x^2} - \frac{2}{x^2}} = \lim_{x \rightarrow +\infty} \frac{3}{\frac{1}{x} - \frac{2}{x^2}} = \frac{3}{0} = +\infty$$

$$23) \lim_{x \rightarrow +\infty} \left(\frac{x^2 + 3}{3x^2 - 5} \right)^{\frac{x^2}{2-x}} = \left[\lim_{x \rightarrow +\infty} \left(\frac{x^2 + 3}{3x^2 - 5} \right) \right]^{\lim_{x \rightarrow +\infty} \left(\frac{x^2}{2-x} \right)} = \left(\frac{1}{3} \right)^{-\infty} = 3^{+\infty} = +\infty$$

$$\blacksquare \lim_{x \rightarrow +\infty} \left(\frac{x^2 + 3}{3x^2 - 5} \right) = \frac{+\infty}{+\infty} \text{ (I)} = \lim_{x \rightarrow +\infty} \frac{\frac{x^2}{x^2} + \frac{3}{x^2}}{\frac{3x^2}{x^2} - \frac{5}{x^2}} = \lim_{x \rightarrow +\infty} \frac{1 + \frac{3}{x^2}}{3 - \frac{5}{x^2}} = \frac{1+0}{3-0} = \frac{1}{3}$$

$$\blacksquare \lim_{x \rightarrow +\infty} \left(\frac{3x^2}{2-x} \right) = \frac{+\infty}{-\infty} \text{ (I)} = \lim_{x \rightarrow +\infty} \frac{\frac{3x^2}{x^2}}{\frac{2}{x^2} - \frac{x}{x^2}} = \lim_{x \rightarrow +\infty} \frac{3}{\frac{2}{x^2} - \frac{1}{x}} = \frac{3}{0} = -\infty$$

$$24) \lim_{x \rightarrow +\infty} \left(\frac{x^2 + 3}{3x^2 - 5} \right)^{\frac{x}{x-2}} = \left[\lim_{x \rightarrow +\infty} \left(\frac{x^2 + 3}{3x^2 - 5} \right) \right]^{\lim_{x \rightarrow +\infty} \left(\frac{x}{x-2} \right)} = \left(\frac{1}{3} \right)^1 = \frac{1}{3}$$

$$\blacksquare \lim_{x \rightarrow +\infty} \left(\frac{x^2 + 3}{3x^2 - 5} \right) = \frac{+\infty}{+\infty} \text{ (I)} = \lim_{x \rightarrow +\infty} \frac{\frac{x^2}{x^2} + \frac{3}{x^2}}{\frac{3x^2}{x^2} - \frac{5}{x^2}} = \lim_{x \rightarrow +\infty} \frac{1 + \frac{3}{x^2}}{3 - \frac{5}{x^2}} = \frac{1+0}{3-0} = \frac{1}{3}$$

$$\blacksquare \lim_{x \rightarrow +\infty} \left(\frac{x}{x-2} \right) = \frac{+\infty}{+\infty} \text{ (I)} = \lim_{x \rightarrow +\infty} \frac{\frac{x}{x}}{\frac{x}{x} - \frac{2}{x}} = \lim_{x \rightarrow +\infty} \frac{1}{1 - \frac{2}{x}} = \frac{1}{1-0} = 1$$

$$25) \lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-1} = \frac{0}{0} \text{ (I)} = \lim_{x \rightarrow 1} \frac{(x-1)(\sqrt{x}+1)}{(\sqrt{x}-1)(\sqrt{x}+1)} = \lim_{x \rightarrow 1} \frac{(x-1)(\sqrt{x}+1)}{(\sqrt{x})^2 - 1^2} = \lim_{x \rightarrow 1} \frac{(x-1)(\sqrt{x}+1)}{(x-1)} = \lim_{x \rightarrow 1} (\sqrt{x}+1) = 1+1 = 2$$

$$26) \lim_{x \rightarrow 0} \frac{2 - \sqrt{4-x}}{x} = \frac{0}{0} \text{ (I)} = \lim_{x \rightarrow 0} \frac{(2 - \sqrt{4-x})(2 + \sqrt{4-x})}{x(2 + \sqrt{4-x})} = \lim_{x \rightarrow 0} \frac{2^2 - (\sqrt{4-x})^2}{x(2 + \sqrt{4-x})} = \lim_{x \rightarrow 0} \frac{4 - (4-x)}{x(2 + \sqrt{4-x})} =$$

