INTEGRACION POR PARTES

Existe una variedad de integrales que se pueden desarrollar, usando la relación: $\int udv = uv - \int vdu$.

El problema es elegir u y dv, por lo cual es útil la siguiente identificación:

I: Función trigonométrica inversa.
L: Función logarítmica.
A: Función algebraica.
T: Función trigonométrica.
E: Función exponencial.
Se usa de la manera siguiente:

EJERCICIOS DESARROLLADOS

4.1.-Encontrar: $\int x\cos xdx$

Solución.- I L A T E

$$\begin{array}{cc} \downarrow & \downarrow \\ x & \cos x \end{array}$$

$$\therefore \begin{array}{ll} u = x & dv = \cos xdx \\ du = dx & v = \operatorname{sen} x \end{array}$$

$$\therefore \int x\cos xdx = x\operatorname{sen} x - \int \operatorname{sen} xdx = x\operatorname{sen} x + \cos x + c$$

Respuesta: $\int x\cos xdx = x\operatorname{sen} x + \cos x + c$

4.2.-Encontrar: $\int x\sec^2 xdx$

Solución.- I L A T E

$$\begin{array}{cc} \downarrow & \downarrow \\ x & \sec^2 3x \end{array}$$

$$\therefore \begin{array}{ll} u = x & dv = \sec^2 3xdx \\ du = dx & v = \frac{1}{3}\tau g 3x \end{array}$$

$$\therefore \int x\sec^2 xdx = \frac{1}{3}x\tau g3x - \frac{1}{3}\int \tau g3xdx = \frac{x\tau g3x}{3} - \frac{1}{9}\ell\eta\left|\sec 3x\right| + c$$

Respuesta: $\int x\sec^2 xdx = \frac{x\tau g3x}{3} - \frac{1}{9}\ell\eta\left|\sec 3x\right| + c$

4.3.-Encontrar: $\int x^2\operatorname{sen} xdx$

Solución.- I L A T E

$$\begin{array}{cc} \downarrow & \downarrow \\ x^2 & \operatorname{sen} x \end{array}$$

$\therefore \quad \begin{matrix} u = x^2 & dv = \operatorname{sen} x dx \\ du = 2x dx & v = -\cos x \end{matrix}$

$\therefore \int x^2 \operatorname{sen} x dx = -x^2 \cos x + 2\int x \cos x dx$ **,** integrando por partes la segunda integral:

$\int x \cos x dx$**;** $\quad \begin{matrix} u = x & dv = \cos x dx \\ du = dx & v = \operatorname{sen} x \end{matrix}$

$\therefore \int x^2 \operatorname{sen} x dx = -x^2 \cos x + 2\left[x \operatorname{sen} x - \int \operatorname{sen} x dx \right] = -x^2 \cos x + 2x \operatorname{sen} x + 2\cos x + c$

Respuesta: $\int x^2 \operatorname{sen} x dx = -x^2 \cos x + 2x \operatorname{sen} x + 2\cos x + c$

4.4.-Encontrar: $\int (x^2 + 5x + 6) \cos 2x dx$

Solución.- I L A T E

$\quad\quad\quad\quad\quad\quad\quad \downarrow \quad \searrow$

$x^2 + 5x + 6 \quad \cos 2x$

$\therefore \quad \begin{matrix} u = x^2 + 5x + 6 & dv = \cos 2x dx \\ du = (2x + 5) dx & v = \frac{1}{2} \operatorname{sen} 2x \end{matrix}$

$\therefore \int (x^2 + 5x + 6) \cos 2x dx = \frac{(x^2 + 5x + 6)}{2} \operatorname{sen} 2x - \frac{1}{2} \int (2x + 5) \operatorname{sen} 2x dx$

Integrando por partes la segunda integral:

I L A T E

$\swarrow \quad \searrow$

$2x + 5 \quad \operatorname{sen} 2x$

$\therefore \quad \begin{matrix} u = 2x + 5 & dv = \operatorname{sen} 2x dx \\ du = 2dx & v = -\frac{1}{2} \cos 2x \end{matrix}$

$\therefore \int (x^2 + 5x + 6) \cos 2x dx = \frac{1}{2} \operatorname{sen} 2x (x^2 + 5x + 6) - \frac{1}{2} \left[(2x + 5)(-\tfrac{1}{2} \cos 2x) + \int \cos 2x dx \right]$

$= \frac{x^2 + 5x + 6}{2} \operatorname{sen} 2x + \frac{1}{4} \cos 2x (2x + 5) - \frac{1}{2} \int \cos 2x dx$

$= \frac{x^2 + 5x + 6}{2} \operatorname{sen} 2x + \frac{2x + 5}{4} \cos 2x - \frac{1}{4} \operatorname{sen} 2x + c$

Respuesta: $\int (x^2 + 5x + 6) \cos 2x dx = \frac{x^2 + 5x + 6}{2} \operatorname{sen} 2x + \frac{2x + 5}{4} \cos 2x - \frac{1}{4} \operatorname{sen} 2x + c$

Nota.-Ya se habrá dado cuenta el lector, que la elección conveniente para el u y el dv, dependerá de la ubicación de los términos funcionales en la palabra ILATE. El de la izquierda corresponde al u, y el otro será el dv.

4.5.-Encontrar: $\int \ell \eta x dx$

Solución.- I L A T E

$\quad\quad\quad\quad\quad \downarrow \quad \downarrow$

$\ell \eta x \quad 1$

$$\therefore \quad \begin{array}{ll} u = \ell\eta x & dv = 1dx \\ du = \dfrac{dx}{x} & v = x \end{array}$$

$$\therefore \int \ell\eta x dx = x\ell\eta x - \int dx = x\ell\eta x - x + c = x(\ell\eta x - 1) + c$$

Respuesta: $\int \ell\eta x dx = x(\ell\eta x - 1) + c$

4.6.-Encontrar: $\int \ell\eta(a^2 + x^2)dx$

Solución.- I L A T E

$\downarrow \quad \searrow$

$\ell\eta(a^2 + x^2) \quad 1$

$$\therefore \quad \begin{array}{ll} u = \ell\eta x & dv = 1dx \\ du = \dfrac{dx}{x} & v = x \end{array}$$

$$\therefore \int \ell\eta(a^2 + x^2)dx = x\ell\eta(a^2 + x^2) - \int \frac{2x^2dx}{a^2 + x^2} = x\ell\eta(a^2 + x^2) - \int (2 - \frac{2a^2}{x^2 + a^2})dx$$

$$= x\ell\eta(a^2 + x^2) - 2\int dx + 2a^2 \int \frac{dx}{x^2 + a^2} = x\ell\eta(a^2 + x^2) - 2x + \frac{2\cancel{a^2}}{\cancel{a}} \operatorname{arc}\tau g \tfrac{x}{a} + c$$

$$= x\ell\eta(a^2 + x^2) - 2x + 2a \operatorname{arc}\tau g \tfrac{x}{a} + c$$

Respuesta: $\int \ell\eta(a^2 + x^2)dx = x\ell\eta(a^2 + x^2) - 2x + 2a \operatorname{arc}\tau g \tfrac{x}{a} + c$

4.7.-Encontrar: $\int \ell\eta \left| x + \sqrt{x^2 - 1} \right| dx$

Solución.- I L A T E

$\downarrow \quad \searrow$

$\ell\eta \left| x + \sqrt{x^2 - 1} \right| \quad 1$

$$\therefore \quad \begin{array}{ll} u = \ell\eta \left| x + \sqrt{x^2 - 1} \right| & dv = 1dx \\ du = \dfrac{1 + \dfrac{x}{\sqrt{x^2 - 1}}}{x + \sqrt{x^2 - 1}} d \Rightarrow du = \dfrac{\dfrac{\cancel{\sqrt{x^2 - 1} + x}}{\sqrt{x^2 - 1}}}{\cancel{x + \sqrt{x^2 - 1}}} dx \Rightarrow du = \dfrac{dx}{\sqrt{x^2 - 1}} & v = x \end{array}$$

$$\therefore \int \ell\eta \left| x + \sqrt{x^2 - 1} \right| dx = x\ell\eta \left| x + \sqrt{x^2 - 1} \right| - \int \frac{xdx}{\sqrt{x^2 - 1}}$$

Sea : $w = x^2 + 1, dw = 2xdx$.

Luego: $x\ell\eta \left| x + \sqrt{x^2 - 1} \right| - \frac{1}{2} \int (x^2 - 1)^{-\frac{1}{2}} 2xdx = x\ell\eta \left| x + \sqrt{x^2 - 1} \right| - \frac{1}{2} \int w^{-\frac{1}{2}} dw$

$$= x\ell\eta \left| x + \sqrt{x^2 - 1} \right| - \frac{1}{2} \frac{w^{\frac{1}{2}}}{\frac{1}{2}} + c = x\ell\eta \left| x + \sqrt{x^2 - 1} \right| - w^{\frac{1}{2}} + c = x\ell\eta \left| x + \sqrt{x^2 - 1} \right| - \sqrt{x^2 - 1} + c$$

Respuesta: $\int \ell\eta \left| x + \sqrt{x^2 - 1} \right| dx = x\ell\eta \left| x + \sqrt{x^2 - 1} \right| - \sqrt{x^2 - 1} + c$

4.8.-Encontrar: $\int \ell\eta^2 x dx$

Solución.- I L A T E

$$\begin{array}{cc} \downarrow & \downarrow \\ \ell\eta^2 x & 1 \end{array}$$

$$\therefore \begin{array}{ll} u = \ell\eta^2 x & dv = 1dx \\ du = 2\ell\eta x \dfrac{1}{x} dx & v = x \end{array}$$

$$\therefore \int \ell\eta^2 x dx = x\ell\eta^2 x - 2\int \ell\eta x \frac{1}{x} x dx = x\ell\eta^2 x - 2\int \ell\eta x dx$$

Por ejercicio 4.5, se tiene: $\int \ell\eta x dx = x(\ell\eta x - 1) + c$

Luego: $\int \ell\eta^2 x dx = x\ell\eta^2 x - 2\left[x(\ell\eta x - 1) + c\right] = x\ell\eta^2 x - 2x(\ell\eta x - 1) + c$

Respuesta: $\int \ell\eta^2 x dx = x\ell\eta^2 x - 2x(\ell\eta x - 1) + c$

4.9.-Encontrar: $\int \operatorname{arc}\tau g x dx$

Solución.- I L A T E

$$\begin{array}{cc} \downarrow & \downarrow \\ \operatorname{arc}\tau g x & 1 \end{array}$$

$$\therefore \begin{array}{ll} u = \operatorname{arc}\tau g x & dv = 1dx \\ du = \dfrac{dx}{1+x^2} & v = x \end{array}$$

$$\therefore \int \operatorname{arc}\tau g x dx = x \operatorname{arc}\tau g x - \int \frac{xdx}{1+x^2}$$

Sea: $w = 1 + x^2, dw = 2xdx$

Luego: $x \operatorname{arc}\tau g x - \frac{1}{2}\int \frac{2xdx}{1+x^2} = x \operatorname{arc}\tau g x - \frac{1}{2}\int \frac{dw}{w} = x \operatorname{arc}\tau g x - \frac{1}{2}\ell\eta|w| + c$

$$= x \operatorname{arc}\tau g x - \frac{1}{2}\ell\eta\left|1 + x^2\right| + c$$

Respuesta: $\int \operatorname{arc}\tau g x dx = x \operatorname{arc}\tau g x - \frac{1}{2}\ell\eta\left|1 + x^2\right| + c$

4.10.- $\int x^2 \operatorname{arc}\tau g x dx$

Solución.- I L A T E

$$\begin{array}{cc} \downarrow & \downarrow \\ \operatorname{arc}\tau g x & x^2 \end{array}$$

$$\therefore \begin{array}{ll} u = \operatorname{arc}\tau g x & dv = x^2 dx \\ du = \dfrac{dx}{1+x^2} & v = \dfrac{x^3}{3} \end{array}$$

$$\therefore \int x^2 \operatorname{arc}\tau g x dx = \frac{x^3}{3}\operatorname{arc}\tau g x - \frac{1}{3}\int \frac{x^2 dx}{1+x^2} = \frac{x^3}{3}\operatorname{arc}\tau g x - \frac{1}{3}\int (x - \frac{x}{x^2+1})dx$$

$$= \frac{x^3}{3}\operatorname{arc}\tau gx - \frac{1}{3}\int xdx - \frac{1}{3}\int \frac{x}{x^2+1}dx$$

