

1. Resuelve las siguientes ecuaciones:

1. $2(2x + 1)^2 - 3(2x - 1)^2 + 5(2x + 1)(2x - 1) = 0$ (1,5 puntos)

2. $4x^4 - 25x^2 = 0$ (1 punto)

3. $\frac{1}{x+2} - \frac{x+2}{x} = \frac{-7}{4}$ (1 punto)

4. $3^x + 3^{x+1} - 3^{x-1} = 297$ (1 punto)

5. $\log_2(6x^2 - 2x) = 2$ (1 punto)

6. $x^4 - x^3 - x^2 + x = 0$ (1 punto)

7. $\sqrt{x+7} - x = 1$ (1 punto)

2. Escribe dos ecuaciones cuyas soluciones sean 1, -1 y -3. (1 punto)

3. Halla tres números enteros consecutivos tales que su producto sea 5 veces su suma. (1,5 puntos)

1) Resuelve:

$$1. \quad 2(2x+1)^2 - 3(2x-1)^2 + 5(2x+1)(2x-1) = 0$$

$$2(4x^2 + 1 + 4x) - 3(4x^2 + 1 - 4x) + 5(4x^2 - 1) = 0$$

$$8x^2 + 2 + 8x - 12x^2 - 3 + 12x + 20x^2 - 5 = 0$$

$$16x^2 + 20x - 6 = 0$$

$$x = \frac{-20 \pm \sqrt{20^2 - 4 \cdot 16 \cdot (-6)}}{2 \cdot 16} = \frac{-20 \pm \sqrt{784}}{32} = \frac{-20 \pm 28}{32}$$

$$\left\{ \begin{array}{l} \frac{-20-28}{32} = \frac{-48}{32} = \boxed{-\frac{3}{2}} \\ \frac{-20+28}{32} = \frac{8}{32} = \boxed{\frac{1}{4}} \end{array} \right.$$

$$2. \quad 4x^4 - 25x^2 = 0$$

$$\text{Cambio } x^2 = t; \quad 4t^2 - 25t = 0; \quad t(4t - 25) = 0$$

$$\boxed{t = 0}$$

$$4t - 25 = 0$$

$$\boxed{t = \frac{25}{4}}$$

Deshacemos el cambio:

$$\text{Si } x^2 = 0, \quad \boxed{x = 0}$$

$$\text{Si } x^2 = \frac{25}{4}, \quad \boxed{x = \pm \sqrt{\frac{25}{4}} = \pm \frac{5}{2}}$$

$$3. \quad \frac{1}{x+2} - \frac{x+2}{x} = -\frac{7}{4}$$

$$\frac{4x}{4x(x+2)} = \frac{-7x(x+2)}{4x(x+2)}$$

$$4x - 4(x^2 + 4x + 4) = -7x^2 - 14x$$

$$4x - 4x^2 - 16x - 16 + 7x^2 + 14x = 0$$

$$3x^2 + 2x - 16 = 0; \quad x = \frac{-2 \pm \sqrt{4 - 4 \cdot 3 \cdot (-16)}}{2 \cdot 3} = \frac{-2 \pm \sqrt{196}}{6} =$$

$$= \frac{-2 \pm 14}{6} \quad \left/ \quad \begin{array}{l} \frac{12}{6} = 2 \\ -\frac{16}{6} = -\frac{8}{3} \end{array} \right.$$

$$\boxed{4.} \quad 3^x + 3^{x+1} - 3^{x-1} = 297$$

$$3^x + 3 \cdot 3^x - \frac{3^x}{3} = 297 \quad \text{Cambio } 3^x = t$$

$$t + 3t - \frac{t}{3} = 297; \quad \frac{3t + 9t - t}{3} = \frac{891}{3}; \quad 11t = 891$$

$$t = \frac{891}{11} = 81 \quad \text{Deshacemos el cambio}$$

$$3^x = 81 = 3^4; \quad \boxed{x = 4}$$

$$\boxed{5.} \quad \log_2(6x^2 - 2x) = 2 \cdot \log_2 2$$

$$\log_2(6x^2 - 2x) = \log_2 2^2; \quad 6x^2 - 2x = 4$$

$$6x^2 - 2x - 4 = 0 \quad x = \frac{2 \pm \sqrt{4 - 4 \cdot 6(-4)}}{2 \cdot 6} = \frac{2 \pm \sqrt{100}}{12}$$

$$= \frac{2 \pm 10}{12} \quad \left/ \quad \begin{array}{l} \frac{12}{12} = 1 \\ -\frac{8}{12} = -\frac{2}{3} \end{array} \right.$$