$$= \lim_{x \rightarrow 0} \frac{4 - 4 + x}{x(2 + \sqrt{4-x})} = \lim_{x \rightarrow 0} \frac{x}{x(2 + \sqrt{4-x})} \stackrel{\text{simplificar}}{=} = \lim_{x \rightarrow 0} \frac{1}{2 + \sqrt{4-x}} = \frac{1}{2 + \sqrt{4}} = \frac{1}{2+2} = \frac{1}{4}$$

$$27) \lim_{x \rightarrow +\infty} (\sqrt{x^3 + x + 1} - x) = \infty - \infty \text{ (I)} = \lim_{x \rightarrow +\infty} (\sqrt{x^3} - x) = \lim_{x \rightarrow +\infty} (x^{\frac{3}{2}} - x^1) = +\infty$$

$$28) \lim_{x \rightarrow +\infty} (x - \sqrt{4x^3 + x + 1}) = \infty - \infty \text{ (I)} = \lim_{x \rightarrow +\infty} (x - \sqrt{4x^3}) = \lim_{x \rightarrow +\infty} (x - 2x^{\frac{3}{2}}) = -\infty$$

$$29) \lim_{x \rightarrow +\infty} (\sqrt{4x^2 + x + 1} - 5x) = \infty - \infty \text{ (I)} = \lim_{x \rightarrow +\infty} (\sqrt{4x^2} - 5x) = \lim_{x \rightarrow +\infty} (2x - 5x) = -\infty$$

$$\begin{aligned}
30) \lim_{x \rightarrow +\infty} (\sqrt{x^2 + x + 1} - x) &= \infty - \infty (I) = \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2 + x + 1} - x)(\sqrt{x^2 + x + 1} + x)}{(\sqrt{x^2 + x + 1} + x)} = \\
&= \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2 + x + 1})^2 - x^2}{(\sqrt{x^2 + x + 1} + x)} = \lim_{x \rightarrow +\infty} \frac{x^2 + x + 1 - x^2}{(\sqrt{x^2 + x + 1} + x)} = \lim_{x \rightarrow +\infty} \frac{x + 1}{\sqrt{x^2 + x + 1} + x} = \frac{+\infty}{+\infty} (I) = \lim_{x \rightarrow +\infty} \frac{\frac{x}{x} + \frac{1}{x}}{\sqrt{\frac{x^2}{x^2} + \frac{x}{x^2} + \frac{1}{x^2} + \frac{x}{x}}} = \\
&= \lim_{x \rightarrow +\infty} \frac{1 + \frac{1}{x}}{\sqrt{1 + \frac{1}{x} + \frac{1}{x^2} + 1}} = \frac{1}{\sqrt{1 + 0 + 0 + 1}} = \frac{1}{\sqrt{1 + 1}} = \frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
31) \lim_{x \rightarrow +\infty} (\sqrt{x+2} - \sqrt{x-2}) &= \infty - \infty (I) = \lim_{x \rightarrow +\infty} \frac{(\sqrt{x+2} - \sqrt{x-2})(\sqrt{x+2} + \sqrt{x-2})}{(\sqrt{x+2} + \sqrt{x-2})} = \\
&= \lim_{x \rightarrow +\infty} \frac{(\sqrt{x+2})^2 - (\sqrt{x-2})^2}{(\sqrt{x+2} + \sqrt{x-2})} = \lim_{x \rightarrow +\infty} \frac{x+2 - (x-2)}{(\sqrt{x+2} + \sqrt{x-2})} = \lim_{x \rightarrow +\infty} \frac{x+2 - x + 2}{(\sqrt{x+2} + \sqrt{x-2})} = \lim_{x \rightarrow +\infty} \frac{4}{(\sqrt{x+2} + \sqrt{x-2})} = \\
&= \frac{4}{\infty + \infty} = \frac{4}{+\infty} = 0
\end{aligned}$$