Por ejercicio 4.9, se tiene: $\int \frac{xdx}{x^2+1} = \frac{1}{2}\ell\eta\left|x^2+1\right| + c$

Luego: $\frac{x^3}{3}\operatorname{arc}\tau gx - \frac{1}{3}\int xdx + \frac{1}{6}\ell\eta\left|x^2+1\right| + c = \frac{x^3}{3}\operatorname{arc}\tau gx - \frac{x^2}{6} + \frac{1}{6}\ell\eta\left|x^2+1\right| + c$

Respuesta: $\int x^2 \operatorname{arc}\tau gxdx = \frac{x^3}{3}\operatorname{arc}\tau gx - \frac{x^2}{6} + \frac{1}{6}\ell\eta\left|x^2+1\right| + c$

4.11.-Encontrar: $\int \operatorname{arc}\cos 2xdx$

Solución.- I L A T E

$\downarrow \quad \downarrow$

$\operatorname{arc}\cos 2x \quad 1$

$$\therefore \quad u = \operatorname{arc}\cos 2x \qquad dv = 1dx$$
$$du = -\frac{2dx}{\sqrt{1-4x^2}} \qquad v = x$$

$$\therefore \int \operatorname{arc}\cos 2xdx = x\operatorname{arc}\cos 2x + 2\int \frac{xdx}{\sqrt{1-4x^2}}$$

Sea: $w = 1-4x^2, dw = -8xdx$

Luego: $x\operatorname{arc}\cos 2x - \frac{2}{8}\int \frac{-8xdx}{\sqrt{1-4x^2}} = x\operatorname{arc}\cos 2x - \frac{1}{4}\int w^{-\frac{1}{2}}dw = x\operatorname{arc}\cos 2x - \frac{1}{4}\frac{w^{\frac{1}{2}}}{\frac{1}{2}} + c$

$$= x\operatorname{arc}\cos 2x - \frac{1}{2}\sqrt{1-4x^2} + c$$

Respuesta: $\int \operatorname{arc}\cos 2xdx = x\operatorname{arc}\cos 2x - \frac{1}{2}\sqrt{1-4x^2} + c$

4.12.-Encontrar: $\int \frac{\operatorname{arcsen}\sqrt{x}}{\sqrt{x}}dx$

Solución.- I L A T E

$\downarrow \quad \searrow$

$\operatorname{arcsen}\sqrt{x} \quad 1$

$$\therefore \quad u = \operatorname{arcsen}\sqrt{x} \qquad dv = x^{-\frac{1}{2}}dx$$
$$du = \frac{1}{\sqrt{1-x}}\frac{dx}{\sqrt{x}} \qquad v = 2\sqrt{x}$$

$$\therefore \int \operatorname{arcsen}\sqrt{x}\,x^{-\frac{1}{2}}dx = 2\sqrt{x}\operatorname{arcsen}\sqrt{x} - \int \frac{dx}{\sqrt{1-x}}$$

Sea: $w = 1-x, dw = -dx$

Luego: $2\sqrt{x}\operatorname{arcsen}\sqrt{x} + \int \frac{-dx}{\sqrt{1-x}} = 2\sqrt{x}\operatorname{arcsen}\sqrt{x} + \int w^{-\frac{1}{2}}dw$

$$= 2\sqrt{x}\operatorname{arcsen}\sqrt{x} + 2w^{\frac{1}{2}} + c = 2\sqrt{x}\operatorname{arcsen}\sqrt{x} + 2\sqrt{1-x} + c$$

Respuesta: $\int \frac{\operatorname{arcsen}\sqrt{x}}{\sqrt{x}}dx = 2\sqrt{x}\operatorname{arcsen}\sqrt{x} + 2\sqrt{1-x} + c$

4.13.-Encontrar: $\int x \operatorname{arcsen} 2x^2 dx$

Solución.- I L A T E

$\downarrow \quad \searrow$

$\operatorname{arcsen} 2x^2 \qquad x$

$$\therefore \quad \begin{array}{ll} u = \operatorname{arcsen} 2x^2 & dv = xdx \\ du = \dfrac{4xdx}{\sqrt{1-4x^4}} & v = \dfrac{x^2}{2} \end{array}$$

$$\therefore \int x \operatorname{arcsen} 2x^2 dx = \frac{x^2}{2}\operatorname{arcsen} 2x^2 - 2\int \frac{x^3 dx}{\sqrt{1-4x^4}}$$

Sea: $w = 1-4x^4, dw = -16x^3 dx$

Luego: $\frac{x^2}{2}\operatorname{arcsen} 2x^2 + \frac{2}{16}\int \frac{(-16x^3dx)}{\sqrt{1-4x^4}} = \frac{x^2}{2}\operatorname{arcsen} 2x^2 + \frac{1}{8}\int w^{-1/2}dw$

$$= \frac{x^2}{2}\operatorname{arcsen} 2x^2 + \frac{1}{8}\frac{w^{1/2}}{1/2} + c = \frac{x^2}{2}\operatorname{arcsen} 2x^2 + \frac{1}{4}w^{1/2} + c$$

$$= \frac{x^2}{2}\operatorname{arcsen} 2x^2 + \frac{1}{4}\sqrt{1-4x^4} + c$$

Respuesta: $\int x \operatorname{arcsen} 2x^2 dx = \frac{x^2}{2}\operatorname{arcsen} 2x^2 + \frac{1}{4}\sqrt{1-4x^4} + c$

4.14.-Encontrar: $\int x e^{x/a} dx$

Sea: $w = \frac{x}{a}, dw = \frac{dx}{a}$

Luego: $\int x e^{x/a} dx = a^2 \int \frac{x}{a} e^{x/a} \frac{dx}{a} = a^2 \int w e^w dw$, integrando por partes se tiene:

Solución.- I L A T E

$\downarrow \quad \downarrow$

$w \qquad e^w$

$$\therefore \quad \begin{array}{ll} u = w & dv = e^w dw \\ du = dw & v = e^w \end{array}$$

$$\therefore a^2 \int we^w dw = a^2\left(we^w - \int e^w dw\right) = a^2\left(we^w - e^w + c\right) = a^2\left(we^w - e^w\right) + c$$

$$= a^2\left(\frac{x}{a}e^{x/a} - e^{x/a}\right) + c = a^2 e^{x/a}\left(\frac{x}{a} - 1\right) + c$$

Respuesta: $\int x e^{x/a} dx = a^2 e^{x/a}\left(\frac{x}{a} - 1\right) + c$

4.15.-Encontrar: $\int x^2 e^{-3x} dx$

Solución.- I L A T E

$$\begin{array}{cc} \downarrow & \downarrow \\ x^2 & e^{-3x} \end{array}$$

$$\therefore \begin{array}{ll} u = x^2 & dv = e^{-3x}dx \\ du = 2xdx & v = -\frac{1}{3}e^{-3x} \end{array}$$

$\therefore \int x^2e^{-3x}dx = -\frac{1}{3}x^2e^{-3x} + \frac{2}{3}\int xe^{-3x}dx$, integrando por partes la segunda integral:

I L A T E

$$\begin{array}{cc} \downarrow & \downarrow \\ x & e^{-3x} \end{array}$$

$$\therefore \begin{array}{ll} u = x & dv = e^{-3x}dx \\ du = dx & v = -\frac{1}{3}e^{-3x} \end{array}$$

$$\therefore \int x^2e^{-3x}dx = -\frac{1}{3}x^2e^{-3x} + \frac{2}{3}\left(-\frac{1}{3}xe^{-3x} + \frac{1}{3}\int e^{-3x}dx\right) = -\frac{x^2e^{-3x}}{3} - \frac{2}{9}xe^{-3x} + \frac{2}{9}\int e^{-3x}dx$$

$$= -\frac{x^2e^{-3x}}{3} - \frac{2}{9}xe^{-3x} - \frac{2}{27}e^{-3x} + c$$

Respuesta: $\int x^2e^{-3x}dx = \frac{-e^{-3x}}{3}\left(x^2 + \frac{2}{3}x + \frac{2}{9}\right) + c$

4.16.-Encontrar: $\int x^3e^{-x^2}dx$

Solución.- $\int x^3e^{-x^2}dx = \int x^2e^{-x^2}xdx$

Sea: $w = -x^2, dw = -2xdx$, además: $x^2 = -w$

Luego: $\int x^2e^{-x^2}xdx = -\frac{1}{2}\int x^2e^{-x^2}x(-2xdx) = -\frac{1}{2}\int -we^wdw = \frac{1}{2}\int we^wdw$, integrando por Partes se tiene:

I L A T E

$$\begin{array}{cc} \downarrow & \downarrow \\ w & e^{w} \end{array}$$

$$\therefore \begin{array}{ll} u = w & dv = e^wdw \\ du = dw & v = e^w \end{array}$$

$$\therefore \frac{1}{2}\int we^wdw = \frac{1}{2}\left(we^w - \int e^wdw\right) = \frac{1}{2}we^w - \frac{1}{2}\int e^wdw = \frac{1}{2}we^w - \frac{1}{2}e^w + c$$

$$= -\frac{1}{2}x^2e^{-x^2} - \frac{1}{2}e^{-x^2} + c = -\frac{1}{2}e^{-x^2}(x^2+1) + c$$

Respuesta: $\int x^3e^{-x^2}dx = -\frac{1}{2}e^{-x^2}(x^2+1) + c$

4.17.-Encontrar: $\int (x^2 - 2x + 5)e^{-x}dx$

Solución.- I L A T E

$$\begin{array}{cc} \downarrow & \downarrow \end{array}$$

$x^2-2x+5 \qquad e^{-x}$

$\therefore \begin{array}{ll} u = x^2-2x+5 & dv = e^{-x}dx \\ du = (2x-2)dx & v = -e^{-x} \end{array}$

$\therefore \int (x^2-2x+5)e^{-x}dx = -e^{-x}(x^2-2x+5) + \int (2x-2)e^{-x}dx$ **,** integrando por partes la segunda integral:

I L A T E

$\downarrow \quad \downarrow$

$2x-2 \quad e^{-x}$

$\therefore \begin{array}{ll} u = 2x-2 & dv = e^{-x}dx \\ du = 2dx & v = -e^{-x} \end{array}$

$\therefore \int (x^2-2x+5)e^{-x}dx = -e^{-x}(x^2-2x+5) + \left[-e^{-x}(2x-2) + 2\int e^{-x}dx\right]$

$= -e^{-x}(x^2-2x+5) - e^{-x}(2x-2) + 2\int e^{-x}dx = -e^{-x}(x^2-2x+5) - e^{-x}(2x-2) - 2e^{-x} + c$

$= -e^{-x}(x^2 \cancel{-2x} + 5 \cancel{+2x} \cancel{-2+2}) + c = -e^{-x}(x^2+5) + c$

Respuesta: $\int (x^2-2x+5)e^{-x}dx = -e^{-x}(x^2+5) + c$

4.18.-Encontrar: $\int e^{ax}\cos bx dx$

Solución.- I L A T E

$\swarrow \quad \downarrow$

$\cos bx \quad e^{ax}$

$\therefore \begin{array}{ll} u = \cos bx & dv = e^{ax}dx \\ du = -b\,\mathrm{sen}\, bx dx & v = \frac{1}{a}e^{ax} \end{array}$

$\therefore \int e^{ax}\cos bx dx = \frac{e^{ax}\cos bx}{a} + \frac{b}{a}\int e^{ax}\,\mathrm{sen}\, bx dx$, Nótese que la segunda integral es semejante a la primera, salvo en la parte trigonométrica; integrando por partes la segunda integral:

I L A T E

$\swarrow \quad \downarrow$

$\mathrm{sen}\, bx \quad e^{ax}$

$\therefore \begin{array}{ll} u = \mathrm{sen}\, bx & dv = e^{ax}dx \\ du = b\cos bx dx & v = \frac{1}{a}e^{ax} \end{array}$

