## *INTEGRACION POR SUSTITUCION*

A veces es conveniente hacer un cambio de variable, para transformar la integral dada en otra, de forma conocida. La técnica en cuestión recibe el nombre de método de sustitución.

## *EJERCICIOS DESARROLLADOS*

**2.1.-Encontrar:** $\int \frac{e^{\ell\eta x}dx}{x^2+7}$

Solución.- Como: $e^{\ell\eta x}$ = x, se tiene: $\int \frac{e^{\ell\eta x}dx}{x^2+7} = \int \frac{xdx}{x^2+7}$

Sea la sustitución: $u = x^2+7$, donde: $du = 2xdx$, Dado que: $\int \frac{xdx}{x^2+7} = \frac{1}{2}\int \frac{2xdx}{x^2+7}$,

Se tiene: $\frac{1}{2}\int \frac{2xdx}{x^2+7} = \frac{1}{2}\int \frac{du}{u}$, integral que es inmediata.

Luego: $= \frac{1}{2}\int \frac{du}{u}\ \frac{1}{2}\ell\eta\left|u\right| + c = \frac{1}{2}\ell\eta\left|x^2+7\right| + c$

**Respuesta:** $\int \frac{e^{\ell\eta x}dx}{x^2+7} = \frac{1}{2}\ell\eta\left|x^2+7\right| + c$

**2.2.-Encontrar:** $\int \frac{e^{\ell\eta x^2}dx}{x^3+8}$

Solución.- Como: $e^{\ell\eta x^2} = x^2$, se tiene: $\int \frac{e^{\ell\eta x^2}dx}{x^3+8} = \int \frac{x^2dx}{x^3+8}$

Sea la sustitución: $w = x^3+8$, donde: $dw = 3x^2dx$, Dado que: $\int \frac{x^2dx}{x^3+8} = \frac{1}{3}\int \frac{3x^2dx}{x^3+8}$,

Se tiene: $\frac{1}{3}\int \frac{3x^2dx}{x^3+8} = \frac{1}{3}\int \frac{dw}{w}$ integral que es inmediata.

Luego: $\frac{1}{3}\int \frac{dw}{w} = \frac{1}{3}\ell\eta\left|w\right| + c = \frac{1}{3}\ell\eta\left|x^3+8\right| + c$

**Respuesta:** $\int \frac{e^{\ell\eta x^2}dx}{x^3+8} = \frac{1}{3}\ell\eta\left|x^3+8\right| + c$

**2.3.-Encontrar:** $\int (x+2)\,\mathrm{s}e\mathrm{n}(x^2+4x-6)dx$

Solución.- Sea la sustitución: $u = x^2+4x-6$, donde: $du = (2x+4)dx$

Dado que: $\int (x+2)\,\mathrm{s}e\mathrm{n}(x^2+4x-6)dx = \frac{1}{2}\int (2x+4)\,\mathrm{s}e\mathrm{n}(x^2+4x-6)dx$**,** se tiene:

$= \frac{1}{2}\int (2x+4)\operatorname{sen}(x^2+4x-6)dx = \frac{1}{2}\int \operatorname{sen} u du$, integral que es inmediata.

Luego: $= \frac{1}{2}\int \operatorname{sen} u du = \frac{1}{2}(-\cos u) + c = -\frac{1}{2}\cos u + c = -\frac{1}{2}\cos(x^2+4x-6) + c$

**Respuesta:** $\int (x+2)\operatorname{sen}(x^2+4x-6)dx = -\frac{1}{2}\cos(x^2+4x-6) + c$

**2.4.-Encontrar:** $\int x\operatorname{sen}(1-x^2)dx$

Solución.-Sea la sustitución: $w = 1-x^2$, donde: $dw = -2xdx$

Dado que: $\int x\operatorname{sen}(1-x^2)dx = -\frac{1}{2}\int (-2x)\operatorname{sen}(1-x^2)dx$

Se tiene que: $-\frac{1}{2}\int (-2x)\operatorname{sen}(1-x^2)dx = -\frac{1}{2}\operatorname{sen} w dw$, integral que es inmediata.

Luego: $-\frac{1}{2}\int \operatorname{sen} w dw = -\frac{1}{2}(-\cos w)dw + c = \frac{1}{2}\cos w + c = \frac{1}{2}\cos(1-x^2) + c$

**Respuesta:** $\int x\operatorname{sen}(1-x^2)dx = \frac{1}{2}\cos(1-x^2) + c$

**2.5.-Encontrar:** $\int x \operatorname{co}\tau g(x^2+1)dx$

Solución.-Sea la sustitución: $u = x^2+1$, donde: $du = 2xdx$

Dado que: $\int x \operatorname{co}\tau g(x^2+1)dx = \frac{1}{2}\int 2x \operatorname{co}\tau g(x^2+1)dx$

Se tiene que: $\frac{1}{2}\int 2x \operatorname{co}\tau g(x^2+1)dx = \frac{1}{2}\int \operatorname{co}\tau g u du$, integral que es inmediata.

Luego: $\frac{1}{2}\int \operatorname{co}\tau g u du = \frac{1}{2}\ell\eta\left|\operatorname{sen} u\right| + c = \frac{1}{2}\ell\eta\left|\operatorname{sen}(x^2+1)\right| + c$

**Respuesta:** $\int x \operatorname{co}\tau g(x^2+1)dx = \frac{1}{2}\ell\eta\left|\operatorname{sen}(x^2+1)\right| + c$

**2.6.-Encontrar:** $\int \sqrt{1+y^4}\, y^3 dy$

Solución.-Sea la sustitución: $w = 1+y^4$, donde: $dw = 4y^3dy$

Dado que: $\int \sqrt{1+y^4}\, y^3 dy = \frac{1}{4}\int (1+y^4)^{1/2} 4y^3 dy$

Se tiene que: $\frac{1}{4}\int (1+y^4)^{1/2} 4y^3 dy = \frac{1}{4}\int w^{1/2} dw$, integral que es inmediata.

Luego: $\frac{1}{4}\int w^{1/2} dw = \frac{1}{4}\frac{w^{3/2}}{\frac{3}{2}} + c = \frac{1}{6}w^{3/2} + c = \frac{1}{6}(1+y^4)^{3/2} + c$

**Respuesta:** $\int \sqrt{1+y^4}\, y^3 dy = \frac{1}{6}(1+y^4)^{3/2} + c$

**2.7.-Encontrar:** $\int \frac{3tdt}{\sqrt[3]{t^2+3}}$

Solución.-Sea la sustitución: $u = t^2+3$, donde: $du = 2tdt$

Dado que: $\int \frac{3tdt}{\sqrt[3]{t^2+3}} = \frac{3}{2}\int \frac{2tdt}{(t^2+3)^{1/3}}$

Se tiene que: $\frac{3}{2}\int \frac{2tdt}{(t^2+3)^{1/3}} = \frac{3}{2}\int \frac{du}{u^{1/3}}$, integral que es inmediata

Luego: $\frac{3}{2}\int \frac{du}{u^{1/3}} = \frac{3}{2}\int u^{-1/3}du = \frac{3}{2}\frac{u^{2/3}}{\frac{2}{3}} + c = \frac{9}{4}u^{2/3} + c = \frac{9}{4}(t^2+3)^{2/3} + c$

**Respuesta:** $\int \frac{3tdt}{\sqrt[3]{t^2+3}} = \frac{9}{4}(t^2+3)^{2/3} + c$

**2.8.-Encontrar:** $\int \frac{dx}{(a+bx)^{1/3}}$**,** a y b constantes.

Solución.- Sea: $w = a + bx$, donde: $dw = bdx$

Luego: $\int \frac{dx}{(a+bx)^{1/3}} = \frac{1}{b}\int \frac{bdx}{(a+bx)^{1/3}} = \frac{1}{b}\int \frac{dw}{w^{1/3}} = \frac{1}{b}\int w^{-1/3} = \frac{1}{b}\frac{w^{2/3}}{\frac{2}{3}} + c = \frac{3}{2b}w^{2/3} + c$

$= \frac{3}{2b}(a+bx)^{2/3} + c$

**Respuesta:** $\int \frac{dx}{(a+bx)^{1/3}} = \frac{3}{2b}(a+bx)^{2/3} + c$

**2.9.-Encontrar:** $\int \frac{\sqrt{\operatorname{arcsen} x}}{1-x^2}dx$

Solución.- $\int \frac{\sqrt{\operatorname{arcsen} x}}{1-x^2}dx = \int \sqrt{\operatorname{arcsen} x}\,\frac{dx}{\sqrt{1-x^2}}$,

Sea: $u = \operatorname{arcsen} x$, donde: $du = \frac{dx}{\sqrt{1-x^2}}$

Luego: $\int \sqrt{\operatorname{arcsen} x}\,\frac{dx}{\sqrt{1-x^2}} = \int u^{1/2}du = \frac{2}{3}u^{3/2} + c = \frac{2}{3}\sqrt{(\operatorname{arcsen} x)^3} + c$

**Respuesta:** $\int \frac{\sqrt{\operatorname{arcsen} x}}{1-x^2}dx = \frac{2}{3}\sqrt{(\operatorname{arcsen} x)^3} + c$

**2.10.-Encontrar:** $\int \frac{\operatorname{arc\tau g}\frac{x}{2}}{4+x^2}dx$

Solución.- Sea: $w = \operatorname{arc\tau g}\frac{x}{2}$, donde: $dw = \frac{1}{1+(\frac{x}{2})^2}(\frac{1}{2})dx = \frac{2dx}{4+x^2}$

Luego: $\int \frac{\operatorname{arc\tau g}\frac{x}{2}}{4+x^2}dx = \frac{1}{2}\int \operatorname{arc\tau g}\left(\frac{x}{2}\right)\frac{2dx}{4+x^2} = \frac{1}{2}\int wdw = \frac{1}{4}w^2 + c = \frac{1}{4}\left(\operatorname{arc\tau g}\frac{x}{2}\right)^2 + c$

**Respuesta:** $\int \frac{\operatorname{arc\tau g}\frac{x}{2}}{4+x^2}dx = \frac{1}{4}\left(\operatorname{arc\tau g}\frac{x}{2}\right)^2 + c$

**2.11.-Encontrar:** $\int \frac{x-\operatorname{arc}\tau g2x}{1+4x^2}dx$

Solución.- $\int \frac{x-\operatorname{arc}\tau g2x}{1+4x^2}dx = \int \frac{xdx}{1+4x^2} - \int \frac{\sqrt{\operatorname{arc}\tau g2x}}{1+4x^2}$

Sea: $u = 1+4x^2$, donde: $du = 8xdx$; $w = \operatorname{arc}\tau g2x$, donde: $dw = \frac{2dx}{1+4x^2}$

Luego: $\int \frac{xdx}{1+4x^2} - \int \frac{\sqrt{\operatorname{arc}\tau g2x}}{1+4x^2} = \frac{1}{8}\int \frac{8xdx}{1+4x^2} - \frac{1}{2}\int \sqrt{\operatorname{arc}\tau g2x}\,\frac{2dx}{1+4x^2}$

$= \frac{1}{8}\int \frac{du}{u} - \frac{1}{2}\int w^{\frac{1}{2}}dw = \frac{1}{8}\ell\eta|u| - \frac{1}{3}w^{\frac{3}{2}} + c = \frac{1}{8}\ell\eta|1+4x^2| - \frac{1}{3}(\operatorname{arc}\tau g2x)^{\frac{3}{2}} + c$

**Respuesta:** $\int \frac{x-\operatorname{arc}\tau g2x}{1+4x^2}dx = \frac{1}{8}\ell\eta|1+4x^2| - \frac{1}{3}(\operatorname{arc}\tau g2x)^{\frac{3}{2}} + c$

**2.12.-Encontrar:** $\int \frac{dx}{\sqrt{(1+x^2)\ell\eta\left|x+\sqrt{1+x^2}\right|}}$

Solución.- $\int \frac{dx}{\sqrt{(1+x^2)\ell\eta\left|x+\sqrt{1+x^2}\right|}} = \int \frac{dx}{\sqrt{1+x^2}\sqrt{\ell\eta\left|x+\sqrt{1+x^2}\right|}}$

Sea: $u = \ell\eta\left|x+\sqrt{1+x^2}\right|$, donde: $du = \frac{1}{x+\sqrt{1+x^2}}(1+\frac{2x}{2\sqrt{1+x^2}}) \Rightarrow du = \frac{dx}{\sqrt{1+x^2}}$

Luego: $\int \frac{dx}{\sqrt{1+x^2}\sqrt{\ell\eta\left|x+\sqrt{1+x^2}\right|}} = \int \frac{du}{\sqrt{u}} = \int u^{-\frac{1}{2}}du = 2u^{\frac{1}{2}} + c = 2\sqrt{\ell\eta\left|x+\sqrt{1+x^2}\right|} + c$

**Respuesta:** $\int \frac{dx}{\sqrt{(1+x^2)\ell\eta\left|x+\sqrt{1+x^2}\right|}} = 2\sqrt{\ell\eta\left|x+\sqrt{1+x^2}\right|} + c$

**2.13.-Encontrar:** $\int \frac{\operatorname{co}\tau g(\ell\eta x)}{x}dx$

Solución.- Sea: $w = \ell\eta x$, donde: $dw = \frac{dx}{x}$

Luego: $\int \frac{\operatorname{co}\tau g(\ell\eta x)}{x}dx = \int \operatorname{co}\tau gwdw = \ell\eta|\operatorname{sen} w| + c = \ell\eta|\operatorname{sen}(\ell\eta x)| + c$

**Respuesta:** $\int \frac{\operatorname{co}\tau g(\ell\eta x)}{x}dx = \ell\eta|\operatorname{sen}(\ell\eta x)| + c$

**2.14.-Encontrar:** $\int \frac{dx}{x(\ell\eta x)^3}$

Solución.- Sea: $u = \ell\eta x$, donde: $du = \frac{dx}{x}$

Luego: $\int \frac{dx}{x(\ell\eta x)^3} = \int \frac{du}{u^3} = \int u^{-3}du = \frac{u^{-2}}{2} + c = \frac{1}{2u^2} + c = \frac{1}{2(\ell\eta x)^2} + c$

**Respuesta:** $\int \frac{dx}{x(\ell\eta x)^3} = \frac{1}{2(\ell\eta x)^2} + c$

**2.15.-Encontrar:** $\int \frac{e^{1/x^2}}{x^3} dx$

Solución.- Sea: $w = \frac{1}{x^2}$, donde: $dw = -\frac{2}{x^3} dx$

Luego: $\int \frac{e^{1/x^2}}{x^3} dx = -\frac{1}{2}\int e^{1/x^2} \frac{-2dx}{x^3} = -\frac{1}{2}\int e^w dw = -\frac{1}{2} e^w + c = -\frac{1}{2} e^{1/x^2} + c$

**Respuesta:** $\int \frac{e^{1/x^2}}{x^3} dx = -\frac{1}{2} e^{1/x^2} + c$

**2.16.-Encontrar:** $\int e^{-x^2+2} xdx$

Solución.- Sea: $u = -x^2 + 2$, donde: $du = -2xdx$

Luego: $\int e^{-x^2+2} xdx = -\frac{1}{2}\int e^{-x^2+2}(-2xdx) = -\frac{1}{2}\int e^u du = -\frac{1}{2} e^u + c = -\frac{1}{2} e^{-x^2+2} + c$

**Respuesta:** $\int e^{-x^2+2} xdx = -\frac{1}{2} e^{-x^2+2} + c$

**2.17.-Encontrar:** $\int x^2 e^{x^3} dx$

Solución.- Sea: $w = x^3$, donde: $dw = 3x^2 dx$

Luego: $\int x^2 e^{x^3} dx = \frac{1}{3}\int 3x^2 e^{x^3} dx = \frac{1}{3}\int e^w dw = \frac{1}{3} e^{x^3} + c$