$$\begin{aligned}
32) \lim_{x \rightarrow +\infty} (x - \sqrt{x^2 + x + 1}) &= \infty - \infty (I) = \lim_{x \rightarrow +\infty} \frac{(x - \sqrt{x^2 + x + 1})(x + \sqrt{x^2 + x + 1})}{(x + \sqrt{x^2 + x + 1})} = \\
&= \lim_{x \rightarrow +\infty} \frac{(x)^2 - (\sqrt{x^2 + x + 1})^2}{(x + \sqrt{x^2 + x + 1})} = \lim_{x \rightarrow +\infty} \frac{x^2 - (x^2 + x + 1)}{(x + \sqrt{x^2 + x + 1})} = \lim_{x \rightarrow +\infty} \frac{x^2 - x^2 - x - 1}{(x + \sqrt{x^2 + x + 1})} = \lim_{x \rightarrow +\infty} \frac{-x - 1}{(x + \sqrt{x^2 + x + 1})} = \\
&= \frac{-\infty}{+\infty} (I) = \lim_{x \rightarrow +\infty} \frac{-\frac{x}{x} - \frac{1}{x}}{\frac{x}{x} + \sqrt{\frac{x^2}{x^2} + \frac{x}{x^2} + \frac{1}{x^2}}} = \lim_{x \rightarrow +\infty} \frac{-1 - \frac{1}{x}}{1 + \sqrt{1 + \frac{1}{x} + \frac{1}{x^2}}} = \frac{-1 + 0}{1 + \sqrt{1 + 0 + 0}} = \frac{-1}{1 + \sqrt{1}} = -\frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
33) \lim_{x \rightarrow 0} \frac{x}{\sqrt{1+x} - \sqrt{1-x}} &= \frac{0}{0} (I) = \lim_{x \rightarrow 0} \frac{x(\sqrt{1+x} + \sqrt{1-x})}{(\sqrt{1+x} - \sqrt{1-x})(\sqrt{1+x} + \sqrt{1-x})} = \lim_{x \rightarrow 0} \frac{x(\sqrt{1+x} + \sqrt{1-x})}{(\sqrt{1+x})^2 - (\sqrt{1-x})^2} = \\
&= \lim_{x \rightarrow 0} \frac{x(\sqrt{1+x} + \sqrt{1-x})}{(1+x) - (1-x)} = \lim_{x \rightarrow 0} \frac{x(\sqrt{1+x} + \sqrt{1-x})}{1+x-1+x} = \lim_{x \rightarrow 0} \frac{x(\sqrt{1+x} + \sqrt{1-x})}{2x} = \lim_{x \rightarrow 0} \frac{\sqrt{1+x} + \sqrt{1-x}}{2} = \\
&= \frac{\sqrt{1+0} + \sqrt{1-0}}{2} = \frac{2}{2} = 1
\end{aligned}$$

$$34) \lim_{x \rightarrow 3} \frac{\sqrt[3]{x^3 - 27}}{\sqrt[3]{x^2 + 6x - 27}} = \lim_{x \rightarrow 3} \sqrt[3]{\frac{x^3 - 27}{x^2 + 6x - 27}} = \sqrt[3]{\lim_{x \rightarrow 3} \frac{x^3 - 27}{x^2 + 6x - 27}} = \sqrt[3]{\frac{9}{4}}$$

$$\lim_{x \rightarrow 3} \frac{x^3 - 27}{x^2 + 6x - 27} = \frac{0}{0} \text{ (I)} = \lim_{x \rightarrow 3} \frac{(x-3)(x^2 + 3x + 9)}{(x-3)(x+9)} = \lim_{x \rightarrow 3} \frac{x^2 + 3x + 9}{x+9} = \frac{(3)^2 + 3(3) + 9}{3+9} = \frac{9+9+9}{12} = \frac{27}{12} = \frac{9}{4}$$

$$\begin{array}{c|cccc} & 1 & 0 & 0 & -27 \\ 3 & & +3 & +9 & +27 \\ \hline & 1 & +3 & +9 & \boxed{0} \end{array} \Rightarrow x^3 - 27 = (x-3)(x^2 + 3x + 9)$$