$\therefore = \frac{e^{ax}\cos bx}{a} + \frac{b}{a}\left(\frac{e^{ax}\,\mathrm{sen}\, bx}{a} - \frac{b}{a}\int e^{ax}\cos bx dx\right)$

$= \frac{e^{ax}\cos bx}{a} + \frac{be^{ax}\,\mathrm{sen}\, bx}{a^2} - \frac{b^2}{a^2}\int e^{ax}\cos bx dx$, Nótese que:

$\int e^{ax}\cos bx dx = \frac{e^{ax}\cos bx}{a} + \frac{be^{ax}\,\mathrm{sen}\, bx}{a^2} - \frac{b^2}{a^2}\int e^{ax}\cos bx dx$, la integral a encontrar aparece con coeficiente 1 en el primer miembro, y en el segundo con coeficiente:

$-\dfrac{b^2}{a^2}$. Transponiendo éste término al primer miembro y dividiendo por el nuevo coeficiente: $1+\dfrac{b^2}{a^2}=\dfrac{a^2+b^2}{a^2}$, se tiene:

$$\left(\frac{a^2+b^2}{a^2}\right)\int e^{ax}\cos bx dx=\frac{ae^{ax}\cos bx+be^{ax}\operatorname{sen} bx}{a^2}+c$$

$$\int e^{ax}\cos bx dx=\frac{\dfrac{ae^{ax}\cos bx+be^{ax}\operatorname{sen} bx}{\cancel{a^2}}}{\left(\dfrac{a^2+b^2}{\cancel{a^2}}\right)}+c=\frac{e^{ax}(a\cos bx+b\operatorname{sen} bx)}{a^2+b^2}+c$$

Respuesta: $\int e^{ax}\cos bx dx=\dfrac{e^{ax}(a\cos bx+b\operatorname{sen} bx)}{a^2+b^2}+c$

4.19.-Encontrar: $\int e^{x}\cos 2x dx$

Solución.- Este ejercicio es un caso particular del ejercicio anterior, donde: $a=1$ y $b=2$. Invitamos al lector, resolverlo por partes, aún cuando la respuesta es inmediata.

Respuesta: $\int e^{x}\cos 2x dx=\dfrac{e^{x}(\cos 2x+2\operatorname{sen} 2x)}{5}+c$

4.20.-Encontrar: $\int e^{ax}\operatorname{sen} bx dx$

Solución.- I L A T E

$$\swarrow \quad \downarrow$$

$$\operatorname{sen} bx \quad e^{ax}$$

$$\therefore \quad \begin{array}{ll} u=\operatorname{sen} bx & dv=e^{ax}dx \\ du=b\cos bx dx & v=\dfrac{1}{a}e^{ax} \end{array}$$

$\therefore \int e^{ax}\operatorname{sen} bx dx=\dfrac{e^{ax}\operatorname{sen} bx}{a}-\dfrac{b}{a}\int e^{ax}\cos bx dx$, integrando por partes la segunda integral:

I L A T E

$$\swarrow \quad \downarrow$$

$$\cos bx \quad e^{ax}$$

$$\therefore \quad \begin{array}{ll} u=\cos bx & dv=e^{ax}dx \\ du=-b\operatorname{sen} bx dx & v=\dfrac{1}{a}e^{ax} \end{array}$$

$$\therefore \int e^{ax}\operatorname{sen} bx dx=\frac{e^{ax}\operatorname{sen} bx}{a}-\frac{b}{a}\left(\frac{e^{ax}\cos bx}{a}+\frac{b}{a}\int e^{ax}\operatorname{sen} bx dx\right)$$

$$=\frac{e^{ax}\operatorname{sen} bx}{a}-\frac{be^{ax}\cos bx}{a^2}-\frac{b^2}{a^2}\int e^{ax}\operatorname{sen} bx dx,$$

Como habrá notado el lector, la integral a encontrar aparece con coeficiente 1 en el primer miembro, y en el segundo con coeficiente: $-\frac{b^2}{a^2}$. Transponiendo éste término al primer miembro y dividiendo por el nuevo coeficiente: $1+\frac{b^2}{a^2}=\frac{a^2+b^2}{a^2}$, se tiene:

$$\left(\frac{a^2+b^2}{a^2}\right)\int e^{ax}\operatorname{sen}bxdx=\frac{ae^{ax}\operatorname{sen}bx-be^{ax}\cos bx}{a^2}+c$$

$$\int e^{ax}\operatorname{sen}bxdx=\frac{\frac{ae^{ax}\operatorname{sen}bx-be^{ax}\cos bx}{\cancel{a^2}}}{\left(\frac{a^2+b^2}{\cancel{a^2}}\right)}+c=\int e^{ax}\operatorname{sen}bxdx=\frac{e^{ax}(a\operatorname{sen}bx-b\cos bx)}{a^2+b^2}+c$$

Respuesta: $\int e^{ax}\operatorname{sen}bxdx=\frac{e^{ax}(a\operatorname{sen}bx-b\cos bx)}{a^2+b^2}+c$

4.21.-Encontrar: $\int x\sqrt{1+x}dx$

Solución.- Cuando el integrando, está formado por el producto de funciones algebraicas, es necesario tomar como dv, la parte más fácil integrable y u como la parte más fácil derivable. Sin embargo, la opción de "más fácil" quedará a criterio del lector.

$$\therefore\quad \begin{array}{ll} u=x & dv=(1+x)^{1/2}dx \\ du=dx & v=\frac{2}{3}(1+x)^{3/2} \end{array}$$

$$\therefore\int x\sqrt{1+x}dx=\frac{2}{3}x(1+x)^{3/2}-\frac{2}{3}\int(1+x)^{3/2}dx=\frac{2}{3}x(1+x)^{3/2}-\frac{2}{3}\frac{(1+x)^{5/2}}{5/2}+c$$

$$=\frac{2}{3}x(1+x)^{3/2}-\frac{4(1+x)^{5/2}}{15}+c$$

Respuesta: $\int x\sqrt{1+x}dx=\frac{2}{3}x(1+x)^{3/2}-\frac{4(1+x)^{5/2}}{15}+c$

4.22.-Encontrar: $\int\frac{x^2dx}{\sqrt{1+x}}$

Solución.- $\int\frac{x^2dx}{\sqrt{1+x}}=\int x^2(1+x)^{-1/2}dx$

$$\therefore\quad \begin{array}{ll} u=x^2 & dv=(1+x)^{-1/2}dx \\ du=2xdx & v=2(1+x)^{1/2} \end{array}$$

$\therefore\int\frac{x^2dx}{\sqrt{1+x}}=2x^2\sqrt{1+x}-4\int x\sqrt{1+x}dx$, integrando por partes la segunda integral:

$$\therefore \quad \begin{array}{ll} u = x & dv = (1+x)^{1/2}dx \\ du = dx & v = \frac{2}{3}(1+x)^{3/2} \end{array}$$

$$\int \frac{x^2dx}{\sqrt{1+x}} = 2x^2\sqrt{1+x} - 4\left[\frac{2}{3}x(1+x)^{3/2} - \frac{2}{3}\int (1+x)^{3/2}dx\right]$$

$$= 2x^2\sqrt{1+x} - \frac{8}{3}x(1+x)^{3/2} + \frac{8}{3}\frac{(1+x)^{5/2}}{5/2} + c = 2x^2\sqrt{1+x} - \frac{8}{3}x(1+x)^{3/2} + \frac{16}{15}(1+x)^{5/2} + c$$

Respuesta: $\int \frac{x^2dx}{\sqrt{1+x}} = 2x^2\sqrt{1+x} - \frac{8}{3}x(1+x)^{3/2} + \frac{16}{15}(1+x)^{5/2} + c$

4.23.-Encontrar: $\int \frac{xdx}{e^x}$

Solución.- $\int \frac{xdx}{e^x} = \int xe^{-x}dx$

$$\begin{array}{ccccc} I & L & A & T & E \\ & & \downarrow & & \downarrow \\ & & x & & e^{-x} \end{array}$$

$$\therefore \quad \begin{array}{ll} u = x & dv = e^{-x}dx \\ du = dx & v = -e^{-x} \end{array}$$

$$\therefore \int xe^{-x}dx = -xe^{-x} + \int e^{-x}dx = -xe^{-x} - e^{-x} + c = e^{-x}(-x-1) + c = -e^{-x}(x+1) + c$$

Respuesta: $\int \frac{xdx}{e^x} = -e^{-x}(x+1) + c$

4.24.-Encontrar: $\int x^2\ell\eta\left|\sqrt{1-x}\right|dx$

Solución.- $\therefore \quad \begin{array}{ll} u = \ell\eta\left|\sqrt{1-x}\right| & dv = x^2dx \\ du = \frac{1}{\left|\sqrt{1-x}\right|}\frac{1}{2}(1-x)^{-1/2}(-1)dx \Rightarrow du = \frac{-dx}{2(1-x)} & v = \frac{x^3}{3} \end{array}$

$$\therefore \int x^2\ell\eta\left|\sqrt{1-x}\right|dx = \frac{x^3}{3}\ell\eta\left|\sqrt{1-x}\right| + \frac{1}{6}\int \frac{x^3}{1-x}dx = \frac{x^3}{3}\ell\eta\left|\sqrt{1-x}\right| - \frac{1}{6}\int\left(x^2 + x + 1 - \frac{1}{1-x}\right)dx$$

$$= \frac{x^3}{3}\ell\eta\left|\sqrt{1-x}\right| - \frac{1}{6}\frac{x^3}{3} - \frac{1}{6}\frac{x^2}{2} - \frac{1}{6}x - \frac{1}{6}\ell\eta\left|1-x\right| + c$$

$$= \frac{x^3}{3}\ell\eta\left|\sqrt{1-x}\right| - \frac{1}{6}\ell\eta\left|1-x\right| - \frac{x^3}{18} - \frac{x^2}{12} - \frac{x}{6} + c$$

Respuesta: $\int x^2\ell\eta\left|\sqrt{1-x}\right|dx = \frac{x^3}{3}\ell\eta\left|\sqrt{1-x}\right| - \frac{1}{6}\ell\eta\left|1-x\right| - \frac{x^3}{18} - \frac{x^2}{12} - \frac{x}{6} + c$

4.25.-Encontrar: $\int x\,\mathrm{sen}^2 xdx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = x & dv = \operatorname{sen}^2 x dx \\ du = dx & v = \frac{1}{2}x - \frac{1}{4}\operatorname{sen} 2x \end{array} \quad \left(v = \int \frac{1-\cos 2x}{2} dx \right)$$

$$\therefore \int x \operatorname{sen}^2 x dx = \frac{1}{2}x^2 - \frac{1}{4}x \operatorname{sen} 2x - \frac{1}{2}\int x dx + \frac{1}{4}\int \operatorname{sen} 2x dx$$

$$= \frac{1}{2}x^2 - \frac{1}{4}x \operatorname{sen} 2x - \frac{1}{4}x^2 - \frac{1}{8}\cos 2x + c = \frac{1}{4}x^2 - \frac{1}{4}x \operatorname{sen} 2x - \frac{1}{8}\cos 2x + c$$

Respuesta: $\int x \operatorname{sen}^2 x dx = \frac{x^2}{4} - \frac{x \operatorname{sen} 2x}{4} - \frac{\cos 2x}{8} + c$

Otra solución.-

$$\int x \operatorname{sen}^2 x dx = \int x \frac{1-\cos 2x}{2} dx = \frac{1}{2}\int x dx - \frac{1}{2}\int x \cos 2x dx = \frac{1}{2}\frac{x^2}{2} - \frac{1}{2}\int x \cos 2x dx$$

$= \frac{x^2}{4} - \frac{1}{2}\int x \cos 2x dx$ **;** integrando por partes, la segunda integral:

$$\therefore \quad \begin{array}{ll} u = x & dv = \cos 2x dx \\ du = dx & v = \frac{1}{2}\operatorname{sen} 2x \end{array}$$

$$\int x \operatorname{sen}^2 x dx = \frac{x^2}{4} - \frac{1}{2}\left(\frac{x}{2}\operatorname{sen} 2x - \frac{1}{2}\int \operatorname{sen} 2x dx \right) = \frac{x^2}{4} - \frac{x}{4}\operatorname{sen} 2x + \frac{1}{4}\int \operatorname{sen} 2x dx$$

$$= \frac{x^2}{4} - \frac{x}{4}\operatorname{sen} 2x + \frac{1}{4}(-\frac{1}{2}\cos 2x) + c = \frac{x^2}{4} - \frac{x}{4}\operatorname{sen} 2x - \frac{\cos 2x}{8} + c$$