**Respuesta:** $\int x^2 e^{x^3} dx = \frac{1}{3} e^{x^3} + c$

**2.18.-Encontrar:** $\int (e^x + 1)^2 e^x dx$

Solución.- Sea: $u = e^x + 1$, donde: $du = e^x dx$

Luego: $\int (e^x + 1)^2 e^x dx = \int u^2 du = \frac{u^3}{3} + c = \frac{(e^x + 1)^3}{3} + c$

**Respuesta:** $\int (e^x + 1)^2 e^x dx = \frac{(e^x + 1)^3}{3} + c$

**2.19.-Encontrar:** $\int \frac{e^x - 1}{e^x + 1} dx$

Solución.- $\int \frac{e^x - 1}{e^x + 1} dx = \int \frac{e^x}{e^x + 1} dx - \int \frac{1}{e^x + 1} dx = \int \frac{e^x}{e^x + 1} dx - \int \frac{e^x e^{-x}}{e^x + 1} dx$

$= \int \frac{e^x}{e^x + 1} dx - \int \frac{e^{-x}}{e^{-x}(e^x + 1)} dx = \int \frac{e^x}{e^x + 1} dx - \int \frac{e^{-x}}{1 + e^x} dx$

Sea: $u = e^x + 1$, donde: $du = e^x dx$ ; $w = 1 + e^{-x}$ ,donde: $dw = -e^{-x} dx$

Luego: $\int \frac{e^x}{e^x + 1} dx - \int \frac{e^{-x}}{1 + e^x} dx = \int \frac{e^x}{e^x + 1} dx - \int \frac{-e^{-x}}{1 + e^{-x}} dx = \int \frac{du}{u} + \int \frac{dw}{w}$

$= \ell\eta|u| + c_1 + \ell\eta|w| + c_2 = \ell\eta|e^x+1| + \ell\eta|1+e^{-x}| + C = \ell\eta\left[|e^x+1||1+e^{-x}|\right] + c$

**Respuesta:** $\int \frac{e^x-1}{e^x+1}dx = \ell\eta\left[|(e^x+1)(1+e^{-x})|\right] + c$ , otra respuesta seria:

$\int \frac{e^x-1}{e^x+1}dx = \ell\eta|e^x+1|^2 - x + c$

**2.20.-Encontrar:** $\int \frac{e^{2x}-1}{e^{2x}+3}dx$

Solución.- $\int \frac{e^{2x}-1}{e^{2x}+3}dx = \int \frac{e^{2x}}{e^{2x}+3}dx - \int \frac{e^0}{e^{2x}+3}dx$

$= \int \frac{e^{2x}}{e^{2x}+3}dx - \int \frac{e^{2x}e^{-2x}}{e^{2x}+3}dx = \int \frac{e^{2x}}{e^{2x}+3}dx - \int \frac{e^{-2x}}{e^{-2x}(e^{2x}+3)}dx = \int \frac{e^{2x}}{e^{2x}+3}dx - \int \frac{e^{-2x}}{1+3e^{-2x}}dx$

Sea: $u = e^{2x}+3$, donde: $du = 2e^{2x}dx$ ; $w = 1+3e^{-2x}$ ,donde: $dw = -6e^{-2x}dx$

Luego: $\int \frac{e^{2x}}{e^{2x}+3}dx - \int \frac{e^{-2x}}{1+3e^{-2x}}dx = \frac{1}{2}\int \frac{2e^{2x}}{e^{2x}+3}dx + \frac{1}{6}\int \frac{-6e^{-2x}}{1+3e^{-2x}}dx = \frac{1}{2}\int \frac{du}{u} + \frac{1}{6}\int \frac{dw}{w}$

$\frac{1}{2}\ell\eta|u| + \frac{1}{6}\ell\eta|w| + c = \frac{1}{2}\ell\eta|e^{2x}+3| + \frac{1}{6}\ell\eta|1+3e^{-2x}| + c = \frac{1}{2}\ell\eta|e^{2x}+3| + \frac{1}{6}\ell\eta\left|1+\frac{3}{e^{2x}}\right| + c$

$= \frac{1}{2}\ell\eta|e^{2x}+3| + \frac{1}{6}\ell\eta\left|\frac{e^{2x}+3}{e^{2x}}\right| + c = \frac{1}{2}\ell\eta|e^{2x}+3| + \frac{1}{6}\ell\eta|e^{2x}+3| - \frac{1}{6}\ell\eta e^{2x} + c$

$= \ell\eta\left(e^{2x}+3\right)^{1/2} + \ell\eta\left(e^{2x}+3\right)^{1/6} - \frac{1}{6}2x + c = \ell\eta\left[\left(e^{2x}+3\right)^{1/2}\left(e^{2x}+3\right)^{1/6}\right] - \frac{x}{3} + c$

$= \ell\eta\left(e^{2x}+3\right)^{2/3} - \frac{x}{3} + c$

**Respuesta:** $\int \frac{e^{2x}-1}{e^{2x}+3}dx = \ell\eta\left(e^{2x}+3\right)^{2/3} - \frac{x}{3} + c$

**2.22.-Encontrar:** $\int \frac{x^2+1}{x-1}dx$

Solución.- Cuando el grado del polinomio dividendo es MAYOR o IGUAL que el grado del polinomio divisor, es necesario efectuar previamente la división de polinomios. El resultado de la división dada es:

$\frac{x^2+1}{x-1} = (x+1) + \frac{2}{x-1}$, Luego: $\int \frac{x^2+1}{x-1}dx = \int\left(x+1+\frac{2}{x-1}\right)dx = \int xdx + \int dx + 2\int \frac{dx}{x-1}$

Sea $u = x-1$, donde $du = dx$

Luego: $\int xdx + \int dx + 2\int \frac{dx}{x-1} = \int xdx + \int dx + 2\int \frac{du}{u} = \frac{x^2}{2} + x + \ell\eta|x-1| + c$

**Respuesta:** $\int \frac{x^2+1}{x-1}dx = \frac{x^2}{2} + x + \ell\eta|x-1| + c$

**2.23.-Encontrar:** $\int \frac{x+2}{x+1}dx$

Solución.- $\frac{x+2}{x+1}=1+\frac{1}{x+1}$, Luego: $\int\frac{x+2}{x+1}dx=\int\left(1+\frac{1}{x+1}\right)dx=\int dx+\int\frac{dx}{x+1}$

Sea $u=x+1$, donde $du=dx$

$\int dx+\int\frac{du}{u}=x+\ell\eta\left|u\right|+c=x+\ell\eta\left|x+1\right|+c$

**Respuesta:** $\int\frac{x+2}{x+1}dx=x+\ell\eta\left|x+1\right|+c$

**2.24.-Encontrar:** $\int\tau g^5x\sec^2 xdx$

Solución.- Sea: $w=\tau gx$, donde: $dw=\sec^2 x$

Luego: $\int\tau g^5x\sec^2 xdx=\int(\tau gx)^5\sec^2 xdx=\int w^5dw=\frac{w^6}{6}+c=\frac{(\tau gx)^6}{6}+c=\frac{\tau g^6x}{6}+c$

**Respuesta:** $\int\tau g^5x\sec^2 xdx=\frac{\tau g^6x}{6}+c$

**2.25.-Encontrar:** $\int \operatorname{sen} x\sec^2 xdx$

Solución.- $\int \operatorname{sen} x\sec^2 xdx=\int \operatorname{sen} x\frac{1}{\cos^2 x}dx=\int\frac{\operatorname{sen} x}{\cos^2 x}dx$

Sea: $u=\cos x$, donde: $du=-\operatorname{sen} x$

Luego: $\int\frac{\operatorname{sen} x}{\cos^2 x}dx=-\int\frac{-\operatorname{sen} xdx}{\cos^2 x}=-\int\frac{du}{u}=-\int u^{-2}du=-\frac{u^{-1}}{-1}+c=\frac{1}{u}+c=\frac{1}{\cos x}+c$

**Respuesta:** $\int \operatorname{sen} x\sec^2 xdx=\sec x+c$

**2.26.-Encontrar:** $\int\frac{\sec^2 3xdx}{1+\tau g3x}$

Solución.- Sea: $u=1+\tau g3xdx$, donde: $du=3\sec^2 3xdx$

Luego: $\int\frac{\sec^2 3xdx}{1+\tau g3x}=\frac{1}{3}\int\frac{3\sec^2 3xdx}{1+\tau g3x}=\frac{1}{3}\int\frac{du}{u}=\frac{1}{3}\ell\eta\left|u\right|+c=\frac{1}{3}\ell\eta\left|1+\tau g3x\right|+c$

**Respuesta:** $\int\frac{\sec^2 3xdx}{1+\tau g3x}=\frac{1}{3}\ell\eta\left|1+\tau g3x\right|+c$

**2.27.-Encontrar:** $\int \operatorname{sen}^3 x\cos xdx$

Solución.- Sea: $w=\operatorname{sen} x$, donde: $dw=\cos xdx$

Luego: $\int \operatorname{sen}^3 x\cos xdx=\int(\operatorname{sen} x)^3\cos xdx=\int w^3dw=\int\frac{w^4}{4}+c=\int\frac{\operatorname{sen}^4 x}{4}+c$

**Respuesta:** $\int \operatorname{sen}^3 x\cos xdx=\int\frac{\operatorname{sen}^4 x}{4}+c$

**2.28.-Encontrar:** $\int\cos^4 x\operatorname{sen} xdx$

Solución.- Sea: $u=\cos x$, donde: $du=-\operatorname{sen} x$

Luego: $\int\cos^4 x\operatorname{sen} xdx=\int(\cos x)^4\operatorname{sen} xdx=-\int(\cos x)^4(-\operatorname{sen} x)dx=-\int u^4du$

$$= -\frac{u^5}{5} + c = -\frac{\cos x^5}{5} + c = -\frac{\cos^5 x}{5} + c$$

**Respuesta:** $\int \cos^4 x \operatorname{sen} x dx = -\frac{\cos^5 x}{5} + c$

**2.29.-Encontrar:** $\int \frac{\sec^5}{\cos ecx} dx$

Solución.- $\int \frac{\sec^5}{\cos ecx} dx = \int \frac{\frac{1}{\cos^5 x}}{\frac{1}{\operatorname{sen} x}} dx = \int \frac{\operatorname{sen} x}{(\cos x)^5} dx$

Sea: $w = \cos x$, donde: $dw = -\operatorname{sen} x dx$

Luego: $\int \frac{\operatorname{sen} x}{(\cos x)^5} dx = -\int \frac{dw}{w^5} = -\int w^{-5} dw = -\frac{w^{-4}}{-4} + c = \frac{1}{4} \frac{1}{w^4} + c = \frac{1}{4\cos^4 x} + c$

$$= \frac{\sec^4 x}{4} + c$$

**Respuesta:** $\int \frac{\sec^5}{\cos ecx} dx = \frac{\sec^4 x}{4} + c$

**2.30.-Encontrar:** $\int e^{\tau g 2x} \sec^2 2x dx$

Solución.- Sea: $u = \tau g 2x$, donde: $du = 2\sec^2 2x dx$

Luego: $\int e^{\tau g 2x} \sec^2 2x dx = \frac{1}{2} \int e^{\tau g 2x} (2\sec^2 2x dx) = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + c = \frac{1}{2} e^{\tau g 2x} + c$

**Respuesta:** $\int e^{\tau g 2x} \sec^2 2x dx = \frac{1}{2} e^{\tau g 2x} + c$

**2.31.-Encontrar:** $\int \frac{2x-5}{3x^2-2} dx$

Solución.- Sea: $w = 3x^2 - 2$, donde: $dw = 6x dx$

Luego: $\int \frac{2x-5}{3x^2-2} dx = \frac{1}{3} \int \frac{3(2x-5)}{3x^2-2} dx = \frac{1}{3} \int \frac{6x-15}{3x^2-2} dx = \frac{1}{3} \int \frac{6x dx}{3x^2-2} - \frac{15}{3} \int \frac{dx}{3x^2-2}$

$$= \frac{1}{3} \int \frac{6x dx}{3x^2-2} - 5 \int \frac{dx}{3(x^2 - \frac{2}{3})} = \frac{1}{3} \int \frac{6x dx}{3x^2-2} - \frac{5}{3} \int \frac{dx}{(x^2 - \frac{2}{3})} = \frac{1}{3} \int \frac{6x dx}{3x^2-2} - \frac{5}{3} \int \frac{dx}{x^2 - (\sqrt{\frac{2}{3}})^2}$$

$\frac{1}{3} \int \frac{dw}{w} - \frac{5}{3} \int \frac{dx}{x^2 - (\sqrt{\frac{2}{3}})^2} = \frac{1}{3} \ell\eta |w| + c_1 - \frac{5}{3} \int \frac{dx}{x^2 - (\sqrt{\frac{2}{3}})^2}$ ; Sea: $v = x$, donde: $dv = dx$

Además: $a = \sqrt{2/3}$ ; se tiene: $\frac{1}{3} \ell\eta |w| + c_1 - \frac{5}{3} \int \frac{dv}{v^2 - a^2}$

$$= \frac{1}{3} \ell\eta |3x^2 - 2| + c_1 - \frac{5}{3} \frac{1}{2a} \ell\eta \left| \frac{v-a}{v+a} \right| + c_2 = \frac{1}{3} \ell\eta |3x^2 - 2| - \frac{5}{3} \left[ \frac{1}{2\sqrt{2/3}} \ell\eta \left| \frac{x - \sqrt{2/3}}{x + \sqrt{2/3}} \right| \right] + C$$

$$= \frac{1}{3} \ell\eta |3x^2 - 2| - \frac{5}{\sqrt{3} 2\sqrt{2}} \ell\eta \left| \frac{\sqrt{3}x - \sqrt{2}}{\sqrt{3}x + \sqrt{2}} \right| + C = \frac{1}{3} \ell\eta |3x^2 - 2| - \frac{5}{2\sqrt{6}} \ell\eta \left| \frac{\sqrt{3}x - \sqrt{2}}{\sqrt{3}x + \sqrt{2}} \right| + C$$

**Respuesta:** $\int \frac{2x-5}{3x^2-2}dx = \frac{1}{3}\ell\eta\left|3x^2-2\right| - \frac{5}{2\sqrt{6}}\ell\eta\left|\frac{\sqrt{3}x-\sqrt{2}}{\sqrt{3}x+\sqrt{2}}\right| + C$

**2.32.-Encontrar:** $\int \frac{dx}{x\sqrt{4-9\ell\eta^2 x}}$

Solución.- $\int \frac{dx}{x\sqrt{4-9\ell\eta^2 x}} = \int \frac{dx}{x\sqrt{2^2-(3\ell\eta x)^2}}$

Sea: $u = 3\ell\eta x$ , donde: $du = \frac{3dx}{x}$

Luego: $\int \frac{dx}{x\sqrt{2^2-(3\ell\eta x)^2}} = \frac{1}{3}\int \frac{3dx}{x\sqrt{2^2-(3\ell\eta x)^2}} = \frac{1}{3}\int \frac{du}{\sqrt{2^2-(u)^2}} = \frac{1}{3}\operatorname{arcs} e\,\mathrm{n}\frac{u}{2} + c$

$= \frac{1}{3}\operatorname{arcs} e\,\mathrm{n}\frac{3\ell\eta x}{2} + c = \frac{1}{3}\operatorname{arcs} e\,\mathrm{n}\,\ell\eta\left|x\right|^{3/2} + c$

**Respuesta:** $\int \frac{dx}{x\sqrt{4-9\ell\eta^2 x}} = \frac{1}{3}\operatorname{arcs} e\,\mathrm{n}\,\ell\eta\left|x\right|^{3/2} + c$