Comprobación:

$$\text{Si } \boxed{x=1} \quad \log_2(6 \cdot 1^2 - 2 \cdot 1) = \log_2 4 = 2 \quad \text{Si}$$

$$\text{Si } \boxed{x=-\frac{2}{3}} \quad \log_2\left(6 \cdot \left(\frac{2}{3}\right)^2 - 2 \cdot \frac{2}{3}\right) = \log_2\left(\frac{24}{9} - \frac{4}{3}\right) =$$

$$= \log_2\left(\frac{24}{9} - \frac{12}{9}\right) = \log_2\left(\frac{12}{9}\right) = \log_2\left(\frac{4}{3}\right) =$$

$$= \log_2 4 - \log_2 3 \neq 2. \quad \boxed{x = -\frac{2}{3} \text{ No es solución}}$$

$$⑥ \quad x^4 - x^3 - x^2 + x = 0$$

$$x(x^3 - x^2 - x + 1) = 0 \quad \boxed{x=0}$$

$$x^3 - x^2 - x + 1 = 0$$

$$\begin{array}{r|rrrr} 1 & 1 & -1 & -1 & +1 \\ & & 1 & 0 & -1 \\ \hline & 1 & 0 & -1 & 0 \end{array}$$

$$(x-1)(x^2-1)=0 \quad x-1=0; \quad \boxed{x=1}$$

$$(x+1)(x-1)=0 \quad \boxed{x=1}$$

$$\boxed{x=-1}$$

$$⑦ \quad \sqrt{x+7} - x = 1; \quad (\sqrt{x+7})^2 = (1+x)^2; \quad x+7 = 1+x^2+2x;$$

$$x^2 + x - 6 = 0$$

$$x = \frac{-1 \pm \sqrt{1-4(-6)}}{2} = \frac{-1 \pm \sqrt{25}}{2} = \frac{-1 \pm 5}{2} \quad \sqrt{\frac{4}{2}} = 2$$

$$\sqrt{\frac{-6}{2}} = -3$$

Comprobación:

$$\boxed{x=2} \quad \sqrt{2+7} - 2 = \sqrt{9} - 2 = 3 - 2 = 1 \quad \underline{\underline{SI}}$$

$$x = -3; \quad \sqrt{-3+7} - 2 = \sqrt{4} - 2 = 2 - 2 = 0 \neq 1 \quad \underline{\underline{NO}}$$

② Escribe dos ecuaciones cuyas soluciones sean 1, -1 y -3

$$(x-1)(x+1)(x-3) = 0$$

$$(x^2-1)(x-3) = 0 \quad \boxed{x^3 - 3x^2 - x + 3 = 0}$$

$$\boxed{2x^3 - 6x^2 - 2x + 6 = 0}$$

O cualquier otra
que se obtiene
multiplicando
cada término por
un n° real.

$$⑤ \quad x \leftarrow 2^\circ \text{ Número}$$

$$x+1 \leftarrow 3^\circ \text{ Número}$$

$$x+1 \leftarrow 1^\circ \text{ Número}$$

$$x(x-1)(x+1) = 5(x+x-1+x+1)$$

$$x(x^2-1) = 5 \cdot 3x; \quad x^3 - x = 15x; \quad x^3 - 16x = 0;$$

$$x(x^2-16) = 0 \quad \boxed{x=0}$$

$$x^2-16=0; \quad x^2=16; \quad \boxed{x=\pm 4}$$

$$\text{Si } \boxed{x=0} \Rightarrow \begin{array}{l} 0-1 \rightarrow -1 \\ 0 \\ 0+1 \rightarrow 1 \end{array} \quad \left\{ \begin{array}{l} 0 \cdot (-1) \cdot 1 = 5 \cdot 0 \\ \underline{\underline{\text{Si}}} \end{array} \right.$$

$$\text{Si } \boxed{x=-4} \Rightarrow \begin{array}{l} -4-1 \rightarrow -5 \\ -4 \\ -4+1 \rightarrow -3 \end{array} \quad \begin{array}{l} -5 \cdot (-4) \cdot -3 = 5 \cdot (-5-4-3) \\ -60 = -60 \\ \underline{\underline{\text{Si}}} \end{array}$$

$$\text{Si } \boxed{x=4} \Rightarrow \begin{array}{l} 4-1 \rightarrow 3 \\ 4 \\ 4+1 \rightarrow 5 \end{array} \quad \begin{array}{l} 3 \cdot 4 \cdot 5 = 5 \cdot (3+4+5) \\ 60 = 60 \\ \underline{\underline{\text{Si}}} \end{array}$$

Así, son posibles soluciones:

$$\boxed{\begin{array}{l} -1, 0, 1 \\ -5, -4, -3 \\ 3, 4, 5 \end{array}}$$