Respuesta: $\int x \operatorname{sen}^2 x dx = \frac{x^2}{4} - \frac{x \operatorname{sen} 2x}{4} - \frac{\cos 2x}{8} + c$

4.26.-Encontrar: $\int x(3x+1)^7 dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = x & dv = (3x+1)^7 dx \\ du = dx & v = \frac{1}{24}(3x+1)^8 \end{array} \quad \left(v = \int (3x+1)^7 dx \right)$$

$$\therefore \int x(3x+1)^7 dx = \frac{x}{24}(3x+1)^8 - \frac{1}{24}\int (3x+1)^8 dx = \frac{x}{24}(3x+1)^8 - \frac{1}{24}\frac{1}{3}\frac{(3x+1)^9}{9} + c$$

$$= \frac{x}{24}(3x+1)^8 - \frac{(3x+1)^9}{648} + c$$

Respuesta: $\int x(3x+1)^7 dx = \frac{x}{24}(3x+1)^8 - \frac{(3x+1)^9}{648} + c$

EJERCICIOS PROPUESTOS

Usando esencialmente el mecanismo presentado, encontrar las integrales siguientes:

4.27.- $\int x(2x+5)^{10}dx$

4.28.- $\int \operatorname{arcsen} x dx$

4.29.- $\int x \operatorname{sen} x dx$

4.30.- $\int x\cos 3x dx$

4.31.- $\int x2^{-x}dx$

4.32.- $\int x^2e^{3x}dx$

4.33.- $\int x^3e^{-x/3}dx$

4.34.- $\int x \operatorname{sen} x\cos x dx$

4.35.- $\int x^2\ell\eta x dx$

4.36.- $\int \frac{\ell\eta x}{x^3}dx$

4.37.- $\int \frac{\ell\eta x}{\sqrt{x}}dx$

4.38.- $\int x \operatorname{arc}\tau g x dx$

4.39.- $\int x \operatorname{arcsen} x dx$

4.40.- $\int \frac{xdx}{\operatorname{sen}^2 x}$

4.41.- $\int e^x \operatorname{sen} x dx$

4.42.- $\int 3^x\cos x dx$

4.43.- $\int \operatorname{sen}(\ell\eta x)dx$

4.44.- $\int (x^2-2x+3)\ell\eta x dx$

4.45.- $\int x\ell\eta\left|\frac{1-x}{1+x}\right|dx$

4.46.- $\int \frac{\ell\eta^2 x}{x^2}dx$

4.47.- $\int x^2 \operatorname{arc}\tau g 3x dx$

4.48.- $\int x(\operatorname{arc}\tau g x)^2dx$

4.49.- $\int (\operatorname{arcsen} x)^2dx$

4.50.- $\int \frac{\operatorname{arcsen} x}{x^2}dx$

4.51.- $\int \frac{\operatorname{arcsen}\sqrt{x}}{\sqrt{1-x}}dx$

4.52.- $\int \frac{\operatorname{sen}^2 x}{e^x}dx$

4.53.- $\int \tau g^2 x\sec^3 x dx$

4.54.- $\int x^3\ell\eta^2 x dx$

4.55.- $\int x\ell\eta(9+x^2)dx$

4.56.- $\int \operatorname{arcsen}\sqrt{x}dx$

4.57.- $\int x \operatorname{arc}\tau g(2x+3)dx$

4.58.- $\int e^{\sqrt{x}}dx$

4.59.- $\int \cos^2(\ell\eta x)dx$

4.60.- $\int \frac{\ell\eta(\ell\eta x)}{x}dx$

4.61.- $\int \ell\eta|x+1|dx$

4.62.- $\int x^2e^x dx$

4.63.- $\int \cos^n x dx$

4.64.- $\int \operatorname{sen}^n x dx$

4.65.- $\int x^m(\ell\eta x)^n dx$

4.66.- $\int x^3(\ell\eta x)^2dx$

4.67.- $\int x^ne^x dx$

4.68.- $\int x^3e^x dx$

4.69.- $\int \sec^n x dx$

4.70.- $\int \sec^3 x dx$

4.71.- $\int x\ell\eta x dx$

4.72.- $\int x^n\ell\eta|ax|dx, n\neq -1$

4.73.- $\int \operatorname{arcsen} ax dx$

4.74.- $\int x \operatorname{sen} ax dx$

4.75.- $\int x^2\cos ax dx$

4.76.- $\int x\sec^2 ax dx$

4.77.- $\int \cos(\ell\eta x)dx$

4.78.- $\int \ell\eta(9+x^2)dx$

4.79.- $\int x\cos(2x+1)dx$

4.80.- $\int x \operatorname{arcsec} x dx$

4.81.- $\int \operatorname{arcsec}\sqrt{x}dx$

4.82.- $\int \sqrt{a^2-x^2}dx$

4.83.- $\int \ell\eta|1-x|dx$

4.84.- $\int \ell\eta(x^2+1)dx$

4.85.- $\int \operatorname{arc}\tau g\sqrt{x}dx$

4.86.- $\int \frac{x \operatorname{arcsen} x}{\sqrt{1-x^2}}dx$

4.87.- $\int x \operatorname{arc}\tau g\sqrt{x^2-1}dx$

4.88.- $\int \frac{x \operatorname{arc}\tau g x}{(x^2+1)^2}dx$

4.89.- $\int \operatorname{arcsen} x\frac{xdx}{\sqrt{(1-x^2)^3}}$

4.90.- $\int x^2\sqrt{1-x}dx$

RESPUESTAS

4.27.- $\int x(2x+5)^{10}dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = x & dv = (2x+5)^{10}dx \\ du = dx & v = \dfrac{(2x+5)^{11}}{22} \end{array}$$

$$\int x(2x+5)^{10}dx = \frac{x}{22}(2x+5)^{11} - \frac{1}{22}\int (2x+5)^{11}dx = \frac{x}{22}(2x+5)^{11} - \frac{1}{44}(2x+5)^{12} + c$$

$$= \frac{x}{22}(2x+5)^{11} - \frac{1}{528}(2x+5)^{12} + c$$

4.28.- $\int \text{arcs}\,e\,\text{n}\,xdx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = \text{arcs}\,e\,\text{n}\,x & dv = dx \\ du = \dfrac{dx}{\sqrt{1-x^2}} & v = x \end{array}$$

Además: $w = 1-x^2, dw = -2xdx$

$$\int \text{arcs}\,e\,\text{n}\,xdx = x\,\text{arcs}\,e\,\text{n}\,x - \int \frac{xdx}{\sqrt{1-x^2}} = x\,\text{arcs}\,e\,\text{n}\,x + \frac{1}{2}\int \frac{dw}{w^{1/2}} = x\,\text{arcs}\,e\,\text{n}\,x + \sqrt{1-x^2} + c$$

4.29.- $\int x\,\text{s}\,e\,\text{n}\,xdx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = x & dv = \text{s}\,e\,\text{n}\,xdx \\ du = dx & v = -\cos x \end{array}$$

$$\int x\,\text{s}\,e\,\text{n}\,xdx = -x\cos x + \int \cos xdx = -x\cos x + \text{s}\,e\,\text{n}\,x + c$$

4.30.- $\int x\cos 3xdx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = x & dv = \cos 3xdx \\ du = dx & v = \dfrac{1}{3}\text{s}\,e\,\text{n}\,3x \end{array}$$

$$\int x\cos 3xdx = \frac{x}{3}\text{s}\,e\,\text{n}\,3x - \int \frac{1}{3}\text{s}\,e\,\text{n}\,3xdx = \frac{x}{3}\text{s}\,e\,\text{n}\,3x + \frac{\cos 3x}{9} + c$$

4.31.- $\int x2^{-x}dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = x & dv = 2^{-x}dx \\ du = dx & v = -\dfrac{2^{-x}}{\ell\eta 2} \end{array}$$

$$\int x2^{-x}dx = -\frac{x2^{-x}}{\ell\eta 2} + \frac{1}{\ell\eta 2}\int 2^{-x}dx = -\frac{x2^{-x}}{\ell\eta 2} + \frac{1}{\ell\eta 2}\left(\frac{-2^{-x}}{\ell\eta 2}\right) + c = -\frac{x}{2^x\ell\eta 2} - \frac{1}{2^{-x}\ell\eta^2 2} + c$$

4.32.- $\int x^2e^{3x}dx$

Solución.-

$\therefore \quad \begin{array}{ll} u = x^2 & dv = e^{3x}dx \\ du = 2xdx & v = \frac{1}{3}e^{3x} \end{array}$

$\int x^2 e^{3x}dx = \frac{x^2}{3}e^{3x} - \frac{2}{3}\int xe^{3x}dx$, integral la cual se desarrolla nuevamente por partes, esto es: $\quad \therefore \quad \begin{array}{ll} u = x & dv = e^{3x}dx \\ du = dx & v = \frac{1}{3}e^{3x} \end{array}$

$$= \frac{x^2}{3}e^{3x} - \frac{2}{3}\left(\frac{x}{3}e^{3x} - \frac{1}{3}\int e^{3x}dx\right) = \frac{x^2}{3}e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{9}\int e^{3x}dx = \frac{x^2}{3}e^{3x} - \frac{2x}{9}e^{3x} + \frac{2}{27}e^{3x} + c$$

4.33.- $\int x^3 e^{-x/3}dx$

Solución.-

$\therefore \quad \begin{array}{ll} u = x^3 & dv = e^{-x/3}dx \\ du = 3x^2dx & v = -3e^{-x/3} \end{array}$

$\int x^3 e^{-x/3}dx = -3x^3e^{-x/3} + 9\int x^2 e^{-x/3}dx$, integral la cual se desarrolla nuevamente por partes, esto es: $\quad \therefore \quad \begin{array}{ll} u = x^2 & dv = e^{-x/3}dx \\ du = 2xdx & v = -3e^{-x/3} \end{array}$

$$= -3x^3e^{-x/3} + 9\left(-3x^2e^{-x/3} + 6\int xe^{-x/3}dx\right) = -3x^3e^{-x/3} - 27x^2e^{-x/3} + 54\int xe^{-x/3}dx$$

, la nueva integral se desarrolla por partes, esto es:

$\therefore \quad \begin{array}{ll} u = x & dv = e^{-x/3}dx \\ du = dx & v = -3e^{-x/3} \end{array}$

$$= -\frac{3x^3}{e^{x/3}} - \frac{27x^2}{e^{x/3}} + 54\left(-3xe^{-x/3} + 3\int e^{-x/3}dx\right) = -\frac{3x^3}{e^{x/3}} - \frac{27x^2}{e^{x/3}} - \frac{162x}{e^{x/3}} + 162(-3e^{-x/3}) + c$$

$$= -\frac{3x^3}{e^{x/3}} - \frac{27x^2}{e^{x/3}} - \frac{162x}{e^{x/3}} - \frac{486}{e^{x/3}} + c$$

4.34.- $\int x \operatorname{sen} x \cos x dx$

Solución.-

$\therefore \quad \begin{array}{ll} u = x & dv = \operatorname{sen} 2xdx \\ du = dx & v = -\frac{\cos 2x}{2} \end{array}$

$$\int x \operatorname{sen} x \cos x dx = \frac{1}{2}\int x \operatorname{sen} 2xdx = \frac{1}{2}\left(-\frac{x}{2}\cos 2x + \frac{1}{2}\int \cos 2xdx\right)$$

$$= -\frac{x}{4}\cos 2x + \frac{1}{4}\int \cos 2xdx = -\frac{x}{4}\cos 2x + \frac{1}{8}\operatorname{sen} 2x + c$$