**2.33.-Encontrar:** $\int \frac{dx}{\sqrt{e^x-1}}$

Solución.- Sea: $u = \sqrt{e^x-1}$ , donde: $du = \frac{e^x dx}{2\sqrt{e^x-1}}$ ; Tal que: $e^x = u^2+1$

Luego: $\int \frac{dx}{\sqrt{e^x-1}} = \int \frac{2du}{u^2+1} = 2\int \frac{du}{u^2+1} = 2\operatorname{arc}\tau g u + c = 2\operatorname{arc}\tau g\sqrt{e^x+1} + c$

**Respuesta:** $\int \frac{dx}{\sqrt{e^x-1}} = 2\operatorname{arc}\tau g\sqrt{e^x+1} + c$

**2.34.-Encontrar:** $\int \frac{x^2+2x+2}{x+1}dx$

Solución.- $\int \frac{x^2+2x+2}{x+1}dx = \int \frac{(x^2+2x+1)+1}{x+1}dx = \int \frac{(x+1)^2+1}{x+1}dx = \int \frac{(x+1)^2+1}{x+1}dx$

$= \int (x+1+\frac{1}{x+1})dx = \int xdx + \int dx + \int \frac{dx}{x+1}$**,** Sea: $w = x+1$**,** donde: $dw = dx$

Luego: $\int xdx + \int dx + \int \frac{dx}{x+1} = \int xdx + \int dx + \int \frac{dw}{w} = \frac{x^2}{2} + x + \ell\eta\left|w\right| + c$

$= \frac{x^2}{2} + x + \ell\eta\left|x+1\right| + c$

**Respuesta:** $\int \frac{x^2+2x+2}{x+1}dx = \frac{x^2}{2} + x + \ell\eta\left|x+1\right| + c$

**2.35.-Encontrar:** $\int \frac{e^{2x}}{\sqrt{e^x+1}}dx$

Solución.- Sea: $u = e^x+1$, donde: $du = e^x dx$

Luego: $\int \frac{e^{2x}}{\sqrt{e^x+1}}dx = \int \frac{u-1}{u^{1/2}}du = \int (u^{1/2} - u^{-1/2})du = \int u^{1/2}du - \int u^{-1/2}du = \frac{u^{3/2}}{3/2} - \frac{u^{-1/2}}{1/2} + c$

$= \frac{u^{3/2}}{3/2} - \frac{u^{-1/2}}{1/2} + c = \frac{2}{3}u^{3/2} - \frac{1}{2}u^{1/2} + c = \frac{2}{3}\sqrt{(e^x+1)^3} - 2\sqrt{(e^x+1)} + c$

**Respuesta:** $\int \frac{e^{2x}}{\sqrt{e^x+1}}dx = \frac{2}{3}\sqrt{(e^x+1)^3} - 2\sqrt{(e^x+1)} + c$

**2.36.-Encontrar:** $\int \frac{\ell\eta 2x}{\ell\eta 4x}\frac{dx}{x}$

Solución.- Sea: $u = \ell\eta 4x$ , donde: $du = \frac{dx}{x}$ ; además: $\ell\eta 4x = (2\times 2x) = \ell\eta 2 + \ell\eta 2x$

$\Rightarrow u = \ell\eta 2 + \ell\eta 2x \Rightarrow \ell\eta 2x = u - \ell\eta 2$

Luego: $\int \frac{\ell\eta 2x}{\ell\eta 4x}\frac{dx}{x} = \int \frac{u-\ell\eta 2}{u}du = \int du - \int \frac{\ell\eta 2}{u}du = \int du - \ell\eta 2\int \frac{du}{u} = u - \ell\eta 2|u| + c$

$= \ell\eta 4x - \ell\eta 2[\ell\eta(\ell\eta 4x)] + c$

**Respuesta:** $\int \frac{\ell\eta 2x}{\ell\eta 4x}\frac{dx}{x} = \ell\eta 4x - \ell\eta 2[\ell\eta(\ell\eta 4x)] + c$

**2.37.-Encontrar:** $\int x(3x+1)^7 dx$

Solución.- Sea: $w = 3x+1$, donde: $dw = 3dx$ ; además: $w-1 = 3x \Rightarrow x = \frac{w-1}{3}$

Luego: $\int x(3x+1)^7 dx = \int \frac{w-1}{3}w^7\frac{dw}{3} = \frac{1}{9}\int (w-1)w^7 dw = \frac{1}{9}\int (w^8 - w^7)dw$

$= \frac{1}{9}\int w^8 dw - \frac{1}{9}\int w^7 dw = \frac{1}{9}\frac{w^9}{9} - \frac{1}{9}\frac{w^8}{8} + c = \frac{1}{81}w^9 - \frac{1}{72}w^8 + c$

$= \frac{1}{81}(3x+1)^9 - \frac{1}{72}(3x+1)^8 + c$

**Respuesta:** $\int x(3x+1)^7 dx = \frac{(3x+1)^9}{81} - \frac{(3x+1)^8}{72} + c$

**2.38.-Encontrar:** $\int \frac{x^2-5x+6}{x^2+4}dx$

Solución.- $\frac{x^2-5x+6}{x^2+4}dx = 1 + \frac{2-5x}{x^2+4}$

Luego: $\int \frac{x^2-5x+6}{x^2+4}dx = \int (1 + \frac{2-5x}{x^2+4})dx = \int dx + 2\int \frac{dx}{x^2+4} - 5\int \frac{xdx}{x^2+4}$

Sea: $u = x^2+4$ , donde: $du = 2xdx$ ; Entonces:

$= x + \operatorname{arc}\tau g\frac{x}{2} - \frac{5}{2}\int \frac{du}{u} = x + \operatorname{arc}\tau g\frac{x}{2} - \frac{5}{2}\ell\eta|u| + c = x + \operatorname{arc}\tau g\frac{x}{2} - \frac{5}{2}\ell\eta|x^2+4| + c$

**Respuesta:** $\int \frac{x^2-5x+6}{x^2+4}dx = x + \operatorname{arc}\tau g\frac{x}{2} - \frac{5}{2}\ell\eta|x^2+4| + c$

## *EJERCICIOS PROPUESTOS*

Usando Esencialmente la técnica de integración por sustitución, encontrar las siguientes integrales:

**2.39.-** $\int 3^x e^x dx$ **2.40.-** $\int \frac{adx}{a-x}$ **2.41.-** $\int \frac{4t+6}{2t+1} dt$

**2.42.-** $\int \frac{1-3x}{3+2x} dx$ **2.43.-** $\int \frac{xdx}{a+bx}$ **2.44.-** $\int \frac{ax-b}{\alpha x+\beta} dx$

**2.45.-** $\int \frac{3t^2+3}{t-1} dt$ **2.46.-** $\int \frac{x^2+5x+7}{x+3} dx$ **2.47.-** $\int \frac{x^4+x^2+1}{x-1} dx$

**2.48.-** $\int \left( a+\frac{b}{x-a} \right)^2 dx$ **2.49.-** $\int \frac{x}{(x+1)^2} dx$ **2.50.-** $\int \frac{bdy}{\sqrt{1-y}}$

**2.51.-** $\int \sqrt{a-bx}\, dx$ **2.52.-** $\int \frac{xdx}{\sqrt{x^2+1}}$ **2.53.-** $\int \frac{\sqrt{x}+\ell\eta x}{x} dx$

**2.54.-** $\int \frac{dx}{3x^2+5}$ **2.55.-** $\int \frac{x^3 dx}{a^2-x^2}$ **2.56.-** $\int \frac{y^2-5y+6}{y^2+4} dy$

**2.57.-** $\int \frac{6t-15}{3t^2-2} dt$ **2.58.-** $\int \frac{3-2x}{5x^2+7} dx$ **2.59.-** $\int \frac{3x+1}{\sqrt{5x^2+1}} dx$

**2.60.-** $\int \frac{xdx}{x^2-5}$ **2.61.-** $\int \frac{xdx}{2x^2+3}$ **2.62.-** $\int \frac{ax+b}{a^2x^2+b^2} dx$

**2.63.-** $\int \frac{xdx}{\sqrt{a^4-x^4}}$ **2.64.-** $\int \frac{x^2 dx}{1+x^6}$ **2.65.-** $\int \frac{x^2 dx}{\sqrt{x^6-1}}$

**2.66.-** $\int \frac{x-\sqrt{\operatorname{arc}\tau g 3x}}{1+9x^2} dx$ **2.67.-** $\int \sqrt{\frac{\operatorname{arcsen} t}{4-4t^2}} dt$ **2.68.-** $\int \frac{\operatorname{arc}\tau g(\frac{x}{3})}{9+x^2} dx$

**2.69.-** $\int \frac{dt}{\sqrt{(9+9t^2)\ell\eta\left|t+\sqrt{1+t^2}\right|}}$ **2.70.-** $\int ae^{-mx} dx$ **2.71.-** $\int 4^{2-3x} dx$

**2.72.-** $\int (e^t-e^{-t}) dt$ **2.73.-** $\int e^{-(x^2+1)} x dx$ **2.74.-** $\int (e^{x/a}-e^{-x/a})^2 dx$

**2.75.-** $\int \frac{a^{2x}-1}{\sqrt{a^x}} dx$ **2.76.-** $\int \frac{e^{1/x}}{x^2} dx$ **2.77.-** $\int 5^{\sqrt{x}} \frac{dx}{\sqrt{x}}$

**2.78.-** $\int x 7^{x^2} dx$ **2.79.-** $\int \frac{e^t dt}{e^t-1}$ **2.80.-** $\int e^x \sqrt{a-be^x}\, dx$

**2.81.-** $\int (e^{x/a}+1)^{1/3} e^{x/a} dx$ **2.82.-** $\int \frac{dx}{2^x+3}$ **2.83.-** $\int \frac{a^x dx}{1+a^{2x}}; a>0$

**2.84.-** $\int \frac{e^{-bx}}{1-e^{-2bx}} dx$ **2.85.-** $\int \frac{e^t dt}{\sqrt{1-e^{2t}}}$ **2.86.-** $\int \cos\frac{x}{\sqrt{2}} dx$

**2.87.-** $\int \operatorname{sen}(a+bx) dx$ **2.88.-** $\int \cos\sqrt{x} \frac{dx}{\sqrt{x}}$ **2.89.-** $\int \operatorname{sen}(\ell\eta x) \frac{dx}{x}$

**2.90.-** $\int (\cos ax+\operatorname{sen} ax)^2 dx$ **2.91.-** $\int \operatorname{sen}^2 x dx$ **2.92.-** $\int \cos^2 x dx$

**2.93.-** $\int \sec^2(ax+b)dx$

**2.94.-** $\int \cos\tau g^2 axdx$

**2.95.-** $\int \frac{dx}{\mathrm{sen}\frac{x}{a}}$

**2.96.-** $\int \frac{dx}{3\cos(5x-\frac{\pi}{4})}$

**2.97.-** $\int \frac{dx}{\mathrm{sen}(ax+b)}$

**2.98.-** $\int \frac{xdx}{\cos^2 x^2}$

**2.99.-** $\int \mathrm{co}\,\tau g\frac{x}{a-b}dx$

**2.100.-** $\int \tau g\sqrt{x}\frac{dx}{\sqrt{x}}$

**2.101.-** $\int \frac{dx}{\tau g\frac{x}{5}}$

**2.102.-** $\int \left(\frac{1}{\mathrm{sen}\,x\sqrt{2}}-1\right)^2 dx$

**2.103.-** $\int \frac{dx}{\mathrm{sen}\,x\cos x}$

**2.104.-** $\int \frac{\cos ax}{\mathrm{sen}^5 ax}dx$

**2.105.-** $\int t\,\mathrm{sen}(1-2t^2)dt$

**2.106.-** $\int \frac{\mathrm{sen}\,3x}{3+\cos 3x}dx$

**2.107.-** $\int \tau g^3\frac{x}{3}\sec^2\frac{x}{3}dx$

**2.108.-** $\int \frac{\mathrm{sen}\,x\cos x}{\sqrt{\cos^2 x-\mathrm{sen}^2 x}}dx$

**2.109.-** $\int \frac{\sqrt{\tau gx}}{\cos^2 x}dx$

**2.110.-** $\int \cos\frac{x}{a}\mathrm{sen}\frac{x}{a}dx$

**2.111.-** $\int t\,\mathrm{co}\,\tau g(2t^2-3)dt$

**2.112.-** $\int \frac{x^3dx}{x^8+5}$

**2.113.-** $\int \mathrm{sen}^3 6x\cos 6xdx$

**2.114.-** $\int \sqrt{1+3\cos^2 x}\,\mathrm{sen}\,2xdx$

**2.115.-** $\int x\sqrt[5]{5-x^2}dx$

**2.116.-** $\int \frac{1+\mathrm{sen}\,3x}{\cos^2 3x}dx$

**2.117.-** $\int \frac{(\cos ax+\mathrm{sen}\,ax)^2}{\mathrm{sen}\,ax}dx$

**2.118.-** $\int \frac{x^3-1}{x+1}dx$

**2.119.-** $\int \frac{\cos ec^2 3xdx}{b-a\,\mathrm{co}\,\tau g3x}$

**2.120.-** $\int \frac{x^3-1}{x^4-4x+1}dx$

**2.121.-** $\int xe^{-x^2}dx$

**2.122.-** $\int \frac{3-\sqrt{2+3x^2}}{2+3x^2}dx$

**2.123.-** $\int \frac{\tau g3x-\mathrm{co}\,\tau g3x}{\mathrm{sen}\,3x}dx$

**2.124.-** $\int \frac{dx}{\sqrt{e^x}}$

**2.125.-** $\int \frac{1+\mathrm{sen}\,x}{x+\cos x}dx$

**2.126.-** $\int \frac{\sec^2 xdx}{\sqrt{\tau g^2x-2}}$

**2.127.-** $\int \frac{dx}{x\ell\eta^2 x}$

**2.128.-** $\int a^{\mathrm{sen}\,x}\cos xdx$

**2.129.-** $\int \frac{x^2}{\sqrt{x^3+1}}dx$

**2.130.-** $\int \frac{xdx}{\sqrt{1-x^4}}$

**2.131.-** $\int \tau g^2 axdx$

**2.132.-** $\int \frac{\sec^2 xdx}{\sqrt{4-\tau g^2 x}}$

**2.133.-** $\int \frac{dx}{\cos x/a}$

**2.134.-** $\int \frac{\sqrt[3]{1+\ell\eta x}}{x}dx$

**2.135.-** $\int \tau g\sqrt{x-1}\frac{dx}{\sqrt{x-1}}$

**2.136.-** $\int \frac{xdx}{\mathrm{sen}\,x^2}$

**2.137.-** $\int \frac{\mathrm{sen}\,x-\cos x}{\mathrm{sen}\,x+\cos x}dx$

**2.138.-** $\int \frac{e^{\mathrm{arc}\tau gx}+x\ell\eta(1+x^2)+1}{1+x^2}$

**2.139.-** $\int \frac{x^2dx}{x^2-2}$

**2.140.-** $\int e^{\mathrm{sen}^2 x}\mathrm{sen}\,2xdx$

**2.141.-** $\int \frac{(1-\mathrm{sen}\frac{x}{\sqrt{2}})^2}{\mathrm{sen}\frac{x}{\sqrt{2}}}dx$

**2.142.-** $\int \frac{5-3x}{\sqrt{4-3x^2}}dx$

**2.143.-** $\int \frac{ds}{e^s+1}$

**2.144.-** $\int \frac{d\theta}{\mathrm{sen}\,a\theta\cos a\theta}$

**2.145.-** $\int \frac{e^s}{\sqrt{e^{2s}-2}}ds$

**2.146.-** $\int \mathrm{sen}(\frac{2\pi t}{T}+\varphi_0)dt$

**2.147.-** $\int \frac{\arccos \frac{x}{2}}{\sqrt{4-x^2}}dx$

**2.148.-** $\int \frac{dx}{x(4-\ell\eta^2 x)}$

**2.149.-** $\int e^{-\tau g x}\sec^2 x dx$

**2.150.-** $\int \frac{\operatorname{sen} x\cos x}{\sqrt{2-\operatorname{sen}^4 x}}dx$

**2.151.-** $\int \frac{\sec x\tau g x}{\sqrt{\sec^2 x+1}}dx$

**2.152.-** $\int \frac{dt}{\operatorname{sen}^2 t\cos^2 t}$

**2.153.-** $\int \frac{\operatorname{arcsen} x+x}{\sqrt{1-x^2}}dx$

**2.154.-** $\int \frac{xdx}{\sqrt{x+1}}$

**2.155.-** $\int x(5x^2-3)^7 dx$

**2.156.-** $\int \sqrt{\frac{\ell\eta(x+\sqrt{x^2+1})}{x^2+1}}dx$

**2.157.-** $\int \frac{\operatorname{sen}^3 x}{\sqrt{\cos x}}dx$

**2.158.-** $\int \frac{\cos xdx}{\sqrt{1+\operatorname{sen}^2 x}}$

**2.159.-** $\int \frac{(\operatorname{arcsen} x)^2}{\sqrt{1-x^2}}dx$

**2.150.-** $\int e^{x+e^x}dx$

**2.161.-** $\int t(4t+1)^7 dt$

**2.162.-** $\int \frac{2t^2-10t+12}{t^2+4}dt$

**2.163.-** $\int \frac{e^t-e^{-t}}{e^t+e^{-t}}dt$

## *RESPUESTAS*

**2.39.-** $\int 3^x e^x dx$ **,** Sea: $u=x, du=dx, a=3e$

$$\int (3e)^x dx=\int (a)^u du=\frac{a^u}{\ell\eta a}+c=\frac{(3e)^x}{\ell\eta(3e)}+c=\frac{(3e)^x}{\ell\eta 3\ell\eta e}+c=\frac{3^x e^x}{\ell\eta 3+\ell\eta e}+c=\frac{3^x e^x}{\ell\eta 3+1}+c$$