$$\begin{array}{c|ccc} & 1 & +6 & -27 \\ 3 & & +3 & +27 \\ \hline & 1 & +9 & \boxed{0} \end{array} \Rightarrow x^2 + 6x - 27 = (x-3)(x+9)$$

$$35) \lim_{x \rightarrow 5} \frac{3 - \sqrt{4+x}}{x-5} = \frac{0}{0} \text{ (I)} = \lim_{x \rightarrow 5} \frac{(3 - \sqrt{4+x})(3 + \sqrt{4+x})}{(x-5)(3 + \sqrt{4+x})} = \lim_{x \rightarrow 5} \frac{(3)^2 - (\sqrt{4+x})^2}{(x-5)(3 + \sqrt{4+x})} = \lim_{x \rightarrow 5} \frac{9 - (4+x)}{(x-5)(3 + \sqrt{4+x})} =$$

$$= \lim_{x \rightarrow 5} \frac{9 - 4 - x}{(x-5)(3 + \sqrt{4+x})} = \lim_{x \rightarrow 5} \frac{5 - x}{(x-5)(3 + \sqrt{4+x})} = \lim_{x \rightarrow 5} \frac{-(x-5)}{(x-5)(3 + \sqrt{4+x})} \text{ simplificar}$$

$$= \lim_{x \rightarrow 5} \frac{-1}{3 + \sqrt{4+x}} = \frac{-1}{3 + \sqrt{4+5}} = \frac{-1}{3 + \sqrt{9}} = -\frac{1}{6}$$

$$36) \lim_{x \rightarrow 4} \frac{3 - \sqrt{5+x}}{1 - \sqrt{5-x}} = \frac{0}{0} \text{ (I)} = \lim_{x \rightarrow 4} \frac{[(3 - \sqrt{5+x})(3 + \sqrt{5+x})](1 + \sqrt{5-x})}{[(1 - \sqrt{5-x})(1 + \sqrt{5-x})](3 + \sqrt{5+x})} =$$

$$= \lim_{x \rightarrow 4} \frac{[(3)^2 - (\sqrt{5+x})^2](1 + \sqrt{5-x})}{[(1)^2 - (\sqrt{5-x})^2](3 + \sqrt{5+x})} = \lim_{x \rightarrow 4} \frac{[9 - (5+x)](1 + \sqrt{5-x})}{[1 - (5-x)](3 + \sqrt{5+x})} = \lim_{x \rightarrow 4} \frac{(9 - 5 - x)(1 + \sqrt{5-x})}{(1 - 5 + x)(3 + \sqrt{5+x})} =$$

$$= \lim_{x \rightarrow 4} \frac{(4-x)(1 + \sqrt{5-x})}{(x-4)(3 + \sqrt{5+x})} = \lim_{x \rightarrow 4} \frac{-(x-4)(1 + \sqrt{5-x})}{(x-4)(3 + \sqrt{5+x})} \text{ simplificar } = \lim_{x \rightarrow 4} \frac{-(1 + \sqrt{5-x})}{(3 + \sqrt{5+x})} = \frac{-(1 + \sqrt{5-4})}{(3 + \sqrt{5+4})} = -\frac{2}{6} = -\frac{1}{3}$$

$$37) \lim_{x \rightarrow 2} \frac{x - \sqrt{x+2}}{\sqrt{4x+1} - 3} = \frac{0}{0} \text{ (I)} = \lim_{x \rightarrow 2} \frac{[(x - \sqrt{x+2})(x + \sqrt{x+2})](\sqrt{4x+1} + 3)}{[(\sqrt{4x+1} - 3)(\sqrt{4x+1} + 3)](x + \sqrt{x+2})} =$$

$$= \lim_{x \rightarrow 2} \frac{[(x)^2 - (\sqrt{x+2})^2](\sqrt{4x+1} + 3)}{[(\sqrt{4x+1})^2 - (3)^2](x + \sqrt{x+2})} = \lim_{x \rightarrow 2} \frac{[x^2 - (x+2)](\sqrt{4x+1} + 3)}{[4x+1-9](x + \sqrt{x+2})} =$$

$$= \lim_{x \rightarrow 2} \frac{(x^2 - x - 2)(\sqrt{4x+1} + 3)}{(4x-8)(x + \sqrt{x+2})} \text{ factorizar polinomios} = \lim_{x \rightarrow 2} \frac{(x-2)(x+1)(\sqrt{4x+1} + 3)}{4(x-2)(x + \sqrt{x+2})} \text{ simplificar}$$