4.35.- $\int x^2 \ell \eta x dx$

Solución.-

$$\therefore \quad u = \ell\eta x \qquad dv = x^2dx$$
$$du = \frac{dx}{x} \qquad v = \frac{x^3}{3}$$

$$\int x^2\ell\eta x dx = \frac{x^3\ell\eta x}{3} - \frac{1}{3}\int x^2dx = \frac{x^3\ell\eta x}{3} - \frac{x^3}{9} + c$$

4.36.- $\int \frac{\ell\eta x}{x^3}dx$

Solución.-

$$\therefore \quad u = \ell\eta x \qquad dv = x^{-3}dx$$
$$du = \frac{dx}{x} \qquad v = -\frac{1}{2x^2}$$

$$\int \frac{\ell\eta x}{x^3}dx = \int x^{-3}\ell\eta x dx = -\frac{\ell\eta x}{2x^2} + \frac{1}{2}\int x^{-3}dx = -\frac{\ell\eta x}{2x^2} - \frac{1}{4x^2} + c$$

4.37.- $\int \frac{\ell\eta x}{\sqrt{x}}dx$

Solución.-

$$\therefore \quad u = \ell\eta x \qquad dv = x^{-1/2}dx$$
$$du = \frac{dx}{x} \qquad v = 2\sqrt{x}$$

$$\int \frac{\ell\eta x}{\sqrt{x}}dx = \int x^{-1/2}\ell\eta x dx = 2\sqrt{x}\ell\eta x - 2\int x^{-1/2}dx = 2\sqrt{x}\ell\eta x - 4\sqrt{x} + c$$

4.38.- $\int x \operatorname{arc}\tau g x dx$

Solución.-

$$\therefore \quad u = \operatorname{arc}\tau g x \qquad dv = xdx$$
$$du = \frac{dx}{1+x^2} \qquad v = \frac{x^2}{2}$$

$$\int x \operatorname{arc}\tau g x dx = \frac{x^2}{2}\operatorname{arc}\tau g x - \frac{1}{2}\int \frac{x^2dx}{1+x^2} = \frac{x^2}{2}\operatorname{arc}\tau g x - \frac{1}{2}\int\left(1 - \frac{1}{1+x^2}\right)dx$$

$$= \frac{x^2}{2}\operatorname{arc}\tau g x - \frac{1}{2}\int dx + \frac{1}{2}\int \frac{dx}{1+x^2} = \frac{x^2}{2}\operatorname{arc}\tau g x - \frac{1}{2}x + \frac{\operatorname{arc}\tau g x}{2} + c$$

4.39.- $\int x \operatorname{arcsen} x dx$

Solución.-

$$\therefore \quad u = \operatorname{arcsen} x \qquad dv = xdx$$
$$du = \frac{dx}{\sqrt{1+x^2}} \qquad v = \frac{x^2}{2}$$

$\int x \operatorname{arcsen} x dx = \frac{x^2}{2}\operatorname{arcsen} x - \frac{1}{2}\int \frac{x^2dx}{\sqrt{1+x^2}}$, integral para la cual se sugiere la sustitución siguiente: $\therefore$ $x = \operatorname{sen}\theta$, $dx = \cos\theta d\theta$

$$=\frac{x^2}{2}\operatorname{arcsen} x-\frac{1}{2}\int\frac{\operatorname{sen}^2\theta\cancel{\cos\theta}d\theta}{\cancel{\cos\theta}}$$
$$=\frac{x^2}{2}\operatorname{arcsen} x-\frac{1}{2}\int\left(\frac{1-\cos 2\theta}{2}\right)d\theta=\frac{x^2}{2}\operatorname{arcsen} x-\frac{1}{4}\int d\theta+\frac{1}{4}\int\cos 2\theta d\theta$$
$$=\frac{x^2}{2}\operatorname{arcsen} x-\frac{1}{4}\theta+\frac{1}{8}\operatorname{sen} 2\theta+c=\frac{x^2}{2}\operatorname{arcsen} x-\frac{1}{4}\operatorname{arcsen} x+\frac{2\operatorname{sen}\theta\cos\theta}{8}+c$$
Como: $\operatorname{sen}\theta=x, \cos\theta=\sqrt{1-x^2}$; luego:
$$=\frac{x^2}{2}\operatorname{arcsen} x-\frac{1}{4}\operatorname{arcsen} x+\frac{1}{4}x\sqrt{1-x^2}+c$$
4.40.- $\int\frac{xdx}{\operatorname{sen}^2 x}$

Solución.-

$$\therefore\quad \begin{array}{ll} u=x & dv=\operatorname{cosec}^2 xdx \\ du=dx & v=-\operatorname{co\tau g} x \end{array}$$
$$\int\frac{xdx}{\operatorname{sen}^2 x}=\int x\operatorname{cosec}^2 xdx=-x\operatorname{co\tau g} x+\int\operatorname{co\tau g} xdx=-x\operatorname{co\tau g} x+\ell\eta\left|\operatorname{sen} x\right|+c$$
4.41.- $\int e^x\operatorname{sen} xdx$

Solución.-

$$\therefore\quad \begin{array}{ll} u=\operatorname{sen} x & dv=e^x dx \\ du=\cos xdx & v=e^x \end{array}$$
$\int e^x\operatorname{sen} xdx=e^x\operatorname{sen} x-\int e^x\cos xdx$, integral la cual se desarrolla por partes, esto es:

$$\therefore\quad \begin{array}{ll} u=\cos x & dv=e^x dx \\ du=-\operatorname{sen} xdx & v=e^x \end{array}$$
$$=e^x\operatorname{sen} x-\left(e^x\cos x+\int e^x\operatorname{sen} xdx\right)=e^x\operatorname{sen} x-e^x\cos x-\int e^x\operatorname{sen} xdx$$
Luego se tiene: $\int e^x\operatorname{sen} xdx=e^x\operatorname{sen} x-e^x\cos x-\int e^x\operatorname{sen} xdx$, de donde es inmediato:

$$2\int e^x\operatorname{sen} xdx=e^x(\operatorname{sen} x-\cos x)+c$$
$$\int e^x\operatorname{sen} xdx=\frac{e^x}{2}(\operatorname{sen} x-\cos x)+c$$
4.42.- $\int 3^x\cos xdx$

Solución.-

$$\therefore\quad \begin{array}{ll} u=\cos x & dv=3^x dx \\ du=-\operatorname{sen} xdx & v=\dfrac{3^x}{\ell\eta 3} \end{array}$$

$\int 3^x \cos x dx = \cos x \dfrac{3^x}{\ell\eta 3} + \dfrac{1}{\ell\eta 3}\int 3^x \operatorname{sen} x dx$, integral la cual se desarrolla por partes, esto es: $\therefore$

$$u = \operatorname{sen} x \qquad dv = 3^x dx$$
$$du = \cos x dx \qquad v = \frac{3^x}{\ell\eta 3}$$

$$= \cos x \frac{3^x}{\ell\eta 3} + \frac{1}{\ell\eta 3}\left(\frac{3^x}{\ell\eta 3}\operatorname{sen} x - \frac{1}{\ell\eta 3}\int 3^x \cos x dx\right)$$

$= \cos x \dfrac{3^x}{\ell\eta 3} + \dfrac{3^x \operatorname{sen} x}{\ell\eta^2 3} - \dfrac{1}{\ell\eta^2 3}\int 3^x \cos x dx$,luego:

$= \int 3^x \cos x dx = \dfrac{3^x}{\ell\eta}\left(\cos x + \dfrac{\operatorname{sen} x}{\ell\eta 3}\right) - \dfrac{1}{\ell\eta^2 3}\int 3^x \cos x dx$, de donde es inmediato:

$$= \left(1 + \frac{1}{\ell\eta^2 3}\right)\int 3^x \cos x dx = \frac{3^x}{\ell\eta 3}\left(\cos x + \frac{\operatorname{sen} x}{\ell\eta 3}\right) + c$$

$$= \left(\frac{\ell\eta^2 3 + 1}{\ell\eta^{\cancel{2}} 3}\right)\int 3^x \cos x dx = \frac{3^x}{\cancel{\ell\eta 3}}\left(\cos x + \frac{\operatorname{sen} x}{\ell\eta 3}\right) + c$$

$$= \int 3^x \cos x dx = \frac{3^x \ell\eta 3}{\ell\eta^2 3 + 1}\left(\cos x + \frac{\operatorname{sen} x}{\ell\eta 3}\right) + c$$

4.43.- $\int \operatorname{sen}(\ell\eta x) dx$

Solución.-

$$\therefore \quad u = \operatorname{sen}(\ell\eta x) \qquad dv = dx$$
$$du = \frac{\cos(\ell\eta x)}{x} dx \qquad v = x$$

$\int \operatorname{sen}(\ell\eta x) dx = x \operatorname{sen}(\ell\eta x) - \int \cos(\ell\eta x) dx$, integral la cual se desarrolla por partes, esto es:

$$\therefore \quad u = \cos(\ell\eta x) \qquad dv = dx$$
$$du = \frac{-\operatorname{sen}(\ell\eta x)}{x} dx \qquad v = x$$

$$= x \operatorname{sen}(\ell\eta x) - \left[x \cos(\ell\eta x) + \int \operatorname{sen}(\ell\eta x) dx\right] = x \operatorname{sen}(\ell\eta x) - x\cos(\ell\eta x) - \int \operatorname{sen}(\ell\eta x) dx$$

Se tiene por tanto:

$\int \operatorname{sen}(\ell\eta x) dx = x\left[\operatorname{sen}(\ell\eta x) - \cos(\ell\eta x)\right] - \int \operatorname{sen}(\ell\eta x) dx$, de donde es inmediato:

$$2\int \operatorname{sen}(\ell\eta x) dx = x\left[\operatorname{sen}(\ell\eta x) - \cos(\ell\eta x)\right] + c \quad \int \operatorname{sen}(\ell\eta x) dx = \frac{x}{2}\left[\operatorname{sen}(\ell\eta x) - \cos(\ell\eta x)\right] + c$$

4.44.- $\int (x^2 - 2x + 3)\ell\eta x dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u=\ell\eta x & dv=(x^2-2x+3)dx \\ du=\dfrac{dx}{x} & v=\dfrac{x^3}{3}-x^2+3x \end{array}$$

$$\int (x^2-2x+3)\ell\eta x dx=(\frac{x^3}{3}-x^2+3x)\ell\eta x-\int(\frac{x^2}{3}-x+3)dx$$

$$=(\frac{x^3}{3}-x^2+3x)\ell\eta x-\int\frac{x^2}{3}dx-\int xdx+3\int dx=(\frac{x^3}{3}-x^2+3x)\ell\eta x-\frac{x^3}{9}-\frac{x^2}{2}+3x+c$$

4.45.- $\int x\ell\eta\left|\frac{1-x}{1+x}\right|dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u=\ell\eta\left|\dfrac{1-x}{1+x}\right| & dv=xdx \\ du=\dfrac{2dx}{x^2-1} & v=\dfrac{x^2}{2} \end{array}$$

$$\int x\ell\eta\left|\frac{1-x}{1+x}\right|dx=\frac{x^2}{2}\ell\eta\left|\frac{1-x}{1+x}\right|-\int\frac{x^2dx}{x^2-1}=\frac{x^2}{2}\ell\eta\left|\frac{1-x}{1+x}\right|-\int(1+\frac{1}{x^2-1})dx$$

$$=\frac{x^2}{2}\ell\eta\left|\frac{1-x}{1+x}\right|-\int dx-\int\frac{dx}{x^2-1}=\frac{x^2}{2}\ell\eta\left|\frac{1-x}{1+x}\right|-x-\frac{1}{2}\ell\eta\left|\frac{x-1}{x+1}\right|+c$$

4.46.- $\int\frac{\ell\eta^2x}{x^2}dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u=\ell\eta^2x & dv=x^{-2}dx \\ du=\dfrac{2\ell\eta x}{x}dx & v=-\dfrac{1}{x} \end{array}$$