**2.40.-** $\int \frac{adx}{a-x}$ **,** Sea: $u=a-x, du=-dx$

$$\int \frac{adx}{a-x}=-a\int \frac{du}{u}=-a\ell\eta\left|u\right|+c=-a\ell\eta\left|a-x\right|+c$$

**2.41.-** $\int \frac{4t+6}{2t+1}dt$ **,** Sea: $u=2t+1, du=2dt;$ $\quad \frac{2t+3}{2t+1}=1+\frac{2}{2t+1}$

$$\int \frac{4t+6}{2t+1}dt=2\int\left(1+\frac{2}{2t+1}\right)dt=2\int dt+2\int\frac{2}{2t+1}dt=2\int dt+2\int\frac{du}{u}=2t+2\ell\eta\left|u\right|+c$$

$$=2t+2\ell\eta\left|2t+1\right|+c$$

**2.42.-** $\int \frac{1-3x}{3+2x}dx$ **,** Sea: $u=3+2x, du=2dx$ ; $\quad \frac{1-3x}{3+2x}=-\frac{3}{2}+\frac{11/2}{2x+3}$

$$\int \frac{1-3x}{3+2x}dx=\int\left(-\frac{3}{2}+\frac{11/2}{2x+3}\right)dx=-\frac{3}{2}\int dx+\frac{11}{4}\int\frac{dx}{2x+3}=-\frac{3}{2}\int dx+\frac{11}{4}\int\frac{du}{u}$$

$$-\frac{3}{2}x+\frac{11}{4}\ell\eta\left|2x+3\right|+c$$

**2.43.-** $\int \frac{xdx}{a+bx}$ **,** Sea: $u=a+bx, du=bdx$ ; $\quad \frac{x}{a+bx}=\frac{1}{b}-\frac{a/b}{a+bx}$

$$\int \frac{xdx}{a+bx}=\frac{1}{b}\int dx-\frac{a}{b}\int\frac{dx}{a+bx}=\frac{1}{b}\int dx-\frac{a}{b^2}\int\frac{du}{u}=\frac{1}{b}x-\frac{a}{b^2}\ell\eta\left|u\right|+c=\frac{x}{b}-\frac{a}{b^2}\ell\eta\left|a+bx\right|+c$$

**2.44.-** $\int \frac{ax-b}{\alpha x+\beta}dx$ **,** Sea: $u=\alpha x+\beta, du=\alpha dx$; $\frac{ax-b}{ax+b}=\frac{a}{\alpha}-\frac{\frac{\alpha\beta}{\alpha}+b}{\alpha x}$

$$\int \frac{ax-b}{\alpha x+\beta}dx=\int\left(\frac{a}{\alpha}-\frac{\frac{\alpha\beta}{\alpha}+b}{\alpha x}\right)dx=\int\frac{a}{\alpha}dx-\int\frac{\frac{a\beta+\alpha b}{\alpha}}{\alpha x+\beta}dx=\frac{a}{\alpha}\int dx-\frac{a\beta+\alpha b}{\alpha}\int\frac{dx}{a\beta+\alpha b}$$

$$=\frac{a}{\alpha}\int dx-\frac{a\beta+\alpha b}{\alpha^2}\int\frac{du}{u}=\frac{a}{\alpha}x-\frac{a\beta+\alpha b}{\alpha^2}\ell\eta|u|+c=\frac{a}{\alpha}x-\frac{a\beta+\alpha b}{\alpha^2}\ell\eta|\ell x+\beta|+c$$

**2.45.-** $\int\frac{3t^2+3}{t-1}dt$ **,** Sea: $u=t-1, du=dt$; $\frac{t^2+1}{t-1}=t+1+\frac{2}{t-1}$

$$\int\frac{3t^2+3}{t-1}dt=3\int\left(t+1+\frac{2}{t-1}\right)dt=3\int tdt+3\int dt+3\int\frac{2}{t-1}dt=\frac{3}{2}t^2+3t+6\ell\eta|u|+c$$

$$=\frac{3}{2}t^2+3t+6\ell\eta|t-1|+c$$

**2.46.-** $\int\frac{x^2+5x+7}{x+3}dx$ **,** Sea: $u=t-1, du=t+1$; $\frac{x^2+5x+7}{x+3}=x+2+\frac{1}{x+3}$

$$\int\frac{x^2+5x+7}{x+3}dx=\int\left(x+2+\frac{1}{x+3}\right)dx=\int xdx+2\int dx+\int\frac{1}{x+3}dx=\frac{x^2}{2}+2x+\ell\eta|u|+c$$

$$=\frac{x^2}{2}+2x+\ell\eta|u|+c=\frac{x^2}{2}+2x+\ell\eta|x+3|+c$$

**2.47.-** $\int\frac{x^4+x^2+1}{x-1}dx$ **,** Sea: $u=x-1, du=dx$;

$$\int\frac{x^4+x^2+1}{x-1}dx=\int\left(x^3+x^2+2x+2+\frac{3}{x-1}\right)dx=\int x^3dx+\int x^2dx+2\int dx+3\int\frac{dx}{x-1}$$

$$=\frac{x^4}{4}+\frac{x^3}{3}+x^2+2+3\ell\eta|u|+c=\frac{x^4}{4}+\frac{x^3}{3}+x^2+2x+3\ell\eta|x-1|+c$$

**2.48.-** $\int\left(a+\frac{b}{x-a}\right)^2dx$ **,** Sea: $u=x-a, du=dx$

$$\int\left(a+\frac{b}{x-a}\right)^2dx=\int\left(a^2+\frac{2ab}{x-a}+\frac{b^2}{(x-a)^2}\right)dx=a^2\int dx+2ab\int\frac{dx}{x-a}+b^2\int\frac{dx}{(x-a)^2}$$

$$=a^2\int dx+2ab\int\frac{du}{u}+b^2\int\frac{du}{u^2}=a^2x+2ab\ell\eta|u|+b^2\frac{u^{-1}}{-1}+c=a^2x+2ab\ell\eta|x-a|-\frac{b^2}{x-a}+c$$ **2.**

**49.-** $\int\frac{x}{(x+1)^2}dx$ **,** Sea: $u=x+1, du=dx$

$$\int\frac{x}{(x+1)^2}dx=\int\frac{(x+1)-1}{(x+1)^2}dx=\int\frac{x+1}{(x+1)^2}dx-\int\frac{dx}{(x+1)^2}=\int\frac{dx}{u}-\int\frac{dx}{u^2}=\ell\eta|u|-\frac{u^{-1}}{-1}+c$$

$= \ell\eta|x+1| + \frac{1}{x+1} + c$

**2.50.-** $\int \frac{bdy}{\sqrt{1-y}}$ **,** Sea: $u = 1-y, du = -dy$

$\int \frac{bdy}{\sqrt{1-y}} = -b\int \frac{du}{\sqrt{u}} = -b\int u^{-1/2}du = -2bu^{1/2} + c = -2b(1-y)^{1/2} + c$

**2.51.-** $\int \sqrt{a-bx}dx$ **,** Sea: $u = a-bx, du = -bdx$

$\int \sqrt{a-bx}dx = -\frac{1}{b}\int u^{1/2}du = -\frac{1}{b}\frac{u^{3/2}}{\frac{3}{2}} + c = -\frac{2}{3b}u^{3/2} + c = -\frac{3}{2b}(a-bx)^{3/2} + c$

**2.52.-** $\int \frac{xdx}{\sqrt{x^2+1}}$ **,** Sea: $u = x^2+1, du = 2xdx$

$\int \frac{xdx}{\sqrt{x^2+1}} = \frac{1}{2}\int \frac{du}{\sqrt{u}} = \frac{1}{2}\int u^{-1/2}du = \frac{1}{\cancel{2}}\frac{u^{1/2}}{\frac{1}{\cancel{2}}} + c = (x^2+1)^{1/2} + c$

**2.53.-** $\int \frac{\sqrt{x} + \ell\eta x}{x}dx$ **,** Sea: $u = \ell\eta x, du = \frac{dx}{x}$

$\int \frac{\sqrt{x} + \ell\eta x}{x}dx = \int x^{-1/2}dx + \int \frac{\ell\eta x}{x}dx = \int x^{-1/2}dx + \int udu = \frac{x^{1/2}}{1/2} + \frac{u^2}{2} + c$

$= 2\sqrt{x} + \frac{\ell\eta^2 x}{2} + c$

**2.54.-** $\int \frac{dx}{3x^2+5}$ **,** Sea: $u^2 = 3x^2, u = \sqrt{3}x, du = \sqrt{3}dx$; $a^2 = 5; a = \sqrt{5}$

$\int \frac{dx}{3x^2+5} = \frac{1}{\sqrt{3}}\int \frac{du}{u^2+a^2} = \frac{1}{\sqrt{3}}\frac{1}{a}\operatorname{arc}tg\frac{u}{a} + c = \frac{1}{\sqrt{3}}\frac{1}{\sqrt{5}}\operatorname{arc}tg\frac{\sqrt{3}x}{\sqrt{5}} + c = \frac{\sqrt{15}}{15}\operatorname{arc}tg\sqrt{\frac{3x}{5}} + c$

**2.55.-** $\int \frac{x^3dx}{a^2-x^2}$ **,** Sea: $u = x^2 - a^2, du = 2xdx$

$\int \frac{x^3dx}{a^2-x^2} = -\int xdx - \int \frac{a^2xdx}{x^2-a^2} = -\int xdx - a^2\int \frac{xdx}{x^2-a^2} = -\int xdx - \frac{a^2}{2}\int \frac{du}{u}$

$= -\frac{x^2}{2} - \frac{a^2}{2}\ell\eta|u| + c = -\frac{x^2}{2} - \frac{a^2}{2}\ell\eta|x^2-a^2| + c$

**2.56.-** $\int \frac{y^2-5y+6}{y^2+4}dy$ **,** Sea: $u = y^2+4, du = 2ydy$

$\int \frac{y^2-5y+6}{y^2+4}dy = \int (1 + \frac{-5y+2}{y^2+4})dy = \int dy + \int \frac{-5y+2}{y^2+4}dy = \int dy - 5\int \frac{ydy}{y^2+4} + 2\int \frac{dy}{y^2+2^2}$

$= y - \frac{5}{2}\ell\eta|u| + \cancel{2}\,\frac{1}{\cancel{2}}\operatorname{arc}\tau g\frac{y}{2} + c = y - \frac{5}{2}\ell\eta|y^2+4| + \operatorname{arc}\tau g\frac{y}{2} + c$

**2.57.-** $\int \frac{6t-15}{3t^2-2}dt$ **,** Sea: $u = 3t^2-2, du = 6tdt; w = \sqrt{3}t, dw = \sqrt{3}dt$

$$\int\frac{6t-15}{3t^2-2}dt=6\int\frac{tdt}{3t^2-2}-15\int\frac{dt}{3t^2-2}=6\int\frac{tdt}{3t^2-2}-15\int\frac{dt}{(\sqrt{3t})^2-(\sqrt{2})^2}$$

$$=\int\frac{du}{u}-\frac{15}{\sqrt{3}}\int\frac{dw}{w^2-(\sqrt{2})^2}=\ell\eta|u|-\frac{15\sqrt{3}}{3}\frac{1}{2\sqrt{2}}\ell\eta\left|\frac{w-\sqrt{2}}{w+\sqrt{2}}\right|+c$$

$$=\ell\eta\left|3t^2-2\right|-\frac{5\sqrt{6}}{4}\ell\eta\left|\frac{t\sqrt{3}-\sqrt{2}}{t\sqrt{3}+\sqrt{2}}\right|+c$$

**2.58.-** $\int\frac{3-2x}{5x^2+7}dx$, Sea: $u=5x^2+7, du=10xdx; w=\sqrt{5}x, dw=\sqrt{5}dx$

$$\int\frac{3-2x}{5x^2+7}dx=3\int\frac{dx}{5x^2+7}-2\int\frac{dx}{5x^2+7}=3\int\frac{dx}{(\sqrt{5}x)^2+(\sqrt{7})^2}-\frac{2}{10}\int\frac{du}{u}$$

$$=\frac{3}{\sqrt{5}}\int\frac{dw}{w^2+(\sqrt{7})^2}-\frac{1}{5}\int\frac{du}{u}=\frac{3}{\sqrt{5}}\frac{1}{\sqrt{7}}\operatorname{arc}\tau g\frac{x\sqrt{5}}{\sqrt{7}}-\frac{1}{5}\ell\eta|u|+c$$

$$=\frac{3\sqrt{35}}{35}\operatorname{arc}\tau g x\sqrt{\frac{5}{7}}-\frac{1}{5}\ell\eta\left|5x^2+7\right|+c$$

**2.59.-** $\int\frac{3x+1}{\sqrt{5x^2+1}}dx$, Sea: $u=5x^2+1, du=10xdx; w=x\sqrt{5}, dw=\sqrt{5}dx$

$$\int\frac{3x+1}{\sqrt{5x^2+1}}dx=3\int\frac{xdx}{\sqrt{5x^2+1}}+\int\frac{dx}{\sqrt{(x\sqrt{5})^2+1^2}}=3\int\frac{xdx}{\sqrt{5x^2+1}}+\int\frac{dx}{\sqrt{(x\sqrt{5})^2+1^2}}$$

$$=\frac{3}{10}\int\frac{du}{\sqrt{u}}+\frac{1}{\sqrt{5}}\int\frac{dw}{\sqrt{w^2+1^2}}=\frac{3}{10}\frac{u^{1/2}}{1/2}+\frac{1}{\sqrt{5}}\ell\eta\left|w+\sqrt{w^2+1}\right|+c$$

$$=\frac{3}{5}\sqrt{5x^2+1}+\frac{1}{\sqrt{5}}\ell\eta\left|x\sqrt{5}+\sqrt{5x^2+1}\right|+c$$