$$= \lim_{x \rightarrow 2} \frac{(x+1)(\sqrt{4x+1} + 3)}{4(x + \sqrt{x+2})} = \frac{(2+1)(\sqrt{4 \cdot 2 + 1} + 3)}{4(2 + \sqrt{2+2})} = \frac{3 \cdot (\sqrt{9} + 3)}{4 \cdot (2 + \sqrt{4})} = \frac{3 \cdot 6}{4 \cdot 4} = \frac{3 \cdot 3 \cdot 2}{4 \cdot 2 \cdot 2} = \frac{9}{8}$$

$$\begin{aligned}
 38) \lim_{x \rightarrow 4} \frac{\sqrt{3x+4} - x}{x - \sqrt{5x-4}} &= \frac{0}{0} \text{ (I)} = \lim_{x \rightarrow 4} \frac{[(\sqrt{3x+4} - x)(\sqrt{3x+4} + x)](x + \sqrt{5x-4})}{[(x - \sqrt{5x-4})(x + \sqrt{5x-4})](\sqrt{3x+4} + x)} = \\
 &= \lim_{x \rightarrow 4} \frac{[(\sqrt{3x+4})^2 - (x)^2](x + \sqrt{5x-4})}{[(x)^2 - (\sqrt{5x-4})^2](\sqrt{3x+4} + x)} = \lim_{x \rightarrow 4} \frac{[(3x+4) - x^2](x + \sqrt{5x-4})}{[x^2 - (5x-4)](\sqrt{3x+4} + x)} = \\
 &= \lim_{x \rightarrow 4} \frac{(-x^2 + 3x + 4)(x + \sqrt{5x-4})}{(x^2 - 5x + 4)(\sqrt{3x+4} + x)} = \lim_{x \rightarrow 4} \frac{-(x-4)(x+1)(x + \sqrt{5x-4})}{(x-4)(x-1)(\sqrt{3x+4} + x)} \stackrel{\text{factorizar polinomios}}{=} \lim_{x \rightarrow 4} \frac{-(x-4)(x+1)(x + \sqrt{5x-4})}{(x-4)(x-1)(\sqrt{3x+4} + x)} \stackrel{\text{simplificar}}{=} \\
 &= \lim_{x \rightarrow 4} \frac{-(x+1)(x + \sqrt{5x-4})}{(x-1)(\sqrt{3x+4} + x)} = \frac{-(4+1)(4 + \sqrt{5(4)-4})}{(4-1)(\sqrt{3(4)+4+4})} = \frac{-5 \cdot 8}{3 \cdot 8} = -\frac{5}{3}
 \end{aligned}$$

$$39) \lim_{x \rightarrow +\infty} \frac{\sqrt{3x^2+1}}{2+5x} = \frac{+\infty}{+\infty} = \lim_{x \rightarrow +\infty} \frac{\sqrt{\frac{3x^2}{x^2} + \frac{1}{x^2}}}{\frac{2}{x} + \frac{5x}{x}} = \lim_{x \rightarrow +\infty} \frac{\sqrt{3 + \frac{1}{x^2}}}{\frac{2}{x} + 5} = \frac{\sqrt{3+0}}{0+5} = \frac{\sqrt{3}}{5}$$

$$\begin{aligned}
 40) \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2-3x+1}}{2+5x} &= \lim_{x \rightarrow +\infty} \frac{\sqrt{(-x)^2-3(-x)+1}}{2+5(-x)} = \lim_{x \rightarrow +\infty} \frac{\sqrt{x^2+3x+1}}{2-5x} = \frac{+\infty}{-\infty} \text{ (I)} = \lim_{x \rightarrow +\infty} \frac{\sqrt{\frac{x^2}{x^2} + \frac{3x}{x^2} + \frac{1}{x^2}}}{\frac{2}{x} - \frac{5x}{x}} = \\
 &= \lim_{x \rightarrow +\infty} \frac{\sqrt{1 + \frac{3}{x} + \frac{1}{x^2}}}{\frac{2}{x} - 5} = \frac{\sqrt{1+0+0}}{0-5} = -\frac{1}{5}
 \end{aligned}$$