**2.60.-** $\int\frac{xdx}{x^2-5}$, Sea: $u=x^2+5, du=2xdx$

$$\int\frac{xdx}{x^2-5}=\frac{1}{2}\int\frac{du}{u}=\frac{1}{2}\ell\eta|u|+c=\frac{1}{2}\ell\eta\left|x^2-5\right|+c$$

**2.61.-** $\int\frac{xdx}{2x^2+3}$, Sea: $u=2x^2+3, du=4xdx$

$$\int\frac{xdx}{2x^2+3}=\frac{1}{4}\int\frac{du}{u}=\frac{1}{4}\ell\eta|u|+c=\frac{1}{4}\ell\eta\left|2x^2+3\right|+c$$

**2.62.-** $\int\frac{ax+b}{a^2x^2+b^2}dx$, Sea: $u=a^2x^2+b^2, du=2a^2xdx; w=ax, dw=adx$

$$\int\frac{ax+b}{a^2x^2+b^2}dx=a\int\frac{xdx}{a^2x^2+b^2}+b\int\frac{dx}{a^2x^2+b^2}=\frac{a}{2a^2}\int\frac{du}{u}+\frac{b}{a}\int\frac{dw}{w^2+b^2}$$

$$=\frac{1}{2}\ell\eta|u|+\frac{\cancel{b}}{a}\frac{1}{\cancel{b}}\operatorname{arc}\tau g\frac{w}{b}+c=\frac{1}{2}\ell\eta\left|a^2x^2+b^2\right|+\frac{1}{a}\operatorname{arc}\tau g\frac{ax}{b}+c$$

**2.63.-** $\int \frac{xdx}{\sqrt{a^4-x^4}}$, Sea: $u=x^2, du=2xdx$

$$\int \frac{xdx}{\sqrt{a^4-x^4}}=\int \frac{xdx}{\sqrt{(\sqrt{a^2})^2-(\sqrt{x^2})^2}}=\frac{1}{2}\int \frac{du}{\sqrt{(\sqrt{a^2})^2-u^2}}=\frac{1}{2}\operatorname{arcsen}\frac{u}{a^2}+c$$

$$=\frac{1}{2}\operatorname{arcsen}\frac{x^2}{a^2}+c$$

**2.64.-** $\int \frac{x^2dx}{1+x^6}$, Sea: $u=x^3, du=3x^2dx$

$$\int \frac{x^2dx}{1+x^6}=\int \frac{x^2dx}{1+(x^3)^2}=\frac{1}{3}\int \frac{du}{1+u^2}=\frac{1}{3}\operatorname{arc}\tau g\left|u\right|+c=\frac{1}{3}\operatorname{arc}\tau g x^3+c$$

**2.65.-** $\int \frac{x^2dx}{\sqrt{x^6-1}}$, Sea: $u=x^3, du=3x^2dx$

$$\int \frac{x^2dx}{\sqrt{x^6-1}}=\int \frac{x^2dx}{\sqrt{(x^3)^2-1}}=\frac{1}{3}\int \frac{du}{\sqrt{u^2-1}}=\frac{1}{3}\ell\eta\left|u+\sqrt{u^2-1}\right|+c=\frac{1}{3}\ell\eta\left|x^3+\sqrt{x^6-1}\right|+c$$

**2.66.-** $\int \frac{x-\sqrt{\operatorname{arc}\tau g 3x}}{1+9x^2}dx$, Sea: $u=1+9x^2, du=18xdx; w=\operatorname{arc}\tau g 3x, dw=\frac{3dx}{1+9x^2}$

$$\int \frac{x-\sqrt{\operatorname{arc}\tau g 3x}}{1+9x^2}dx=\int \frac{xdx}{1+9x^2}-\int \frac{\sqrt{\operatorname{arc}\tau g 3x}}{1+9x^2}dx=\frac{1}{18}\int \frac{du}{u}-\frac{1}{3}\int w^{1/2}dw$$

$$=\frac{1}{18}\ell\eta\left|u\right|-\frac{1}{3}\frac{w^{3/2}}{3/2}+c=\frac{1}{18}\ell\eta\left|1+9x^2\right|-\frac{2(\operatorname{arc}\tau g 3x)^{3/2}}{9}+c$$

**2.67.-** $\int \sqrt{\frac{\operatorname{arcsen} t}{4-4t^2}}dt$, Sea: $u=\operatorname{arcsen} t, du=\frac{dt}{\sqrt{1-t^2}}$

$$\int \sqrt{\frac{\operatorname{arcsen} t}{4-4t^2}}dt=\frac{1}{2}\int \sqrt{\frac{\operatorname{arcsen} t}{1-t^2}}dt=\frac{1}{2}\int \frac{\sqrt{\operatorname{arcsen} t}}{\sqrt{1-t^2}}dt=\frac{1}{2}\int \sqrt{u}du=\frac{1}{\cancel{2}}\frac{u^{3/2}}{3/\cancel{2}}+c=\frac{1}{3}u^{3/2}+c$$

$$=\frac{1}{3}\sqrt{(\operatorname{arcsen} t)^3}+c$$

**2.68.-** $\int \frac{\operatorname{arc}\tau g(\frac{x}{3})}{9+x^2}dx$, Sea: $u=\operatorname{arc}\tau g\frac{x}{3}, du=\frac{3dx}{9+x^2}$

$$\int \frac{\operatorname{arc}\tau g(\frac{x}{3})}{9+x^2}dx=\frac{1}{3}\int udu=\frac{1}{3}\frac{u^2}{2}+c=\frac{1}{6}u^2+c=\frac{\operatorname{arc}\tau g(\frac{x}{3})^2}{6}+c$$

**2.69.-** $\int \frac{dt}{\sqrt{(9+9t^2)\ell\eta\left|t+\sqrt{1+t^2}\right|}}$, Sea: $u=\ell\eta\left|t+\sqrt{1+t^2}\right|, du=\frac{dt}{\sqrt{1+t^2}}$

$$=\frac{1}{3}\int \frac{dt}{\sqrt{(1+t^2)}\sqrt{\ell\eta\left|t+\sqrt{1+t^2}\right|}}=\frac{1}{3}\int \frac{du}{\sqrt{u}}=\frac{1}{3}\frac{u^{1/2}}{1/2}+c=\frac{2}{3}\sqrt{u}+c=\frac{2}{3}\sqrt{\ell\eta\left|t+\sqrt{1+t^2}\right|}+c$$

**2.70.-** $\int ae^{-mx}dx$ **,** Sea: $u=-mx, du=-mdx$

$$\int ae^{-mx}dx = a\int e^{-mx}dx = -\frac{a}{m}\int e^{u}du = -\frac{a}{m}e^{u}+c = -\frac{a}{m}e^{-mx}+c$$

**2.71.-** $\int 4^{2-3x}dx$ **,** Sea: $u=2-3x, du=-3dx; a=4$

$$\int 4^{2-3x}dx = -\frac{1}{3}\int a^{u}du = -\frac{1}{3}\frac{a^{u}}{\ell\eta a}+c = -\frac{4^{2-3x}}{3\ell\eta 4}+c$$

**2.72.-** $\int (e^{t}-e^{-t})dt$ **,** Sea: $u=-t, du=-dt$

$$\int (e^{t}-e^{-t})dt = \int e^{t}dt - \int e^{-t}dt = \int e^{t}dt - \int e^{u}dt = e^{t}+e^{u}+c = e^{t}+e^{-t}+c$$

**2.73.-** $\int e^{-(x^2+1)}xdx$ **,** Sea: $u=-x^2-1, du=-2xdx$

$$\int e^{-(x^2+1)}xdx = \int e^{-x^2-1}xdx = -\frac{1}{2}\int e^{u}du = -\frac{1}{2}e^{u}+c = -\frac{1}{2}e^{-(x^2+1)}+c = -\frac{1}{2e^{x^2+1}}+c$$

**2.74.-** $\int (e^{x/a}-e^{-x/a})^2dx$ **,** Sea: $u=\frac{2x}{a}, du=\frac{2dx}{a}; w=-\frac{2x}{a}, dw=-\frac{2dx}{a}$

$$\int (e^{x/a}-e^{-x/a})^2dx = \int (e^{2x/a}+2e^{x/a}e^{-x/a}+e^{-2x/a})dx = \int e^{2x/a}dx + 2\int dx + \int e^{-2x/a}dx$$

$$= \frac{a}{2}\int e^{u}du + 2\int dx - \frac{a}{2}\int e^{w}dw = \frac{a}{2}e^{u}+2x-\frac{a}{2}e^{w}+c = \frac{a}{2}e^{\frac{2x}{a}}+2x-\frac{a}{2}e^{-\frac{2x}{a}}+c$$

**2.75.-** $\int \frac{a^{2x}-1}{\sqrt{a^{x}}}dx$ **,** Sea: $u=-\frac{x}{2}, du=-\frac{dx}{2}; w=\frac{3x}{2}, dw=\frac{3dx}{2}$

$$\int \frac{a^{2x}-1}{\sqrt{a^{x}}}dx = \int \frac{a^{2x}dx}{\sqrt{a^{x}}} - \int \frac{dx}{\sqrt{a^{x}}} = \int a^{2x-x/2}dx - \int a^{-x/2}dx = \int a^{3x/2}dx - \int a^{-x/2}dx$$

$$= \frac{2}{3}\int a^{w}dw + 2\int a^{u}du = \frac{2}{3}\frac{a^{w}}{\ell\eta a}+2\frac{a^{u}}{\ell\eta a}+c = \frac{2}{3}\frac{a^{3x/2}}{\ell\eta a}+2\frac{a^{-x/2}}{\ell\eta a}+c = \frac{2}{\ell\eta a}\left(\frac{a^{3x/2}}{3}+a^{-x/2}\right)+c$$

**2.76.-** $\int \frac{e^{1/x}}{x^2}dx$ **,** Sea: $u=\frac{1}{x}, du=-\frac{dx}{x^2}$

$$\int \frac{e^{1/x}}{x^2}dx = -\int e^{u}du = -e^{u}+c = -e^{1/x}+c = -\sqrt[x]{e}+c$$

**2.77.-** $\int 5^{\sqrt{x}}\frac{dx}{\sqrt{x}}$ **,** Sea: $u=\sqrt{x}, du=\frac{dx}{2\sqrt{x}}$

$$\int 5^{\sqrt{x}}\frac{dx}{\sqrt{x}} = 2\int 5^{u}du = \frac{2\times 5^{u}}{\ell\eta 5}+c = \frac{2\times 5^{\sqrt{x}}}{\ell\eta 5}+c$$

**2.78.-** $\int x7^{x^2}dx$ **,** Sea: $u=x^2, du=2xdx$

$$\int x7^{x^2}dx = \frac{1}{2}\int 7^{u}du = \frac{1}{2}\frac{7^{u}}{\ell\eta 7}+c = \frac{1}{2}\frac{7^{x^2}}{\ell\eta 7}+c$$

**2.79.-** $\int \frac{e^{t}dt}{e^{t}-1}$ **,** Sea: $u=e^{t}-1, du=e^{t}dt$

$$\int \frac{e^t dt}{e^t-1} = \int \frac{du}{u} = \ell\eta|u| + c = \ell\eta\left|e^t-1\right| + c$$

**2.80.-** $\int e^x\sqrt{a-be^x}\,dx$ **,** Sea: $u = a-be^x, du = -be^x dx$

$$\int e^x\sqrt{a-be^x}\,dx = -\frac{1}{b}\int\sqrt{u}\,du = -\frac{1}{b}\frac{u^{3/2}}{3/2}+c = -\frac{2}{3b}u^{3/2}+c = -\frac{2}{3b}(a-be^x)^{3/2}+c$$

**2.81.-** $\int (e^{x/a}+1)^{1/3}e^{x/a}dx$ **,** Sea: $u = e^{x/a+1}, du = \frac{e^{x/a}}{a}dx$

$$\int (e^{x/a}+1)^{1/3}e^{x/a}dx = \int \sqrt[3]{e^{x/a}+1}e^{x/a}dx = a\int u^{1/3}du = \frac{au^{4/3}}{4/3}+c = \frac{3a(e^{x/a}+1)^{4/3}}{4}+c$$

**2.82.-** $\int \frac{dx}{2^x+3}$ **,** Sea: $u = 2^x+3, du = 2^x\ell\eta 2dx$

$$\int \frac{dx}{2^x+3} = \frac{1}{3}\int\frac{3dx}{2^x+3} = \frac{1}{3}\int\frac{2^x+3-2^x}{2^x+3}dx = \frac{1}{3}\int\frac{2^x+3}{2^x+3}dx - \frac{1}{3}\int\frac{2^x}{2^x+3}dx = \frac{1}{3}\int dx - \frac{1}{3}\int\frac{du}{u}$$

$$= \frac{1}{3}x - \frac{1}{3}\ell\eta|u| + c = \frac{1}{3}x - \frac{1}{3\ell\eta 2}\ell\eta|u| + c = \frac{1}{3}x - \frac{\ell\eta\left|2^x+3\right|}{3\ell\eta 2}+c$$

**2.83.-** $\int \frac{a^x dx}{1+a^{2x}}$ **,** Sea: $u = a^x, du = a^x\ell\eta a dx; a>0$

$$\int\frac{a^x dx}{1+a^{2x}} = \int\frac{a^x dx}{1+(a^x)^2} = \frac{1}{\ell\eta a}\int\frac{du}{1+u^2} = \frac{1}{\ell\eta a}\operatorname{arc}\tau g u + c = \frac{1}{\ell\eta a}\operatorname{arc}\tau g a^x + c$$

**2.84.-** $\int \frac{e^{-bx}}{1-e^{-2bx}}dx$ **,** Sea: $u = e^{-bx}, du = -be^{-bx}dx$

$$\int\frac{e^{-bx}}{1-e^{-2bx}}dx = \int\frac{e^{-bx}}{1-(e^{-bx})^2}dx = -\frac{1}{b}\int\frac{du}{1-u^2} = -\frac{1}{b}\int\frac{du}{(-1)(u^2-1)} = \frac{1}{2b}\ell\eta\left|\frac{u-1}{u+1}\right|+c$$

$$= \frac{1}{2b}\ell\eta\left|\frac{e^{-bx}-1}{e^{-bx}+1}\right|+c\,.$$

**2.85.-** $\int \frac{e^t dt}{\sqrt{1-e^{2t}}}$ **,** Sea: $u = e^t, du = e^t dt$

$$\int\frac{e^t dt}{\sqrt{1-e^{2t}}} = \int\frac{e^t dt}{\sqrt{1-(e^t)^2}} = \int\frac{du}{\sqrt{1-u^2}} = \operatorname{arcsen} u + c = \operatorname{arcsen} e^t + c$$

**2.86.-** $\int \cos\frac{x}{\sqrt{2}}dx$ **,** Sea: $u = \frac{x}{\sqrt{2}}, du = \frac{dx}{\sqrt{2}}$

$$\int\cos\frac{x}{\sqrt{2}}dx = \sqrt{2}\int\cos u\,du = \sqrt{2}\operatorname{sen} u + c = \sqrt{2}\operatorname{sen}\frac{x}{\sqrt{2}}+c$$

**2.87.-** $\int \operatorname{sen}(a+bx)dx$ **,** Sea: $u = a+bx, du = bdx$

$$\int\operatorname{sen}(a+bx)dx = \frac{1}{b}\int\operatorname{sen} u\,du = -\frac{1}{b}\cos u + c = -\frac{1}{b}\cos(a+bx)+c$$