$$41) \lim_{x \rightarrow +\infty} \frac{\sqrt[3]{x^7-2x+4}}{3x^2-1} = \frac{+\infty}{+\infty} \text{ (I)} = \lim_{x \rightarrow +\infty} \frac{\sqrt[3]{x^7}}{3x^2} = \lim_{x \rightarrow +\infty} \frac{x^{\frac{7}{3}}}{x^2} \stackrel{\text{grado N} > \text{grado D}}{=} +\infty$$

$$\begin{aligned}
 42) \lim_{x \rightarrow -\infty} \frac{\sqrt[3]{x^7-2x+4}}{3x^2-1} &= \lim_{x \rightarrow +\infty} \frac{\sqrt[3]{(-x)^7-2(-x)+4}}{3(-x)^2-1} = \lim_{x \rightarrow +\infty} \frac{\sqrt[3]{-x^7+2x+4}}{3x^2-1} = \frac{-\infty}{+\infty} \text{ (I)} = \\
 &= \lim_{x \rightarrow +\infty} \frac{\sqrt[3]{-x^7}}{3x^2} = \lim_{x \rightarrow +\infty} \frac{-x^{\frac{7}{3}}}{x^2} \stackrel{\text{grado N} > \text{grado D}}{=} -\infty
 \end{aligned}$$

$$43) \lim_{x \rightarrow +\infty} \left(\frac{\sqrt{x+9}-3}{x^2} \right) = \frac{+\infty}{+\infty} \text{ (I)} = \lim_{x \rightarrow +\infty} \left(\frac{\sqrt{x}}{x^2} \right) = \lim_{x \rightarrow +\infty} \frac{x^{\frac{1}{2}}}{x^2} \stackrel{\text{grado N} < \text{grado D}}{=} 0$$

$$44) \lim_{x \rightarrow +\infty} 2 \left(\frac{x^3-1}{x^3+x} \right)^{x^2+1} = 2 \lim_{x \rightarrow +\infty} \left(\frac{x^3-1}{x^3+x} \right)^{x^2+1} = 2 \stackrel{(*)}{=} e = e^{\sqrt{2}}$$

$$\text{(*)} \lim_{x \rightarrow +\infty} \left(\frac{x^3-1}{x^3+x} \right)^{x^2+1} = 1^{+\infty} \text{ (I)} = e^{\lim_{x \rightarrow +\infty} \left(\frac{x^3-1}{x^3+x} - 1 \right) \cdot (x^2+1)} \stackrel{(**)}{=} e^{-1} = \frac{1}{e}$$

$$\text{(**)} \lim_{x \rightarrow +\infty} \left(\frac{x^3-1}{x^3+x} - 1 \right) \cdot (x^2+1) = \lim_{x \rightarrow +\infty} \left(\frac{x^3-1-x^3-x}{x(x^2+1)} \right) \cdot (x^2+1) = \lim_{x \rightarrow +\infty} \left(\frac{-1-x}{x(x^2+1)} \right) \cdot (x^2+1) = \lim_{x \rightarrow +\infty} \frac{-1-x}{x} = -1$$

$$45) \lim_{x \rightarrow 1} \left(\frac{x^3 + 1}{x^2 + 1} \right)^{\frac{3}{x-1}} = 1^\infty (\text{I}) = e^{\lim_{x \rightarrow 1} \left(\frac{x^3 + 1}{x^2 + 1} - 1 \right) \cdot \frac{3}{x-1}} = e^{\frac{3}{(*)}} = \sqrt{e^3} = e\sqrt{e}$$

$$(*) \lim_{x \rightarrow 1} \left(\frac{x^3 + 1}{x^2 + 1} - 1 \right) \cdot \frac{3}{x-1} = \lim_{x \rightarrow 1} \left(\frac{x^3 + 1 - x^2 - 1}{x^2 + 1} \right) \cdot \frac{3}{x-1} = \lim_{x \rightarrow 1} \left(\frac{x^3 - x^2}{x^2 + 1} \right) \cdot \frac{3}{x-1} = \lim_{x \rightarrow 1} \frac{x^2(x-1)}{x^2 + 1} \cdot \frac{3}{(x-1)} =$$