**2.88.-** $\int \cos\sqrt{x}\,\frac{dx}{\sqrt{x}}$, Sea: $u=\sqrt{x}, du=\frac{dx}{2\sqrt{x}}$

$$\int \cos\sqrt{x}\,\frac{dx}{\sqrt{x}} = 2\int \cos u du = 2\operatorname{sen} u + c = 2\operatorname{sen}\sqrt{x} + c$$

**2.89.-** $\int \operatorname{sen}(\ell\eta x)\frac{dx}{x}$, Sea: $u=\ell\eta x, du=\frac{dx}{x}$

$$\int \operatorname{sen}(\ell\eta x)\frac{dx}{x} = \int \operatorname{sen} u du = -\cos u + c = -\cos\ell\eta x + c$$

**2.90.-** $\int (\cos ax + \operatorname{sen} ax)^2 dx$, Sea: $u=2ax, du=2adx$

$$\int (\cos ax + \operatorname{sen} ax)^2 dx = \int (\cos^2 ax + 2\cos ax \operatorname{sen} ax + \operatorname{sen}^2 ax)dx$$

$$= \int (1+2\cos ax \operatorname{sen} ax)dx = \int dx + 2\int \cos ax \operatorname{sen} axdx = \int dx + \int \operatorname{sen} 2axdx$$

$$= x - \frac{1}{2a}\cos 2ax + c$$

**2.91.-** $\int \operatorname{sen}^2 xdx$, Sea: $u=2x, du=2dx$

$$\int \operatorname{sen}^2 xdx = \int \frac{1-\cos 2x}{2}dx = \frac{1}{2}\int dx - \frac{1}{2}\int \cos 2xdx = \frac{1}{2}\int dx - \frac{1}{4}\int \cos udu = \frac{1}{2}x - \frac{1}{4}\operatorname{sen} u + c$$

$$= \frac{1}{2}x - \frac{1}{4}\operatorname{sen} 2x + c$$

**2.92.-** $\int \cos^2 xdx$, Sea: $u=2x, du=2dx$

$$\int \cos^2 xdx = \int \frac{1+\cos 2x}{2}dx = \frac{1}{2}\int dx + \frac{1}{2}\int \cos 2xdx = \frac{1}{2}\int dx + \frac{1}{4}\int \cos udu = \frac{1}{2}x + \frac{1}{4}\operatorname{sen} u + c$$

$$= \frac{1}{2}x + \frac{1}{4}\operatorname{sen} 2x + c$$

**2.93.-** $\int \sec^2(ax+b)dx$, Sea: $u=ax+b, du=adx$

$$\int \sec^2(ax+b)dx = \frac{1}{a}\int \sec^2 udu = \frac{1}{a}\tau gu + c = \frac{1}{a}\tau g(ax+b) = +c$$

**2.94.-** $\int \operatorname{co}\tau g^2 axdx$, Sea: $u=ax, du=adx$

$$\int \operatorname{co}\tau g^2 axdx = \frac{1}{a}\int \operatorname{co}\tau g^2 udu = \frac{1}{a}\int (\cos ec^2 u - 1)du = \frac{1}{a}\int \cos ec^2 udu - \frac{1}{a}\int du$$

$$= -\frac{\operatorname{co}\tau gu}{a} - \frac{u}{a} + c = -\frac{\operatorname{co}\tau gax}{a} - \frac{\cancel{a}x}{\cancel{a}} + c = -\frac{\operatorname{co}\tau gax}{a} - x + c$$

**2.95.-** $\int \frac{dx}{\operatorname{sen}\frac{x}{a}}$, Sea: $u = x/a, du = dx/a$

$$\int \frac{dx}{\operatorname{sen}\frac{x}{a}} = \int \cos ec\tfrac{x}{a}dx = a\int \cos ecudu = a\ell\eta\left|\cos ecu - \operatorname{co}\tau gu\right| + c$$

$$= a\ell\eta\left|\cos ec\, x/a - \operatorname{co}\tau g\, x/a\right| + c$$

**2.96.-** $\int \frac{dx}{3\cos(5x-\frac{\pi}{4})}$, Sea: $u=5x-\pi/4, du=5dx$

$$\int \frac{dx}{3\cos(5x-\frac{\pi}{4})}=\frac{1}{3}\int \sec(5x-\frac{\pi}{4})dx=\frac{1}{15}\int \sec u du=\frac{1}{15}\ell\eta\left|\sec u+\tau gu\right|+c$$

$$=\frac{1}{15}\ell\eta\left|\sec(5x-\frac{\pi}{4})+\tau g(5x-\frac{\pi}{4})\right|+c$$

**2.97.-** $\int \frac{dx}{\operatorname{sen}(ax+b)}$, Sea: $u=ax+b, du=adx$

$$\int \frac{dx}{\operatorname{sen}(ax+b)}=\int \cos ec(ax+b)dx=\frac{1}{a}\int \cos ecudu=\frac{1}{a}\ell\eta\left|\cos ecu-\operatorname{co}\tau gu\right|+c$$

$$=\frac{1}{a}\ell\eta\left|\cos ec(ax+b)-\operatorname{co}\tau g(ax+b)\right|+c$$

**2.98.-** $\int \frac{xdx}{\cos^2 x^2}$, Sea: $u=x^2, du=2xdx$

$$\int \frac{xdx}{\cos^2 x^2}=\int x\sec^2 x^2dx=\frac{1}{2}\int \sec^2 udu=\frac{1}{2}\tau gu+c=\frac{1}{2}\tau gx^2+c$$

**2.99.-** $\int \operatorname{co}\tau g\frac{x}{a-b}dx$, Sea: $u=\frac{x}{a-b}, du=\frac{dx}{a-b}$

$$\int \operatorname{co}\tau g\frac{x}{a-b}dx=(a-b)\int \operatorname{co}\tau gudu=(a-b)\ell\eta\left|\operatorname{sen}u\right|+c=(a-b)\ell\eta\left|\operatorname{sen}\frac{x}{a-b}\right|+c$$

**2.100.-** $\int \tau g\sqrt{x}\frac{dx}{\sqrt{x}}$, Sea: $u=\sqrt{x}, du=\frac{dx}{2\sqrt{x}}$

$$\int \tau g\sqrt{x}\frac{dx}{\sqrt{x}}=2\int \tau gudu=2\ell\eta\left|\sec u\right|+c=2\ell\eta\left|\sec\sqrt{x}\right|+c$$

**2.101.-** $\int \frac{dx}{\tau g\frac{x}{5}}$, Sea: $u=x/5, du=dx/5$

$$\int \frac{dx}{\tau g\frac{x}{5}}=\int \operatorname{co}\tau g\tfrac{x}{5}dx=5\int \operatorname{co}\tau gudu=5\ell\eta\left|\operatorname{sen}u\right|+c=5\ell\eta\left|\operatorname{sen}x/5\right|+c$$

**2.102.-** $\int \left(\frac{1}{\operatorname{sen}x\sqrt{2}}-1\right)^2dx$, Sea: $u=x\sqrt{2}, du=\sqrt{2}dx$

$$\int \left(\frac{1}{\operatorname{sen}x\sqrt{2}}-1\right)^2dx=\int (\cos ecx\sqrt{2}-1)^2dx=\int (\cos ec^2x\sqrt{2}-2\cos ecx\sqrt{2}+1)dx$$

$$=\int \cos ec^2x\sqrt{2}dx-2\int \cos ecx\sqrt{2}dx+\int dx=\frac{1}{\sqrt{2}}\int \cos ec^2udu-\frac{2}{\sqrt{2}}\int \cos ecudu+\int dx$$

$$=-\frac{1}{\sqrt{2}}\operatorname{co}\tau gu-\sqrt{2}\ell\eta\left|\cos ecu-\operatorname{co}\tau gu\right|+x+c$$

$$=-\frac{1}{\sqrt{2}}\operatorname{co}\tau gx\sqrt{2}-\sqrt{2}\ell\eta\left|\cos ecx\sqrt{2}-\operatorname{co}\tau gx\sqrt{2}\right|+x+c$$

**2.103.-** $\int \frac{dx}{\operatorname{sen} x \cos x}$, Sea: $u = 2x, du = 2dx$

$$\int \frac{dx}{\operatorname{sen} x \cos x} = \int \frac{dx}{\frac{1}{2}\operatorname{sen} 2x} = 2\int \cos ec 2x dx = \int \cos ecu du = \ell\eta\left|\cos ecu - \operatorname{co}\tau gu\right| + c$$

$$= \ell\eta\left|\cos ec2x - \operatorname{co}\tau g 2x\right| + c$$

**2.104.-** $\int \frac{\cos ax}{\operatorname{sen}^5 ax} dx$, Sea: $u = \operatorname{sen} ax, du = a\cos ax dx$

$$\int \frac{\cos ax}{\operatorname{sen}^5 ax} dx = \frac{1}{a}\int \frac{du}{u^5} = \frac{1}{a}\frac{u^{-4}}{-4} + c = -\frac{u^{-4}}{4a} + c = -\frac{\operatorname{sen}^{-4} ax}{4a} + c = -\frac{1}{4a\operatorname{sen}^4 ax} + c$$

**2.105.-** $\int t\operatorname{sen}(1-2t^2)dt$, Sea: $u = 1-2t^2, du = -4tdt$

$$\int t\operatorname{sen}(1-2t^2)dt = -\frac{1}{4}\int \operatorname{sen} u du = \frac{1}{4}\cos u + c = \frac{1}{4}\cos(1-2t^2) + c$$

**2.106.-** $\int \frac{\operatorname{sen} 3x}{3+\cos 3x} dx$, Sea: $u = 3+\cos 3x, du = -3\operatorname{sen} 3x dx$

$$\int \frac{\operatorname{sen} 3x}{3+\cos 3x} dx = -\frac{1}{3}\int \frac{du}{u} = -\frac{1}{3}\ell\eta|u| + c = -\frac{1}{3}\ell\eta|3+\cos 3x| + c$$

**2.107.-** $\int \tau g^3 \frac{x}{3}\sec^2 \frac{x}{3} dx$, Sea: $u = \tau g(x/3), du = \frac{1}{3}\sec^2(x/3)dx$

$$\int \tau g^3 \tfrac{x}{3}\sec^2 \tfrac{x}{3} dx = 3\int u^3 du = \frac{3u^4}{4} + c = \frac{3\tau g^4(x/3)}{4} + c$$

**2.108.-** $\int \frac{\operatorname{sen} x\cos x}{\sqrt{\cos^2 x - \operatorname{sen}^2 x}} dx$, Sea: $u = \cos 2x, du = 2\operatorname{sen} 2x dx$

$$\int \frac{\operatorname{sen} x\cos x}{\sqrt{\cos^2 x - \operatorname{sen}^2 x}} dx = \int \frac{\operatorname{sen} x\cos x}{\sqrt{\cos 2x}} dx = \frac{1}{4}\int \frac{\operatorname{sen} 2x}{\sqrt{\cos 2x}} = \frac{1}{4}\int \frac{du}{\sqrt{u}} = \frac{1}{4}\frac{u^{1/2}}{1/2} + c = \frac{u^{1/2}}{2} + c$$

$$= \frac{\sqrt{\cos 2x}}{2} + c$$

**2.109.-** $\int \frac{\sqrt{\tau gx}}{\cos^2 x} dx$, Sea: $u = \tau gx, du = \sec^2 x dx$

$$\int \frac{\sqrt{\tau gx}}{\cos^2 x} dx = \int \sqrt{\tau gx}\sec^2 x dx = \int u^{1/2} du = \frac{u^{3/2}}{3/2} + c = \frac{2}{3}u^{3/2} + c = \frac{2}{3}\tau g^{3/2} x + c$$

**2.110.-** $\int \cos\frac{x}{a}\operatorname{sen}\frac{x}{a} dx$, Sea: $u = 2x/a, du = 2dx$

$$\int \cos\tfrac{x}{a}\operatorname{sen}\tfrac{x}{a} dx = \frac{1}{2}\int \operatorname{sen}\tfrac{2x}{a} dx = \frac{a}{4}\int \operatorname{sen} u du = -\frac{a}{4}\cos u + c = -\frac{a}{4}\cos\tfrac{2x}{a} + c$$

**2.111.-** $\int t\operatorname{co}\tau g(2t^2-3)dt$, Sea: $u = 2t^3-3, du = 4tdt$

$$\int t\operatorname{co}\tau g(2t^2-3)dt = \frac{1}{4}\int \operatorname{co}\tau gu du = \frac{1}{4}\ell\eta|\operatorname{sen} u| + c = \frac{1}{4}\ell\eta|\operatorname{sen}(2t^2-3)| + c$$

**2.112.-** $\int \frac{x^3dx}{x^8+5}$ **,** Sea: $u = x^4, du = 4x^3dx$

$$\int \frac{x^3dx}{x^8+5} = \int \frac{x^3dx}{(x^4)^2+(\sqrt{5})^2} = \frac{1}{4}\int \frac{du}{u^2+(\sqrt{5})^2} = \frac{1}{4}\frac{1}{\sqrt{5}} \operatorname{arc}\tau g \frac{u}{\sqrt{5}} + c = \frac{\sqrt{5}}{20} \operatorname{arc}\tau g \frac{x^4}{\sqrt{5}} + c$$

**2.113.-** $\int \operatorname{sen}^3 6x \cos 6x dx$ **,** Sea: $u = \operatorname{sen} 6x, du = 6\cos 6x dx$

$$\int \operatorname{sen}^3 6x \cos 6x dx = \frac{1}{6}\int u^3 du = \frac{1}{6}\frac{u^4}{4} + c = \frac{u^4}{24} + c = \frac{\operatorname{sen}^4 6x}{24} + c$$

**2.114.-** $\int \sqrt{1+3\cos^2 x}\, \operatorname{sen} 2x dx$ **,** Sea: $u = \frac{5+3\cos 2x}{2}, du = -3\operatorname{sen} 2x dx$

$$\int \sqrt{1+3\cos^2 x}\, \operatorname{sen} 2x dx = \int \sqrt{1+3(\frac{1+\cos 2x}{2})}\, \operatorname{sen} 2x dx = \int \sqrt{1+\frac{3+3\cos 2x}{2}}\, \operatorname{sen} 2x dx$$

$$= \int \sqrt{\frac{5+3\cos 2x}{2}}\, \operatorname{sen} 2x dx = -\frac{1}{3}\int u^{1/2} du = -\frac{1}{3}\frac{u^{3/2}}{3/2} + c = -\frac{2}{9}u^{3/2} + c$$

$$= -\frac{2}{9}\left(\frac{5+3\cos 2x}{2}\right)^{3/2} + c$$

**2.115.-** $\int x\sqrt[5]{5-x^2}\, dx$ **,** Sea: $u = 5-x^2, du = -2x dx$

$$\int x\sqrt[5]{5-x^2}\, dx = -\frac{1}{2}\int u^{1/5} du = -\frac{1}{2}\frac{u^{6/5}}{6/5} + c = -\frac{5}{12}u^{6/5} + c = -\frac{5(5-x^2)^{6/5}}{12} + c$$

**2.116.-** $\int \frac{1+\operatorname{sen} 3x}{\cos^2 3x} dx$ **,** Sea: $u = \operatorname{sen} 3x, du = 3dx; w = \cos u, dw = -\operatorname{sen} u du$

$$\int \frac{1+\operatorname{sen} 3x}{\cos^2 3x} dx = \int \frac{dx}{\cos^2 3x} + \int \frac{\operatorname{sen} 3x}{\cos^2 3x} dx = \frac{1}{3}\int \sec^2 u du + \frac{1}{3}\int \frac{\operatorname{sen} u}{\cos^2 u} du$$

$$= \frac{1}{3}\int \sec^2 u du - \frac{1}{3}\int \frac{dw}{w^2} = \frac{1}{3}\tau g u + \frac{1}{3w} + c = \frac{1}{3}\tau g u + \frac{1}{3\cos u} + c = \frac{1}{3}\tau g 3x + \frac{1}{3\cos 3x} + c$$

**2.117.-** $\int \frac{(\cos ax + \operatorname{sen} ax)^2}{\operatorname{sen} ax} dx$ **,** Sea: $u = ax, du = adx$

$$\int \frac{(\cos ax + \operatorname{sen} ax)^2}{\operatorname{sen} ax} dx = \int \frac{\cos^2 ax + 2\cos ax \operatorname{sen} ax + \operatorname{sen}^2 ax}{\operatorname{sen} ax} dx$$

$$= \int \frac{\cos^2 ax}{\operatorname{sen} ax} dx + 2\int \frac{\cos ax \cancel{\operatorname{sen} ax}}{\cancel{\operatorname{sen} ax}} dx + \int \frac{\cancel{\operatorname{sen}^2 ax}}{\cancel{\operatorname{sen} ax}} dx$$

$$= \int \frac{1-\operatorname{sen}^2 ax}{\operatorname{sen} ax} dx + 2\int \cos ax dx + \int \operatorname{sen} ax dx$$

$$= \int \frac{dx}{\operatorname{sen} ax} + 2\int \cos ax dx$$

$$= \int \operatorname{cosec} ax dx + 2\int \cos ax dx = \frac{1}{a}\int \operatorname{cosec} u du + \frac{2}{a}\int \cos u du$$

$$=\frac{1}{a}\ell\eta\left|\operatorname{cosec}u-\operatorname{co}\tau gu\right|+\frac{2}{a}\operatorname{sen}u+c=\frac{1}{a}\ell\eta\left|\operatorname{cosec}ax-\operatorname{co}\tau gax\right|+\frac{2}{a}\operatorname{sen}ax+c$$

**2.118.-** $\int\frac{x^3-1}{x+1}dx$ **,** Sea: $u=x+1, du=dx$

$$\int\frac{x^3-1}{x+1}dx=\int(x^2-x+1-\frac{2}{x+1})dx=\int x^2dx-\int xdx+\int dx-\int\frac{2}{x+1}dx$$

$$=\int x^2dx-\int xdx+\int dx-2\int\frac{du}{u}=\frac{x^3}{3}-\frac{x^2}{2}+x-2\ell\eta\left|x+1\right|+c$$

**2.119.-** $\int\frac{\operatorname{cosec}^2 3xdx}{b-a\operatorname{co}\tau g3x}$ **,** Sea: $u=b-a\operatorname{co}\tau g3x, du=3a\operatorname{cosec}^2 3xdx$

$$\int\frac{\operatorname{cosec}^2 3xdx}{b-a\operatorname{co}\tau g3x}=\frac{1}{3a}\int\frac{du}{u}=\frac{1}{3a}\ell\eta\left|u\right|+c=\frac{1}{3a}\ell\eta\left|b-a\operatorname{co}\tau g3x\right|+c$$

**2.120.-** $\int\frac{x^3-1}{x^4-4x+1}dx$ **,** Sea: $u=x^4-4x+1, du=(4x^3-4)dx$

$$\int\frac{x^3-1}{x^4-4x+1}dx=\frac{1}{4}\int\frac{(4x^3-4)dx}{x^4-4x+1}=\frac{1}{4}\int\frac{du}{u}=\frac{1}{4}\ell\eta\left|u\right|+c=\frac{1}{4}\ell\eta\left|x^4-4x+1\right|+c$$

**2.121.-** $\int xe^{-x^2}dx$ **,** Sea: $u=-x^2, du=-2xdx$

$$\int xe^{-x^2}dx=-\frac{1}{2}\int e^u du=-\frac{1}{2}e^u+c=-\frac{1}{2}e^{-x^2}+c$$

**2.122.-** $\int\frac{3-\sqrt{2+3x^2}}{2+3x^2}dx$ **,** Sea: $u=x\sqrt{3}, du=\sqrt{3}dx; a=\sqrt{2}$

$$\int\frac{3-\sqrt{2+3x^2}}{2+3x^2}dx=3\int\frac{dx}{(\sqrt{2})^2+(\sqrt{3}x)^2}-\int\frac{(2+3x^2)^{1/2}}{2+3x^2}dx$$

$$\frac{3}{\sqrt{3}}\int\frac{\sqrt{3}dx}{(\sqrt{2})^2+(\sqrt{3}x)^2}-\int\frac{\cancel{(2+3x^2)}^{1/2}}{\cancel{2+3x^2}}dx=\frac{3}{\sqrt{3}}\int\frac{\sqrt{3}dx}{(\sqrt{2})^2+(\sqrt{3}x)^2}-\int(2+3x^2)^{-1/2}dx$$

$$=\frac{3}{\sqrt{3}}\int\frac{du}{(a)^2+(u)^2}-\int(2+3x^2)^{-1/2}dx=\sqrt{3}\int\frac{du}{(a)^2+(u)^2}-\int\frac{dx}{\sqrt{(\sqrt{2})^2+(x\sqrt{3})^2}}$$

$$=\sqrt{3}\int\frac{du}{(a)^2+(u)^2}-\frac{1}{\sqrt{3}}\int\frac{du}{\sqrt{a^2+u^2}}=\frac{\sqrt{3}}{a}\operatorname{arc}\tau g\frac{u}{a}-\frac{1}{\sqrt{3}}\ell\eta\left|u+\sqrt{a^2+u^2}\right|+c$$

$$=\frac{\sqrt{3}}{\sqrt{2}}\operatorname{arc}\tau g\frac{x\sqrt{3}}{\sqrt{2}}-\frac{\sqrt{3}}{3}\ell\eta\left|x\sqrt{3}+\sqrt{2+3+x^2}\right|+c$$

**2.123.-** $\int\frac{\tau g3x-\operatorname{co}\tau g3x}{\operatorname{sen}3x}dx$ **,** Sea: $u=3x, du=3dx; w=\operatorname{sen}u, dw=\cos udu$

$$\int\frac{\tau g3x-\operatorname{co}\tau g3x}{\operatorname{sen}3x}dx=\int\frac{\frac{\operatorname{sen}3x}{\cos3x}-\frac{\cos3x}{\operatorname{sen}3x}}{\operatorname{sen}3x}dx=\int\frac{dx}{\cos3x}-\int\frac{\cos3x}{\operatorname{sen}^2 3x}dx$$

$$=\int \sec 3x dx-\int \frac{\cos 3x}{\operatorname{sen}^2 3x}dx=\frac{1}{3}\int \sec u du-\frac{1}{3}\int \frac{\cos u}{\operatorname{sen}^2 u}du=\frac{1}{3}\int \sec u du-\frac{1}{3}\int \frac{dw}{w^2}$$

$$=\frac{1}{3}\ell\eta\left|\sec u+\tau g u\right|-\frac{1}{3}\frac{w^{-1}}{-1}+c=\frac{1}{3}\ell\eta\left|\sec 3x+\tau g 3x\right|+\frac{1}{3\operatorname{sen}3x}+c$$

**2.124.-** $\int \frac{dx}{\sqrt{e^x}}$**,** Sea: $u=-\frac{x}{2}, du=-\frac{dx}{2}$

$$\int \frac{dx}{\sqrt{e^x}}=\int \frac{dx}{(e^x)^{1/2}}=\int e^{-x/2}dx=-2\int e^u du=-2e^u+c=-2e^{-x/2}+c=\frac{-2}{e^{x/2}}+c=\frac{-2}{\sqrt{e^x}}+c$$

**2.125.-** $\int \frac{1+\operatorname{sen}x}{x+\cos x}dx$**,** Sea: $u=x+\cos x, du=(1-\operatorname{sen}x)dx$

$$\int \frac{1+\operatorname{sen}x}{x+\cos x}dx=\int \frac{du}{u}=\ell\eta\left|u\right|+c=\ell\eta\left|x+\cos x\right|+c$$

**2.126.-** $\int \frac{\sec^2 x dx}{\sqrt{\tau g^2 x-2}}$**,** Sea: $u=\tau g x, du=\sec^2 x dx$

$$\int \frac{\sec^2 x dx}{\sqrt{\tau g^2 x-2}}=\int \frac{du}{\sqrt{u^2-2}}=\ell\eta\left|u+\sqrt{u^2-2}\right|+c=\ell\eta\left|\tau g x+\sqrt{\tau g x^2-2}\right|+c$$

**2.127.-** $\int \frac{dx}{x\ell\eta^2 x}$**,** Sea: $u=\ell\eta x, du=\frac{dx}{2}$

$$\int \frac{dx}{x\ell\eta^2 x}=\int \frac{dx}{x(\ell\eta x)^2}=\int \frac{du}{u^2}=\frac{u^{-1}}{-1}+c=-\frac{1}{u}+c=-\frac{1}{\ell\eta\left|x\right|}+c$$

**2.128.-** $\int a^{\operatorname{sen}x}\cos x dx$**,** Sea: $u=\operatorname{sen}x, du=\cos x dx$

$$\int a^{\operatorname{sen}x}\cos x dx=\int a^u du=\frac{a^u}{\ell\eta a}+c=\frac{a^{\operatorname{sen}x}}{\ell\eta a}+c$$

**2.129.-** $\int \frac{x^2}{\sqrt{x^3+1}}dx$**,** Sea: $u=x^3+1, du=3x^2dx$

$$\int \frac{x^2dx}{\sqrt{x^3+1}}=\int \frac{x^2dx}{(x^3+1)^{1/3}}=\frac{1}{3}\int \frac{du}{u^{1/3}}=\frac{1}{\cancel{3}}\frac{u^{2/3}}{2/\cancel{3}}+c=\frac{u^{2/3}}{2}+c=\frac{(x^2+1)^{2/3}}{2}+c=\frac{\sqrt[3]{(x^2+1)^2}}{2}+c$$

**2.130.-** $\int \frac{xdx}{\sqrt{1-x^4}}$**,** Sea: $u=x^2, du=2xdx$

$$\int \frac{xdx}{\sqrt{1-x^4}}=\int \frac{xdx}{\sqrt{1-(x^2)^2}}=\frac{1}{2}\int \frac{2xdx}{\sqrt{1-(x^2)^2}}=\frac{1}{2}\int \frac{2xdx}{\sqrt{1-(u)^2}}=\frac{1}{2}\operatorname{arcsen}u+c$$

$$=\frac{1}{2}\operatorname{arcsen}x^2+c$$

**2.131.-** $\int \tau g^2 ax dx$**,** Sea: $u=ax, du=adx$

$$\int \tau g^2 axdx = \int (\sec^2 ax - 1)dx = \int \sec^2 axdx - \int dx = \frac{1}{a}\int \sec^2 udu - \int dx = \frac{1}{a}\tau gu - x + c$$
$$= \frac{1}{a}\tau gax - x + c$$

**2.132.-** $\int \frac{\sec^2 xdx}{\sqrt{4-\tau g^2 x}}$ **,** Sea: $u = \tau gx, du = \sec^2 xdx$

$$\int \frac{\sec^2 xdx}{\sqrt{4-\tau g^2 x}} = \int \frac{du}{\sqrt{2^2 - u^2}} = \operatorname{arcs} e\operatorname{n} \frac{u}{2} + c = \operatorname{arcs} e\operatorname{n} \frac{\tau gx}{2} + c$$

**2.133.-** $\int \frac{dx}{\cos \frac{x}{a}}$ **,** Sea: $u = \frac{x}{a}, du = \frac{dx}{a}$

$$\int \frac{dx}{\cos \frac{x}{a}} = \int \sec \tfrac{x}{a} dx = a\int \sec udu = a\ell\eta \left|\sec u + \tau gu\right| + c = a\ell\eta\left|\sec \tfrac{x}{a} + \tau g \tfrac{x}{a}\right| + c$$

**2.134.-** $\int \frac{\sqrt[3]{1+\ell\eta x}}{x}dx$ **,** Sea: $u = 1 + \ell\eta x, du = \frac{dx}{x}$

$$\int \frac{\sqrt[3]{1+\ell\eta x}}{x}dx = \int u^{\frac{1}{3}}du = \frac{u^{\frac{4}{3}}}{\frac{4}{3}} + c = \frac{3u^{\frac{4}{3}}}{4} + c = \frac{3(1+\ell\eta x)^{\frac{4}{3}}}{4} + c$$

**2.135.-** $\int \tau g\sqrt{x-1}\frac{dx}{\sqrt{x-1}}$ **,** Sea: $u = \sqrt{x-1}, du = \frac{dx}{2\sqrt{x-1}}$

$$\int \tau g\sqrt{x-1}\frac{dx}{\sqrt{x-1}} = 2\int \tau gu\frac{du}{u} = 2\ell\eta\left|\sec\sqrt{x-1}\right| + c = -2\ell\eta\left|\cos\sqrt{x-1}\right| + c$$

**2.136.-** $\int \frac{xdx}{\operatorname{sen} x^2}$ **,** Sea: $u = x^2, du = 2xdx$

$$\int \frac{xdx}{\operatorname{sen} x^2} = \frac{1}{2}\int \frac{du}{\operatorname{sen} u} = \frac{1}{2}\int \cos ecudu = \frac{1}{2}\ell\eta\left|\cos ecu - \operatorname{co}\tau gu\right| + c$$
$$= \frac{1}{2}\ell\eta\left|\cos ecx^2 - \operatorname{co}\tau gx^2\right| + c$$

**2.137.-** $\int \frac{\operatorname{sen} x - \cos x}{\operatorname{sen} x + \cos x}dx$ **,** Sea: $u = \operatorname{sen} x + \cos x, du = (\cos x - \operatorname{sen} x)dx$

$$\int \frac{\operatorname{sen} x - \cos x}{\operatorname{sen} x + \cos x}dx = -\int \frac{du}{u} = -\ell\eta\left|\operatorname{sen} x + \cos x\right| + c$$

**2.138.-** $\int \frac{e^{\operatorname{arc}\tau gx} + x\ell\eta(1+x^2) + 1}{1+x^2}$ **,** Sea: $u = \operatorname{arc}\tau gx, du = \frac{dx}{1+x^2}; w = \ell\eta(1+x^2)d, dw = \frac{2xdx}{1+x^2}$

$$\int \frac{e^{\operatorname{arc}\tau gx} + x\ell\eta(1+x^2) + 1}{1+x^2} = \int \frac{e^{\operatorname{arc}\tau gx}dx}{1+x^2} + \int \frac{x\ell\eta(1+x^2)dx}{1+x^2} + \int \frac{dx}{1+x^2}$$
$$= \int e^u du + \frac{1}{2}\int wdw + \int \frac{dx}{1+x^2} = e^u + \frac{1}{2}\frac{w^2}{2} + \operatorname{arc}\tau gx + c = e^u + \frac{\ell\eta^2(1+x^2)}{4} + \operatorname{arc}\tau gx + c$$

**2.139.-** $\int \frac{x^2dx}{x^2-2}$ **,**

$$\int \frac{x^2dx}{x^2-2} = \int (1+\frac{2}{x^2-2})dx = \int dx + 2\int \frac{dx}{x^2-2} = x + 2\frac{1}{2\sqrt{2}}\ell\eta\left|\frac{x-\sqrt{2}}{x+\sqrt{2}}\right| + c$$

$$= x + \frac{\sqrt{2}}{2}\ell\eta\left|\frac{x-\sqrt{2}}{x+\sqrt{2}}\right| + c$$

**2.140.-** $\int e^{\operatorname{sen}^2 x}\operatorname{sen} 2xdx$ **,** Sea: $u = \frac{1-\cos 2x}{2}, du = \operatorname{sen} 2xdx$

$$\int e^{\operatorname{sen}^2 x}\operatorname{sen} 2xdx = \int e^{\frac{1-\cos 2x}{2}}\operatorname{sen} 2xdx = \int e^u du = e^u + c = e^{\operatorname{sen}^2 x} + c$$