$$= \lim_{x \rightarrow 1} \frac{3x^2}{x^2 + 1} = \frac{3}{2}$$

$$46) \lim_{x \rightarrow +\infty} \left(\sqrt{\frac{x-2}{x+5}} \right)^x = \lim_{x \rightarrow +\infty} \left(\frac{x-2}{x+5} \right)^{\frac{x}{2}} = 1^\infty (\text{I}) = e^{\lim_{x \rightarrow +\infty} \left(\frac{x-2}{x+5} - 1 \right) \cdot \frac{x}{2}} = e^{\frac{-7}{2}} = \frac{1}{e^{\frac{7}{2}}} = \frac{1}{\sqrt{e^7}} = \frac{1}{e^3 \sqrt{e}}$$

$$(*) \lim_{x \rightarrow +\infty} \left(\frac{x-2}{x+5} - 1 \right) \cdot \frac{x}{2} = \lim_{x \rightarrow +\infty} \left(\frac{x-2-x-5}{x+5} \right) \cdot \frac{x}{2} = \lim_{x \rightarrow +\infty} \left(\frac{-7}{x+5} \right) \cdot \frac{x}{2} = \lim_{x \rightarrow +\infty} \frac{-7x}{2x+10} = \lim_{x \rightarrow +\infty} \frac{-\frac{7x}{x}}{\frac{2x}{x} + \frac{10}{x}} =$$

$$= \lim_{x \rightarrow +\infty} \frac{-7}{2 + \frac{10}{x}} = -\frac{7}{2+0} = -\frac{7}{2}$$

$$47) \lim_{x \rightarrow 2} (x-1)^{\frac{1}{x-2}} = 1^\infty (\text{I}) = e^{\lim_{x \rightarrow 2} (x-1-1) \cdot \frac{1}{x-2}} = e^{\lim_{x \rightarrow 2} \frac{x-2}{x-2}} = e^{\lim_{x \rightarrow 2} 1} = e^1 = e$$

$$48) \lim_{x \rightarrow 1} x^{\frac{2}{1-x}} = 1^\infty (\text{I}) = e^{\lim_{x \rightarrow 1} (x-1) \cdot \frac{2}{1-x}} = e^{\lim_{x \rightarrow 1} \frac{2(x-1)}{1-x}} = e^{\lim_{x \rightarrow 1} \frac{2(x-1)}{-(x-1)}} = e^{\lim_{x \rightarrow 1} (-2)} = e^{-2} = \frac{1}{e^2}$$

$$49) \lim_{x \rightarrow 0} \sqrt[2]{1-4x} = \lim_{x \rightarrow 0} 1 - 4x(1-4x)^{\frac{1}{2x}} = 1^\infty (\text{I}) = e^{\lim_{x \rightarrow 0} (1-4x-1) \cdot \frac{1}{2x}} = e^{\lim_{x \rightarrow 0} \frac{-4x}{2x}} = e^{\lim_{x \rightarrow 0} (-2)} = e^{-2} = \frac{1}{e^2}$$

$$50) \lim_{x \rightarrow 2} \left(\frac{2x-1}{x+1} \right)^{\frac{x}{x-2}} = 1^\infty (\text{I}) = e^{\lim_{x \rightarrow 2} \left(\frac{2x-1}{x+1} - 1 \right) \cdot \frac{x}{x-2}} = e^{\frac{2}{(*)}} = \sqrt[3]{e^2}$$

$$(*) \lim_{x \rightarrow 2} \left(\frac{2x-1}{x+1} - 1 \right) \cdot \frac{x}{x-2} = \lim_{x \rightarrow 2} \left(\frac{2x-1-x-1}{x+1} \right) \cdot \frac{x}{x-2} = \lim_{x \rightarrow 2} \left(\frac{x-2}{x+1} \right) \cdot \frac{x}{x-2} = \lim_{x \rightarrow 2} \frac{x}{x+1} = \frac{2}{3}$$

$$51) \lim_{x \rightarrow +\infty} \frac{x-1}{x^2+x-2} \text{ cuando } x \rightarrow +\infty, x \rightarrow 0, x \rightarrow 1, x \rightarrow 3$$