**2.141.-** $\int \frac{(1-\operatorname{sen}\frac{x}{\sqrt{2}})^2}{\operatorname{sen}\frac{x}{\sqrt{2}}}dx$ **,** Sea: $u = \frac{x}{\sqrt{2}}, du = \frac{dx}{\sqrt{2}}$

$$\int \frac{(1-\operatorname{sen}\frac{x}{\sqrt{2}})^2}{\operatorname{sen}\frac{x}{\sqrt{2}}}dx = \int\left(\frac{1-2\operatorname{sen}\frac{x}{\sqrt{2}}+\operatorname{sen}^2\frac{x}{\sqrt{2}}}{\operatorname{sen}\frac{x}{\sqrt{2}}}\right)dx = \int \cos ec\frac{x}{\sqrt{2}}dx - 2\int dx + \int \operatorname{sen}\frac{x}{\sqrt{2}}dx$$

$$= \sqrt{2}\int \cos ecudu - 2\int dx + \sqrt{2}\int \operatorname{sen} udu = \sqrt{2}\ell\eta\left|\cos ecu - \operatorname{co}\tau gu\right| - 2x - \sqrt{2}\cos u + c$$

$$= \sqrt{2}\ell\eta\left|\cos ec\frac{x}{\sqrt{2}} - \operatorname{co}\tau g\frac{x}{\sqrt{2}}\right| - 2x - \sqrt{2}\cos\frac{x}{\sqrt{2}} + c$$

**2.142.-** $\int \frac{5-3x}{\sqrt{4-3x^2}}dx$ **,** Sea: $u = x\sqrt{3}, du = \sqrt{3}dx; w = 4-3x^2, dw = -6xdx$

$$\int \frac{5-3x}{\sqrt{4-3x^2}}dx = 5\int \frac{dx}{\sqrt{4-3x^2}} - 3\int \frac{xdx}{\sqrt{4-3x^2}} = 5\int \frac{dx}{\sqrt{4-(x\sqrt{3})^2}} - 3\int \frac{xdx}{\sqrt{4-3x^2}}$$

$$= \frac{5}{\sqrt{3}}\int \frac{du}{\sqrt{2^2-u^2}} + \frac{3}{6}\int \frac{dw}{\sqrt{w}} = \frac{5}{\sqrt{3}}\operatorname{arcsen}\frac{u}{2} + \frac{1}{2}\frac{w^{1/2}}{1/2} + c = \frac{5\sqrt{3}}{3}\operatorname{arcsen}\frac{x\sqrt{3}}{2} + \sqrt{4-3x^2} + c$$

**2.143.-** $\int \frac{ds}{e^s+1}$ **,** Sea: $u = 1+e^{-s}, du = -e^{-s}ds$

$$\int \frac{ds}{e^s+1} = \int \frac{e^{-s}ds}{e^{-s}+1} = -\int \frac{du}{u} = -\ell\eta|u| + c = -\ell\eta\left|e^{-s}+1\right| + c$$

**2.144.-** $\int \frac{d\theta}{\operatorname{sen} a\theta\cos a\theta}$ **,** Sea: $u = 2a\theta, du = 2ad\theta$

$$\int \frac{d\theta}{\operatorname{sen} a\theta\cos a\theta} = \int \frac{d\theta}{\frac{1}{2}\operatorname{sen} 2a\theta} = 2\int \cos ec2a\theta d\theta = \frac{2}{2a}\int \cos ecudu$$

$$= \frac{1}{a}\ell\eta\left|\cos ecu - \operatorname{co}\tau gu\right| + c = \frac{1}{a}\ell\eta\left|\cos ec2a\theta - \operatorname{co}\tau g2a\theta\right| + c$$

**2.145.-** $\int \frac{e^s}{\sqrt{e^{2s}-2}}ds$ **,** Sea: $u = e^s, du = e^sds$

$$\int \frac{e^s}{\sqrt{e^{2s}-2}}ds = \int \frac{e^s}{\sqrt{(e^s)^2-2}}ds = -\int \frac{du}{\sqrt{u^2-2}} = \ell\eta\left|u+\sqrt{u^2-2}\right| + c$$

$$= \ell\eta\left|e^s+\sqrt{(e^s)^2-2}\right| + c = \ell\eta\left|e^s+\sqrt{e^{2s}-2}\right| + c$$

**2.146.-** $\int \operatorname{sen}(\frac{2\pi t}{T}+\varphi_0)dt$ , Sea: $u=\dfrac{2\pi t}{T}+\varphi_0, du=\dfrac{2\pi t}{T}dt$

$$\int \operatorname{sen}(\tfrac{2\pi t}{T}+\varphi_0)dt=\frac{T}{2\pi}\int \operatorname{sen} u du=-\frac{T}{2\pi}\cos u+c=-\frac{T}{2\pi}\cos(\frac{2\pi t}{T}+\varphi_0)+c$$

**2.147.-** $\int \dfrac{\arccos x/2}{\sqrt{4-x^2}}dx$ , Sea: $u=\arccos\dfrac{x}{2}, du=-\dfrac{dx}{\sqrt{4-x^2}}$

$$\int \frac{\arccos x/2}{\sqrt{4-x^2}}dx=-\int u du=-\frac{u^2}{2}+c=-\frac{(\arccos x/2)^2}{2}+c$$

**2.148.-** $\int \dfrac{dx}{x(4-\ell\eta^2 x)}$ , Sea: $u=\ell\eta x, du=\dfrac{dx}{x}$

$$\int \frac{dx}{x(4-\ell\eta^2 x)}=\int \frac{dx}{x\left[2^2-(\ell\eta x)^2\right]}=\int \frac{du}{2^2-u^2}=\frac{1}{4}\ell\eta\left|\frac{2+u}{2-u}\right|+c=\frac{1}{4}\ell\eta\left|\frac{2+\ell\eta x}{2-\ell\eta x}\right|+c$$

**2.149.-** $\int e^{-\tau gx}\sec^2 x dx$ , Sea: $u=-\tau gx, du=-\sec^2 x dx$

$$\int e^{-\tau gx}\sec^2 x dx=-\int e^u du=-e^u+c=-e^{-\tau gx}+c$$

**2.150.-** $\int \dfrac{\operatorname{sen} x\cos x}{\sqrt{2-\operatorname{sen}^4 x}}dx$ , Sea: $u=\operatorname{sen}^2 x, du=2\operatorname{sen} x\cos x dx$

$$\int \frac{\operatorname{sen} x\cos x}{\sqrt{2-\operatorname{sen}^4 x}}dx=\int \frac{\operatorname{sen} x\cos x}{\sqrt{2-(\operatorname{sen}^2 x)^2}}dx=\frac{1}{2}\int \frac{du}{\sqrt{2-u^2}}=\frac{1}{2}\operatorname{arcsen}\frac{u}{\sqrt{2}}+c$$

$$=\frac{1}{2}\operatorname{arcsen}\frac{(\operatorname{sen}^2 x)}{\sqrt{2}}+c$$

**2.151.-** $\int \dfrac{\sec x\tau gx}{\sqrt{\sec^2 x+1}}dx$ , Sea: $u=\sec x, du=\sec x\tau gx dx$

$$\int \frac{\sec x\tau gx}{\sqrt{\sec^2 x+1}}dx=\int \frac{du}{\sqrt{u^2+1}}=\ell\eta\left|u+\sqrt{u^2+1}\right|+c=\ell\eta\left|\sec x+\sqrt{\sec^2 x+1}\right|+c$$

**2.152.-** $\int \dfrac{dt}{\operatorname{sen}^2 t\cos^2 t}$ , Sea: $u=2t, du=2dt$

$$\int \frac{dt}{\operatorname{sen}^2 t\cos^2 t}=\int \frac{dt}{(\operatorname{sen} t\cos t)^2}=\int \frac{dt}{(\frac{1}{2}\operatorname{sen} 2t)^2}=4\int \frac{dt}{\operatorname{sen}^2 2t}=4\int \operatorname{cosec}^2 2t dt$$

$$=2\int \operatorname{cosec}^2 u du=-2\operatorname{co}\tau gu+c=-2\operatorname{co}\tau g2t+c$$

**2.153.-** $\int \dfrac{\operatorname{arcsen} x+x}{\sqrt{1-x^2}}dx$ ,

Sea: $u=\operatorname{arcsen} x, du=\dfrac{dx}{\sqrt{1-x^2}}; w=1-x^2, dw=-2xdx$

$$\int \frac{\operatorname{arcsen} x+x}{\sqrt{1-x^2}}dx=\int \frac{\operatorname{arcsen} x}{\sqrt{1-x^2}}dx+\int \frac{x}{\sqrt{1-x^2}}dx=\int u du-\frac{1}{2}\int \frac{dw}{\sqrt{w}}=\int u du-\frac{1}{2}\int w^{-1/2}dw$$

$$=\frac{u^2}{2}-\frac{1}{2}\frac{w^{1/2}}{1/2}+c=\frac{(\operatorname{arcsen} x)^2}{2}-\sqrt{1-x^2}+c$$

**2.154.-** $\int \frac{xdx}{\sqrt{x+1}}$**,** Sea: $t=\sqrt{x+1} \Rightarrow x=t^2-1; dx=2tdt$

$$\int \frac{xdx}{\sqrt{x+1}}=\int \frac{(t^2-1)2tdt}{t}=2\int (t^2-1)dt=2(\frac{t^3}{3}-t)+c=\frac{2\sqrt{(x+1)^3}}{3}-2\sqrt{x+1}+c$$

**2.155.-** $\int x(5x^2-3)^7 dx$**,** Sea: $u=5x^2-3, du=10xdx$

$$\int x(5x^2-3)^7 dx=\frac{1}{10}\int u^7 du=\frac{1}{10}\frac{u^8}{8}+c=\frac{u^8}{80}+c=\frac{(5x^2-3)^8}{80}+c$$

**2.156.-** $\int \sqrt{\frac{\ell\eta(x+\sqrt{x^2+1})}{x^2+1}}dx$**,** Sea: $u=\ell\eta(x+\sqrt{x^2+1}), du=\frac{dx}{\sqrt{x^2+1}}$

$$\int \sqrt{\frac{\ell\eta(x+\sqrt{x^2+1})}{x^2+1}}dx=\int \frac{\sqrt{\ell\eta(x+\sqrt{x^2+1})}}{\sqrt{x^2+1}}dx=\int \sqrt{u}du=\frac{u^{3/2}}{3/2}+c$$

$$=\frac{2\sqrt{\left[\ell\eta(x+\sqrt{x^2+1})\right]^3}}{3}+c$$

**2.157.-** $\int \frac{\operatorname{sen}^3 x}{\sqrt{\cos x}}dx$**,** Sea: $u=\cos x, du=-\operatorname{sen} xdx$

$$\int \frac{\operatorname{sen}^3 x}{\sqrt{\cos x}}dx=\int \frac{\operatorname{sen}^2 x\operatorname{sen} xdx}{\sqrt{\cos x}}=\int \frac{(1-\cos^2 x)\operatorname{sen} xdx}{\sqrt{\cos x}}=\int \frac{\operatorname{sen} xdx}{\sqrt{\cos x}}-\int \frac{\cos^2 x\operatorname{sen} xdx}{\sqrt{\cos x}}$$

$$=\int \cos^{-1/2} x\operatorname{sen} xdx-\int \cos^{3/2} x\operatorname{sen} xdx=-\int u^{1/2}du+\int u^{3/2}du=-\frac{u^{3/2}}{3/2}+\frac{u^{5/2}}{5/2}+c$$

$$=-\frac{2u^{3/2}}{3}+\frac{2u^{5/2}}{5}+c=-\frac{2\cos x^{3/2}}{3}+\frac{2\cos x^{5/2}}{5}+c=-\frac{2\sqrt{\cos^3 x}}{3}+\frac{2\sqrt{\cos^5 x}}{5}+c$$

**2.158.-** $\int \frac{\cos xdx}{\sqrt{1+\operatorname{sen}^2 x}}$**,**

Sea: $t=\sqrt{1+\operatorname{sen}^2 x} \Rightarrow \operatorname{sen}^2 x=t^2-1; 2\operatorname{sen} x\cos xdx=2tdt$

$$\int \frac{\cos xdx}{\sqrt{1+\operatorname{sen}^2 x}}=\int \frac{\frac{t}{\sqrt{t^2-1}}}{t}=\int \frac{dt}{\sqrt{t^2-1}}=\ell\eta\left|\sqrt{1+\operatorname{sen}^2 x}+\operatorname{sen} x\right|+c$$

**2.159.-** $\int \frac{(\operatorname{arcsen} x)^2}{\sqrt{1-x^2}}dx$**,** Sea: $u=\operatorname{arcsen} x, du=\frac{dx}{\sqrt{1-x^2}}$

$$\int \frac{(\operatorname{arcsen} x)^2}{\sqrt{1-x^2}}dx=\int u^2du=\frac{u^3}{3}+c=\frac{(\operatorname{arcsen} x)^3}{3}+c$$

**2.150.-** $\int e^{x+e^x}dx$**,** Sea: $u=e^{e^x}, du=e^{e^x}e^x dx$

$$\int e^{x+e^x}\,dx = \int e^x e^{e^x}\,dx = \int du = u + c = e^{e^x} + c$$

**2.161.-** $\int t(4t+1)^7\,dt$ **,** Sea: $u = 4t+1 \Rightarrow t = \dfrac{u-1}{4}, du = 4dt$

$$\int t(4t+1)^7\,dt = \int \frac{u-1}{4}u^7\frac{du}{4} = \frac{1}{16}\int (u-1)u^7 du = \frac{1}{16}\int (u^8 - u^7)du = \frac{1}{16}\frac{u^9}{9} - \frac{1}{16}\frac{u^8}{8} + c$$

$$= \frac{(4t+1)^9}{144} - \frac{(4t+1)^8}{128} + c$$

**2.162.-** $\int \dfrac{2t^2 - 10t + 12}{t^2+4}dt$ **,** Sea: $u = t^2 + 4, du = du = 2tdt$

$$\int \frac{2t^2 - 10t + 12}{t^2+4}dt = 2\int \frac{t^2 - 5t + 6}{t^2+4}dt = 2\int \left(1 + \frac{2-5t}{t^2+4}\right)dt = 2\int dt + 4\int \frac{dt}{t^2+4} - 10\int \frac{dt}{t^2+4}$$

$$= 2\int dt + 4\int \frac{dt}{t^2+4} - 5\int \frac{du}{u} = 2t + 2\,\mathrm{arc}\,\tau g\,\tfrac{t}{2} - 5\ell\eta|u| + c = 2t + 2\,\mathrm{arc}\,\tau g\,\tfrac{t}{2} - 5\ell\eta\left|t^2+4\right| + c$$

**2.163.-** $\int \dfrac{e^t - e^{-t}}{e^t + e^{-t}}dt$ **,**

Sea: $u = e^{2t} + 1, du = 2e^{2t}dt; w = 1 + e^{-2t}, dw = -2e^{-2t}dt$

$$\int \frac{e^t - e^{-t}}{e^t + e^{-t}}dt = \int \frac{e^t dt}{e^t + e^{-t}} - \int \frac{e^{-t}dt}{e^t + e^{-t}} = \int \frac{e^{2t}dt}{e^{2t}+1} - \int \frac{e^{-2t}dt}{1+e^{-2t}} = \frac{1}{2}\int \frac{du}{u} + \frac{1}{2}\int \frac{dw}{w}$$

$$= \frac{1}{2}(\ell\eta|u| + \ell\eta|w|) + c = \frac{1}{2}\ell\eta|uw| + c = \frac{1}{2}\ell\eta(e^{2t}+1)(1+e^{-2t}) + c$$