$$\blacksquare \lim_{x \rightarrow +\infty} \frac{x-1}{x^2+x-2} = \frac{+\infty}{+\infty} (\text{I}) = \lim_{x \rightarrow +\infty} \frac{\frac{x}{x^2} - \frac{1}{x^2}}{\frac{x}{x^2} + \frac{x}{x^2} - \frac{2}{x^2}} = \lim_{x \rightarrow +\infty} \frac{\frac{1}{x} - \frac{1}{x^2}}{1 + \frac{1}{x} - \frac{2}{x^2}} = \frac{0-0}{1+0-0} = \frac{0}{1} = 0$$

$$\blacksquare \lim_{x \rightarrow 0} \frac{x-1}{x^2+x-2} = \frac{-1}{-2} = \frac{1}{2}$$

$$\blacksquare \lim_{x \rightarrow 1} \frac{x-1}{x^2+x-2} = \frac{0}{0} (\text{I}) = \lim_{x \rightarrow 1} \frac{(x-1)}{(x-1)(x+2)} = \lim_{x \rightarrow 1} \frac{1}{(x+2)} = \frac{1}{3}$$

$$\blacksquare \lim_{x \rightarrow 3} \frac{x-1}{x^2+x-2} = \frac{2}{10} = \frac{1}{5}$$

52) $\lim_{x \rightarrow +\infty, x \rightarrow 0, x \rightarrow -1, x \rightarrow -3} \frac{x^5 + 2x^4 + 4x^3 + 8x^2}{4x^4 + 12x^3 + 13x^2 + 6x + 1}$ cuando $x \rightarrow +\infty, x \rightarrow 0, x \rightarrow -1, x \rightarrow -3$

$$\lim_{x \rightarrow +\infty} \frac{x^5 + 2x^4 + 4x^3 + 8x^2}{4x^4 + 12x^3 + 13x^2 + 6x + 1} = \frac{+\infty}{+\infty} \text{ (I)} = \lim_{x \rightarrow +\infty} \frac{\frac{x^5}{x^4} + \frac{2x^4}{x^5} + \frac{4x^3}{x^5} + \frac{8x^2}{x^5}}{\frac{4x^4}{x^5} + \frac{12x^3}{x^5} + \frac{13x^2}{x^5} + \frac{6x}{x^5} + \frac{1}{x^5}} =$$

$$= \lim_{x \rightarrow +\infty} \frac{x + \frac{2}{x} + \frac{4}{x^2} + \frac{8}{x^3}}{\frac{4}{x} + \frac{12}{x^2} + \frac{13}{x^3} + \frac{6}{x^4} + \frac{1}{x^5}} = \frac{+\infty}{0} = +\infty$$

$$\lim_{x \rightarrow 0} \frac{x^5 + 2x^4 + 4x^3 + 8x^2}{4x^4 + 12x^3 + 13x^2 + 6x + 1} = \frac{0}{1} = 0$$

$$\lim_{x \rightarrow -1} \frac{x^5 + 2x^4 + 4x^3 + 8x^2}{4x^4 + 12x^3 + 13x^2 + 6x + 1} = \frac{5}{0} = \begin{cases} \lim_{x \rightarrow -1^-} \frac{x^5 + 2x^4 + 4x^3 + 8x^2}{4x^4 + 12x^3 + 13x^2 + 6x + 1} = \frac{5}{0^+} = +\infty \\ \lim_{x \rightarrow -1^+} \frac{x^5 + 2x^4 + 4x^3 + 8x^2}{4x^4 + 12x^3 + 13x^2 + 6x + 1} = \frac{5}{0^+} = +\infty \end{cases}$$

$$\lim_{x \rightarrow -3} \frac{x^5 + 2x^4 + 4x^3 + 8x^2}{4x^4 + 12x^3 + 13x^2 + 6x + 1} = \frac{(-3)^5 + 2(-3)^4 + 4(-3)^3 + 8(-3)^2}{4(-3)^4 + 12(-3)^3 + 13(-3)^2 + 6(-3) + 1} = \frac{-243 + 162 - 108 + 72}{324 - 324 + 117 - 18 + 1} = -\frac{117}{100}$$