$\int\frac{\ell\eta^2x}{x^2}dx=-\frac{\ell\eta^2x}{x}+2\int\frac{\ell\eta x}{x^2}dx=-\frac{\ell\eta^2x}{x}+2\int x^{-2}\ell\eta xdx$, integral la cual se desarrolla por partes, esto es:

$$\therefore \quad \begin{array}{ll} u=\ell\eta x & dv=x^{-2}dx \\ du=\dfrac{dx}{x} & v=-\dfrac{1}{x} \end{array}$$

$$=-\frac{\ell\eta^2x}{x}+2\left(-\frac{\ell\eta x}{x}+\int\frac{dx}{x^2}\right)=-\frac{\ell\eta^2x}{x}-\frac{2\ell\eta x}{x}+2\int\frac{dx}{x^2}=-\frac{\ell\eta^2x}{x}-\frac{2\ell\eta x}{x}-\frac{2}{x}+c$$

4.47.- $\int x^2\operatorname{arc}\tau g3xdx$

Solución.-

$$\therefore \quad \begin{array}{ll} u=\operatorname{arc}\tau g3x & dv=x^2dx \\ du=\dfrac{3dx}{1+9x^2} & v=\dfrac{x^3}{3} \end{array}$$

$$\int x^2 \operatorname{arc}\tau g 3x dx = \frac{x^3}{3}\operatorname{arc}\tau g 3x - \int \frac{x^3 dx}{1+9x^2} = \frac{x^3}{3}\operatorname{arc}\tau g 3x - \frac{1}{9}\int \frac{x^3 dx}{1/9 + x^2}$$

$$= \frac{x^3}{3}\operatorname{arc}\tau g 3x - \frac{1}{9}\left[\int\left(x - \frac{1/9\, x}{x^2 + 1/9}\right)dx\right] = \frac{x^3}{3}\operatorname{arc}\tau g 3x - \frac{1}{9}\frac{x^2}{2} + \frac{1}{81}\int \frac{xdx}{x^2 + 1/9}$$

$$= \frac{x^3}{3}\operatorname{arc}\tau g 3x - \frac{x^2}{18} + \frac{1}{162}\ell\eta\left|x^2 + \frac{1}{9}\right| + c$$

4.48.- $\int x(\operatorname{arc}\tau gx)^2 dx$

Solución.-

$$\therefore \quad u = (\operatorname{arc}\tau gx)^2 \qquad dv = xdx$$
$$du = \frac{2\operatorname{arc}\tau gxdx}{1+x^2} \qquad v = \frac{x^2}{2}$$

$\int x(\operatorname{arc}\tau gx)^2 dx = \frac{x^2}{2}(\operatorname{arc}\tau gx)^2 - \int(\operatorname{arc}\tau gx)\frac{x^2 dx}{1+x^2}$, integral la cual se desarrolla por partes, esto es:

$$\therefore \quad u = \operatorname{arc}\tau gx \qquad dv = \frac{x^2 dx}{1+x^2}$$
$$du = \frac{dx}{1+x^2} \qquad v = x - \operatorname{arc}\tau gx$$

$$= \frac{(x\operatorname{arc}\tau gx)^2}{2} - \left[(x - \operatorname{arc}\tau gx)\operatorname{arc}\tau gx - \int(x - \operatorname{arc}\tau gx)\frac{dx}{1+x^2}\right]$$

$$= \frac{(x\operatorname{arc}\tau gx)^2}{2} - x\operatorname{arc}\tau gx + (\operatorname{arc}\tau gx)^2 + \int\frac{xdx}{1+x^2} - \int\frac{\operatorname{arc}\tau gxdx}{1+x^2}$$

$$= \frac{(x\operatorname{arc}\tau gx)^2}{2} - x\operatorname{arc}\tau gx + (\operatorname{arc}\tau gx)^2 + \frac{1}{2}\ell\eta(1+x^2) - \frac{(\operatorname{arc}\tau gx)^2}{2} + c$$

4.49.- $\int(\operatorname{arcs}e\operatorname{n}x)^2 dx$

Solución.-

$$\therefore \quad u = (\operatorname{arcs}e\operatorname{n}x)^2 \qquad dv = dx$$
$$du = \frac{2\operatorname{arcs}e\operatorname{n}xdx}{\sqrt{1-x^2}} \qquad v = x$$

$\int(\operatorname{arcs}e\operatorname{n}x)^2 dx = x(\operatorname{arcs}e\operatorname{n}x)^2 - 2\int\operatorname{arcs}e\operatorname{n}x\frac{xdx}{\sqrt{1-x^2}}$, integral la cual se desarrolla por partes, esto es: $\therefore$

$$u = \operatorname{arcs}e\operatorname{n}x \qquad dv = \frac{xdx}{\sqrt{1-x^2}}$$
$$du = \frac{dx}{\sqrt{1-x^2}} \qquad v = -\sqrt{1-x^2}$$

$$= x(\operatorname{arcs}e\operatorname{n}x)^2 - 2\left[-\sqrt{1-x^2}\operatorname{arcs}e\operatorname{n}x + \int dx\right]$$

$$= x(\operatorname{arcs}e\operatorname{n}x)^2 + 2\sqrt{1-x^2}\operatorname{arcs}e\operatorname{n}x - 2x + c$$

4.50.- $\int \frac{\operatorname{arcsen} x}{x^2}dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = \operatorname{arcsen} x & dv = x^{-2}dx \\ du = \dfrac{dx}{\sqrt{1-x^2}} & v = -\dfrac{1}{x} \end{array}$$

$$\int \frac{\operatorname{arcsen} x}{x^2}dx = \int x^{-2} \operatorname{arcsen} x dx = -\frac{\operatorname{arcsen} x}{x} + \int \frac{dx}{x\sqrt{1-x^2}}$$

$$= -\frac{\operatorname{arcsen} x}{x} + \ell\eta \left| \frac{x}{1+\sqrt{1-x^2}} \right| + c$$

4.51.- $\int \frac{\operatorname{arcsen} \sqrt{x}}{\sqrt{1-x}}dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = \operatorname{arcsen} \sqrt{x} & dv = \dfrac{dx}{\sqrt{1-x}} \\ du = \dfrac{dx}{\sqrt{1-x}} \dfrac{1}{2\sqrt{x}} & v = -2\sqrt{1-x} \end{array}$$

$$\int \frac{\operatorname{arcsen} \sqrt{x}}{\sqrt{1-x}}dx = -2\sqrt{1-x} \operatorname{arcsen} \sqrt{x} + \int \frac{dx}{\sqrt{x}} = -2\sqrt{1-x} \operatorname{arcsen} \sqrt{x} + 2\sqrt{x} + c$$

4.52.- $\int \frac{\operatorname{sen}^2 x}{e^x}dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u = \operatorname{sen}^2 x & dv = e^{-x}dx \\ du = 2\operatorname{sen} x \cos x & v = -e^{-x} \end{array}$$

$$\int \frac{\operatorname{sen}^2 x}{e^x}dx = \int \operatorname{sen}^2 x e^{-x}dx = -e^{-x} \operatorname{sen}^2 x + 2\int \operatorname{sen} x \cos x e^{-x}dx$$

$= -e^{-x} \operatorname{sen}^2 x + \cancel{2}\int \frac{\operatorname{sen} 2x}{\cancel{2}} e^{-x}dx$, $*$Integral la cual se desarrolla por partes, esto es:

$$\therefore \quad \begin{array}{ll} u = \operatorname{sen} 2x & dv = e^{-x}dx \\ du = 2\cos 2x dx & v = -e^{-x} \end{array}$$

$= -e^{-x} \operatorname{sen}^2 x + 2\int \cos 2x e^{-x}dx$, Integral la cual se desarrolla por partes, esto es:

$$\therefore \quad \begin{array}{ll} u = \cos 2x & dv = e^{-x}dx \\ du = -2\operatorname{sen} 2x dx & v = -e^{-x} \end{array}$$

$$\int \operatorname{sen} 2x e^{-x}dx = -e^{-x} \operatorname{sen} 2x + 2\left(-e^{-x} \cos 2x - 2\int \operatorname{sen} 2x e^{-x}dx\right)$$

$\int \operatorname{sen} 2x e^{-x}dx = -e^{-x} \operatorname{sen} 2x - 2e^{-x} \cos 2x - 4\int \operatorname{sen} 2x e^{-x}dx$, de donde:

$$5\int \operatorname{sen} 2x e^{-x}dx = -e^{-x}(\operatorname{sen} 2x + 2\cos 2x) + c$$

$\int \operatorname{sen} 2x e^{-x} dx = \frac{-e^{-x}}{5}(\operatorname{sen} 2x + 2\cos 2x) + c$, Sustituyendo en: $*$

$$\int \frac{\operatorname{sen}^2 x dx}{e^x} = -e^{-x}\operatorname{sen}^2 x - \frac{2e^{-x}}{5}(\operatorname{sen} 2x + 2\cos 2x) + c$$

4.53.- $\int \tau g^2 x \sec^3 x dx = \int (\sec^2 x - 1)\sec^3 x dx = \int \sec^5 x dx(*) - \int \sec^3 x dx(**)$

Solución.-

$*\int \sec^5 x dx$, Sea: $u = \sec^3 x$ $\quad dv = \sec^2 x dx$; $\quad du = 3\sec^3 x\tau g x dx$ $\quad v = \tau g x$

$$\int \sec^5 x dx = \int \sec^3 x \sec^2 x dx = \sec^3 x\tau g x - 3\int \sec^3 x\tau g^2 x dx$$

$**\int \sec^3 x dx$, Sea: $u = \sec x$ $\quad dv = \sec^2 x dx$; $\quad du = \sec x\tau g x dx$ $\quad v = \tau g x$

$$\int \sec^3 x dx = \int \sec x \sec^2 x dx = \sec x\tau g x - \int \sec x\tau g^2 x dx = \sec x\tau g x - \int \sec x(\sec x^2 - 1)dx$$

$= \sec x\tau g x - \int \sec^3 x dx + \int \sec x dx$, luego: $2\int \sec^3 x dx = \sec x\tau g x + \int \sec x dx$

Esto es: $\int \sec^3 x dx = \frac{1}{2}(\sec x\tau g x + \ell n|\sec x\tau g x|) + c$, ahora bien:

$\int \tau g^2 x \sec^3 x dx = \int \sec^5 x dx - \int \sec^3 x dx$, con ($*$ y $**$)

$$\int \tau g^2 x \sec^3 x dx = \sec^3 x\tau g x - 3\int \sec^3 x\tau g^2 x dx - \frac{1}{2}(\sec x\tau g x + \ell n|\sec x\tau g x|) + c$$

De lo anterior: $4\int \tau g^2 x \sec^3 x dx = \sec^3 x\tau g x - \frac{1}{2}(\sec x\tau g x + \ell n|\sec x\tau g x|) + c$

Esto es: $\int \tau g^2 x \sec^3 x dx = \frac{1}{4}\sec^3 x\tau g x - \frac{1}{8}(\sec x\tau g x + \ell n|\sec x\tau g x|) + c$

4.54.- $\int x^3 \ell\eta^2 x dx$

Solución.-

$\therefore$ $u = \ell\eta^2 x$ $\quad dv = x^3 dx$; $\quad du = \frac{2\ell\eta x}{x}dx$ $\quad v = \frac{x^4}{4}$

$\int x^3 \ell\eta^2 x dx = \frac{x^4}{4}\ell\eta^2 x - \frac{1}{2}\int x^3 \ell\eta x dx$, integral la cual se desarrolla por partes, esto es:

$u = \ell\eta x$ $\quad dv = x^3 dx$; $\quad du = \frac{dx}{x}$ $\quad v = \frac{x^4}{4}$

$$= \frac{x^4}{4}\ell\eta^2 x - \frac{1}{2}\left(\frac{x^4}{4}\ell\eta x - \frac{1}{4}\int x^3 dx\right) = \frac{x^4}{4}\ell\eta^2 x - \frac{1}{8}x^4\ell\eta x + \frac{1}{8}\frac{x^4}{4} + c$$

$$= \frac{x^4}{4}\ell\eta^2 x - \frac{1}{8}x^4\ell\eta x + \frac{x^4}{32} + c$$

4.55.- $\int x\ell\eta(9+x^2)dx$

Solución.-

$$\therefore \quad u=\ell\eta(9+x^2) \qquad dv=xdx$$
$$du=\frac{2xdx}{9+x^2} \qquad v=\frac{x^2}{2}$$

$$\int x\ell\eta(9+x^2)dx=\frac{x^2}{2}\ell\eta(9+x^2)-\int\frac{x^3}{9+x^2}dx=\frac{x^2}{2}\ell\eta(9+x^2)-\int\left(x-\frac{9x}{x^2+9}\right)dx$$

$$=\frac{x^2}{2}\ell\eta(9+x^2)-\int xdx+9\int\frac{xdx}{9+x^2}=\frac{x^2}{2}\ell\eta(9+x^2)-\frac{x^2}{2}+\frac{9}{2}\ell\eta(x^2+9)+c$$

$$=\frac{x^2}{2}\left[\ell\eta(9+x^2)-1\right]+\frac{9}{2}\ell\eta(x^2+9)+c$$

4.56.- $\int \text{arcsen}\sqrt{x}dx$

Solución.-

$$\therefore \quad u=\text{arcsen}\sqrt{x}dx \qquad dv=dx$$
$$du=\frac{dx}{\sqrt{1-x^2}}\frac{1}{2\sqrt{x}} \qquad v=x$$

$$\int\text{arcsen}\sqrt{x}dx=x\,\text{arcsen}\sqrt{x}-\int\frac{xdx}{\sqrt{1-x}}\frac{1}{2\sqrt{x}}=x\,\text{arcsen}\sqrt{x}-\frac{1}{2}\int\frac{\sqrt{x}dx}{\sqrt{1-x}}$$

Para la integral resultante, se recomienda la siguiente sustitución: $\sqrt{1-x}=t$, de donde: $x=1-t^2$, y $dx=-2tdt$ (ver capitulo 9)

$$=x\,\text{arcsen}\sqrt{x}-\frac{1}{2}\frac{\sqrt{1-t^2}(-2\cancel{t}dt)dx}{\cancel{t}}=x\,\text{arcsen}\sqrt{x}+\sqrt{1-t^2}dt,$$ Se recomienda la sustitución: $t=\text{sen}\,\theta$, de donde: $\sqrt{1-t^2}=\cos\theta$, y $dt=\cos\theta d\theta$. Esto es:

$$=x\,\text{arcsen}\sqrt{x}+\int\cos^2\theta d\theta=x\,\text{arcsen}\sqrt{x}+\frac{1}{2}\int(1+\cos 2\theta)d\theta$$

$$=x\,\text{arcsen}\sqrt{x}+\frac{1}{2}\theta+\frac{1}{4}\text{sen}\,2\theta+c=x\,\text{arcsen}\sqrt{x}+\frac{1}{2}\theta+\frac{1}{2}\text{sen}\,\theta\cos\theta+c$$

$$=x\,\text{arcsen}\sqrt{x}+\frac{\text{arcsen}\,t}{2}+\frac{t}{2}\sqrt{1-t^2}+c=x\,\text{arcsen}\sqrt{x}+\frac{\text{arcsen}\sqrt{1-x}}{2}+\frac{\sqrt{1-x}}{2}\sqrt{x}+c$$

4.57.- $\int x\,\text{arc}\tau g(2x+3)dx$

Solución.-

$$\therefore \quad u=\text{arc}\tau g(2x+3) \qquad dv=xdx$$
$$du=\frac{2dx}{1+(2x+3)^2} \qquad v=\frac{x^2}{2}$$

$$\int x\,\text{arc}\tau g(2x+3)dx=\frac{x^2}{2}\text{arc}\tau g(2x+3)-\int\frac{x^2dx}{1+4x^2+12x+9}$$

$$=\frac{x^2}{2}\operatorname{arc}\tau g(2x+3)-\int\frac{x^2dx}{4x^2+12x+10}=\frac{x^2}{2}\operatorname{arc}\tau g(2x+3)-\int\left(\frac{1}{4}-\frac{3x+5/2}{4x^2+12x+10}\right)dx$$

$$=\frac{x^2}{2}\operatorname{arc}\tau g(2x+3)-\frac{1}{4}\int dx+\int\frac{3x+5/2}{4x^2+12x+10}dx$$

$$=\frac{x^2}{2}\operatorname{arc}\tau g(2x+3)-\frac{1}{4}x+3\int\frac{x+5/6}{4x^2+12x+10}dx$$

$$=\frac{x^2}{2}\operatorname{arc}\tau g(2x+3)-\frac{1}{4}x+\frac{3}{8}\int\frac{8x+40/6}{4x^2+12x+10}dx$$

$$=\frac{x^2}{2}\operatorname{arc}\tau g(2x+3)-\frac{1}{4}x+\frac{3}{8}\int\frac{8x+12-32/6}{4x^2+12x+10}dx$$

$$=\frac{x^2}{2}\operatorname{arc}\tau g(2x+3)-\frac{1}{4}x+\frac{3}{8}\int\frac{(8x+12)dx}{4x^2+12x+10}-\frac{3}{8}\frac{32}{6}\int\frac{dx}{4x^2+12x+10}$$

$$=\frac{x^2}{2}\operatorname{arc}\tau g(2x+3)-\frac{1}{4}x+\frac{3}{8}\ell\eta\left|4x^2+12x+10\right|-2\int\frac{dx}{4x^2+12x+10}$$

$$=\frac{x^2}{2}\operatorname{arc}\tau g(2x+3)-\frac{1}{4}x+\frac{3}{8}\ell\eta\left|4x^2+12x+10\right|-2\int\frac{dx}{(2x+3)^2+1}$$

$$=\frac{x^2}{2}\operatorname{arc}\tau g(2x+3)-\frac{1}{4}x+\frac{3}{8}\ell\eta\left|4x^2+12x+10\right|-\frac{2}{2}\int\frac{2dx}{(2x+3)^2+1}$$

$$=\frac{x^2}{2}\operatorname{arc}\tau g(2x+3)-\frac{1}{4}x+\frac{3}{8}\ell\eta\left|4x^2+12x+10\right|-\operatorname{arc}\tau g(2x+3)+c$$

$$=\frac{1}{2}\left[(x^2-2)\operatorname{arc}\tau g(2x+3)-\frac{1}{2}x+\frac{3}{4}\ell\eta\left|4x^2+12x+10\right|\right]+c$$

4.58.- $\int e^{\sqrt{x}}dx$

Solución.-

$$\therefore\quad \begin{array}{ll} u=e^{\sqrt{x}} & dv=dx \\ du=\dfrac{e^{\sqrt{x}}dx}{2\sqrt{x}} & v=x \end{array}$$

$\int e^{\sqrt{x}}dx=xe^{\sqrt{x}}-\frac{1}{2}\int\frac{xe^{\sqrt{x}}dx}{2\sqrt{x}}$, Se recomienda la sustitución: $z=\sqrt{x}, dz=\frac{dx}{2\sqrt{x}}$

$=xe^{\sqrt{x}}-\frac{1}{2}\int z^2e^zdz$, Esta integral resultante, se desarrolla por partes:

$$\therefore\quad \begin{array}{ll} u=z^2 & dv=e^zdz \\ du=2zdz & v=e^z \end{array}$$

$=xe^{\sqrt{x}}-\frac{1}{2}\left(z^2e^z-2\int ze^zdz\right)=xe^{\sqrt{x}}-\frac{z^2e^z}{2}+\int ze^zdz$, integral que se desarrolla por partes:

$$\therefore \quad \begin{matrix} u = z & dv = e^z dz \\ du = dz & v = e^z \end{matrix}$$

$$= xe^{\sqrt{x}} - \frac{z^2 e^z}{2} + ze^z - \int e^z dz = xe^{\sqrt{x}} - \frac{z^2 e^z}{2} + ze^z - e^z + c = xe^{\sqrt{x}} - \frac{xe^{\sqrt{x}}}{2} + \sqrt{x}e^{\sqrt{x}} - e^{\sqrt{x}} + c$$

$$= e^{\sqrt{x}}\left(\frac{x}{2} + \sqrt{x} - 1\right) + c$$

4.59.- $\int \cos^2(\ell\eta x)dx$

Solución.-

$$\therefore \quad \begin{matrix} u = \cos(2\ell\eta x) & dv = dx \\ du = -\frac{[\mathrm{s}e\,\mathrm{n}(2\ell\eta x)]2dx}{x} & v = x \end{matrix}$$

$$\int \cos^2(\ell\eta x)dx = \int \frac{1+\cos(2\ell\eta x)}{2}dx = \frac{1}{2}\int dx + \frac{1}{2}\int \cos(2\ell\eta x)dx$$

$$= \frac{1}{2}x + \frac{1}{2}\left[x\cos(2\ell\eta x) + 2\int \mathrm{s}e\,\mathrm{n}(2\ell\eta x)dx\right] = \frac{x}{2} + \frac{x}{2}\cos(2\ell\eta x) + \int \mathrm{s}e\,\mathrm{n}(2\ell\eta x)dx *$$

Integral que se desarrolla por partes:

$$\therefore \quad \begin{matrix} u = \mathrm{s}e\,\mathrm{n}(2\ell\eta x) & dv = dx \\ du = -\frac{[\cos(2\ell\eta x)]2dx}{x} & v = x \end{matrix}$$

$$* = \frac{x}{2} + \frac{x}{2}\cos(2\ell\eta x) + x\,\mathrm{s}e\,\mathrm{n}(2\ell\eta x) - 2\int \cos(2\ell\eta x)dx,$$

Dado que apareció nuevamente: $\int \cos(2\ell\eta x)dx$, igualamos: $*$

$\cancel{\frac{x}{2}} + \frac{1}{2}\int \cos(2\ell\eta x)dx = \cancel{\frac{x}{2}} + \frac{x}{2}\cos(2\ell\eta x) + x\,\mathrm{s}e\,\mathrm{n}(2\ell\eta x) - 2\int \cos(2\ell\eta x)dx$, de donde:

$$\frac{5}{2}\int \cos(2\ell\eta x)dx = \frac{x}{2}\cos(2\ell\eta x) + x\,\mathrm{s}e\,\mathrm{n}(2\ell\eta x) + c$$

$\frac{1}{2}\int \cos(2\ell\eta x)dx = \frac{x}{10}\cos(2\ell\eta x) + \frac{x}{5}\mathrm{s}e\,\mathrm{n}(2\ell\eta x) + c$, Por tanto:

$$\int \cos^2(\ell\eta x)dx = \frac{x}{2} + \frac{x}{10}\cos(2\ell\eta x) + \frac{x}{5}\mathrm{s}e\,\mathrm{n}(2\ell\eta x) + c$$

4.60.- $\int \frac{\ell\eta(\ell\eta x)}{x}dx$, Sustituyendo por: $w = \ell\eta x, dw = \frac{dx}{x}$, Se tiene:

Solución.-

$\int \frac{\ell\eta(\ell\eta x)}{x}dx = \int \ell\eta w dw$, Esta integral se desarrolla por partes:

$$\therefore \quad \begin{matrix} u = \ell\eta w & dv = dw \\ du = \frac{dw}{w} & v = w \end{matrix}$$

$$= w\ell\eta w - \int dw = w\ell\eta w - w + c = w(\ell\eta w - 1) + c = \ell\eta x\left[\ell\eta(\ell\eta x) - 1\right] + c$$

4.61.- $\int \ell\eta\left|x+1\right|dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u=\ell\eta\left|x+1\right| & dv=dx \\ du=\dfrac{dx}{x+1} & v=x \end{array}$$

$$\int \ell\eta\left|x+1\right|dx = x\ell\eta\left|x+1\right| - \int \frac{xdx}{x+1} = x\ell\eta\left|x+1\right| - \int\left(1-\frac{1}{x+1}\right)dx$$

$$= x\ell\eta\left|x+1\right| - x + \ell\eta\left|x+1\right| + c$$

4.62.- $\int x^2 e^x dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u=x^2 & dv=e^x dx \\ du=2xdx & v=e^x \end{array}$$

$$\int x^2 e^x dx = x^2 e^x - 2\int xe^x dx$$

Integral que se desarrolla nuevamente por partes:

$$\therefore \quad \begin{array}{ll} u=x & dv=e^x dx \\ du=dx & v=e^x \end{array}$$

$$= x^2 e^x - 2\left[xe^x - \int e^x dx\right] = x^2 e^x - 2xe^x + 2e^x + c$$

4.63.- $\int \cos^n xdx = \int \cos^{n-1} x\cos xdx$

Solución.-

$$\therefore \quad \begin{array}{ll} u=\cos^{n-1} x & dv=\cos xdx \\ du=(n-1)\cos^{n-2} x(-\operatorname{sen} x)dx & v=\operatorname{sen} x \end{array}$$

$$= \cos^{n-1} x\operatorname{sen} x + (n-1)\int \operatorname{sen}^2 x\cos^{n-2} xdx$$

$$= \cos^{n-1} x\operatorname{sen} x + (n-1)\int (1-\cos^2 x)\cos^{n-2} xdx$$

$= \cos^{n-1} x\operatorname{sen} x + (n-1)\int \cos^{n-2} xdx - (n-1)\int \cos^n xdx$, Se tiene:

$\int \cos^n xdx = \cos^{n-1} x\operatorname{sen} x + (n-1)\int \cos^{n-2} xdx - (n-1)\int \cos^n xdx$, Esto es:

$$n\int \cos^n xdx = \cos^{n-1} x\operatorname{sen} x + (n-1)\int \cos^{n-2} xdx$$

$$\int \cos^n xdx = \frac{\cos^{n-1} x\operatorname{sen} x}{n} + \frac{(n-1)}{n}\int \cos^{n-2} xdx$$

4.64.- $\int \operatorname{sen}^n xdx = \int \operatorname{sen}^{n-1} x\operatorname{sen} xdx$

Solución.-

$$\therefore \quad \begin{array}{ll} u=\operatorname{sen}^{n-1} x & dv=\operatorname{sen} xdx \\ du=(n-1)\operatorname{sen}^{n-2} x(\cos x)dx & v=-\cos x \end{array}$$

$$= -\operatorname{sen}^{n-1} x\cos x + (n-1)\int \cos^2 x\operatorname{sen}^{n-2} xdx$$

$$= -\operatorname{sen}^{n-1} x\cos x + (n-1)\int (1-\operatorname{sen}^2 x)\operatorname{sen}^{n-2} xdx$$

$= -\operatorname{sen}^{n-1} x\cos x + (n-1)\int \operatorname{sen}^{n-2} xdx - (n-1)\int \operatorname{sen}^{n} xdx$, Se tiene:

$$\int \operatorname{sen}^{n} xdx = -\operatorname{sen}^{n-1} x\cos x + (n-1)\int \operatorname{sen}^{n-2} xdx - (n-1)\int \operatorname{sen}^{n} xdx$$

$$n\int \operatorname{sen}^{n} xdx = -\operatorname{sen}^{n-1} x\cos x + (n-1)\int \operatorname{sen}^{n-2} xdx$$

$$\int \operatorname{sen}^{n} xdx = \frac{-\operatorname{sen}^{n-1} x\cos x}{n} + \frac{(n-1)}{n}\int \operatorname{sen}^{n-2} xdx$$

4.65.- $\int x^m(\ell\eta x)^n dx = x^{m+1}(\ell\eta x)^n - n\int x^m(\ell\eta x)^{n-1}dx - m\int x^m(\ell\eta x)^n dx$

Solución.-

$$\therefore \quad u = x^m(\ell\eta x)^n, \quad du = x^m n(\ell\eta x)^{n-1}\frac{dx}{x} + mx^{m-1}(\ell\eta x)^n dx \qquad dv = dx, \quad v = x$$

Se tiene: $(m+1)\int x^m(\ell\eta x)^n dx = x^{m+1}(\ell\eta x)^n - n\int x^m(\ell\eta x)^{n-1}dx$

$$\int x^m(\ell\eta x)^n dx = \frac{x^{m+1}(\ell\eta x)^n}{(m+1)} - \frac{n}{(m+1)}\int x^m(\ell\eta x)^{n-1}dx$$

4.66.- $\int x^3(\ell\eta x)^2 dx$

Solución.-

Puede desarrollarse como caso particular del ejercicio anterior, haciendo: $m = 3, n = 2$

$$\int x^3(\ell\eta x)^2 dx = \frac{x^{3+1}(\ell\eta x)^2}{3+1} - \frac{2}{3+1}\int x^3(\ell\eta x)^{2-1}dx = \frac{x^4(\ell\eta x)^2}{4} - \frac{1}{2}\int x^3(\ell\eta x)dx *$$

Para la integral resultante: $\int x^3(\ell\eta x)dx *$

$$\int x^3(\ell\eta x)dx = \frac{x^4(\ell\eta x)}{4} - \frac{1}{4}\int x^3 dx = \frac{x^4(\ell\eta x)}{4} - \frac{x^4}{16} + c$$, introduciendo en: $*$

$$\int x^3(\ell\eta x)^2 dx = \frac{x^4(\ell\eta x)^2}{4} - \frac{x^4}{8}(\ell\eta x) + \frac{x^4}{32} + c$$

4.67.- $\int x^n e^x dx$

Solución.-

$$\therefore \quad u = x^n, \quad du = nx^{n-1}dx \qquad dv = e^x dx, \quad v = e^x$$

$$\int x^n e^x dx = x^n e^x - n\int x^{n-1}e^x dx$$

4.68.- $\int x^3 e^x dx$

Solución.-

$$\therefore \quad u = x^3, \quad du = 3x^2 dx \qquad dv = e^x dx, \quad v = e^x$$

Puede desarrollarse como el ejercicio anterior, haciendo: $n = 3$

$\int x^3 e^x dx = x^3 e^x - 3\int x^2 e^x dx *$, Además:

$*\int x^2e^xdx = x^2e^x - 2\int xe^xdx**$, Además: $\int xe^xdx = xe^x - \int e^xdx = xe^x - e^x + c$

Reemplazando en$**$ y luego en $*$:

$$\int x^3e^xdx = x^3e^x - 3\left[x^2e^x - 2(xe^x - e^x)\right] + c$$

$$\int x^3e^xdx = e^x(x^3 - 3x^2 + 6x - 6) + c$$

4.69.- $\int \sec^n xdx = \int \sec^{n-2} x\sec^2 xdx$

Solución.-

$$\therefore \begin{array}{ll} u = \sec^{n-2} x & dv = \sec^2 xdx \\ du = (n-2)\sec^{n-3} x\sec x\tau gxdx & v = \tau gx \end{array}$$

$$= \sec^{n-2} x\tau gx - (n-2)\int \tau g^2 x\sec^{n-2} xdx = \sec^{n-2} x\tau gx - (n-2)\int (\sec^2 x - 1)\sec^{n-2} xdx$$

$= \sec^{n-2} x\tau gx - (n-2)\int \sec^n xdx + (n-2)\int \sec^{n-2} xdx$, Se tiene:

$$\int \sec^n xdx = \sec^{n-2} x\tau gx - (n-2)\int \sec^n xdx + (n-2)\int \sec^{n-2} xdx$$

$$(n-1)\int \sec^n xdx = \sec^{n-2} x\tau gx + (n-2)\int \sec^{n-2} xdx$$

$$\int \sec^n xdx = \frac{\sec^{n-2} x\tau gx}{(n-1)} + \frac{(n-2)}{(n-1)}\int \sec^{n-2} xdx$$

4.70.- $\int \sec^3 xdx$

Solución.-

Puede desarrollarse como caso particular del ejercicio anterior, haciendo: $n = 3$

$$\int \sec^3 xdx = \frac{\sec^{3-2} x\tau gx}{3-1} + \frac{3-2}{3-1}\int \sec^{3-2} xdx = \frac{\sec x\tau gx}{2} + \frac{1}{2}\int \sec xdx$$

$$= \frac{\sec x\tau gx}{2} + \frac{1}{2}\ell\eta\left|\sec x\tau gx\right| + c$$

4.71.- $\int x\ell\eta xdx$

Solución.-

$$\therefore \begin{array}{ll} u = \ell\eta x & dv = xdx \\ du = \dfrac{dx}{x} & v = \dfrac{x^2}{2} \end{array}$$

$$\int x\ell\eta xdx = \frac{x^2}{2}\ell\eta x - \int \frac{xdx}{2} = \frac{x^2}{2}\ell\eta x - \frac{1}{4}x^2 + c$$

4.72.- $\int x^n\ell\eta\left|ax\right|dx, n \neq -1$

Solución.-

$$\therefore \begin{array}{ll} u = \ell\eta\left|ax\right| & dv = xdx \\ du = \dfrac{dx}{x} & v = \dfrac{x^{n+1}}{n+1} \end{array}$$

$$\int x^n\ell\eta\left|ax\right|dx = \frac{x^{n+1}}{n+1}\ell\eta\left|ax\right| - \frac{1}{n+1}\int x^ndx = \frac{x^{n+1}}{n+1}\ell\eta\left|ax\right| - \frac{x^{n+1}}{(n+1)^2} + c$$

4.73.- $\int \operatorname{arcs}e\operatorname{n} axdx$

Solución.-

$$\therefore \quad \begin{aligned} u &= \operatorname{arcs}e\operatorname{n} ax \\ du &= \frac{adx}{\sqrt{1-a^2x^2}} \end{aligned} \qquad \begin{aligned} dv &= dx \\ v &= x \end{aligned}$$

$$\int \operatorname{arcs}e\operatorname{n} axdx = x\operatorname{arcs}e\operatorname{n} ax - \int \frac{axdx}{\sqrt{1-a^2x^2}} = x\operatorname{arcs}e\operatorname{n} ax + \frac{1}{2a}\int \frac{(-2a^2x)dx}{\sqrt{1-a^2x^2}}$$

$$= x\operatorname{arcs}e\operatorname{n} ax + \frac{1}{2a}\frac{(1-a^2x^2)^{1/2}}{1/2} + c = x\operatorname{arcs}e\operatorname{n} ax + \frac{1}{a}\sqrt{1-a^2x^2} + c$$

4.74.- $\int x\operatorname{s}e\operatorname{n} axdx$

Solución.-

$$\therefore \quad \begin{aligned} u &= x \\ du &= dx \end{aligned} \qquad \begin{aligned} dv &= \operatorname{s}e\operatorname{n} axdx \\ v &= -\frac{1}{a}\cos ax \end{aligned}$$

$$\int x\operatorname{s}e\operatorname{n} axdx = -\frac{x}{a}\cos ax + \frac{1}{a}\int \cos axdx = -\frac{x}{a}\cos ax + \frac{1}{a^2}\operatorname{s}e\operatorname{n} ax + c$$

$$= \frac{1}{a^2}\operatorname{s}e\operatorname{n} ax - \frac{x}{a}\cos ax + c$$

4.75.- $\int x^2 \cos axdx$

Solución.-

$$\therefore \quad \begin{aligned} u &= x^2 \\ du &= 2xdx \end{aligned} \qquad \begin{aligned} dv &= \cos axdx \\ v &= -\frac{1}{a}\operatorname{s}e\operatorname{n} ax \end{aligned}$$

$\int x^2 \cos axdx = \frac{x^2}{a}\operatorname{s}e\operatorname{n} ax - \frac{2}{a}\int x\operatorname{s}e\operatorname{n} axdx$, aprovechando el ejercicio anterior:

$$= \frac{x^2}{a}\operatorname{s}e\operatorname{n} ax - \frac{2}{a}\left(\frac{1}{a^2}\operatorname{s}e\operatorname{n} ax - \frac{x}{a}\cos ax\right) + c = \frac{x^2}{a}\operatorname{s}e\operatorname{n} ax - \frac{2}{a^3}\operatorname{s}e\operatorname{n} ax - \frac{2x}{a^2}\cos ax + c$$

4.76.- $\int x\sec^2 axdx$

Solución.-

$$\therefore \quad \begin{aligned} u &= x \\ du &= dx \end{aligned} \qquad \begin{aligned} dv &= \sec^2 axdx \\ v &= \frac{1}{a}\tau gax \end{aligned}$$

$$\int x\sec^2 axdx = \frac{x}{a}\tau gax - \frac{1}{a}\int \tau gaxdx = \frac{x}{a}\tau gax - \frac{1}{a}\frac{1}{a}\ell\eta\left|\sec ax\right| + c$$

$$= \frac{x}{a}\tau gax - \frac{1}{a^2}\ell\eta\left|\sec ax\right| + c$$

4.77.- $\int \cos(\ell\eta x)dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u=\cos(\ell\eta x) & dv=dx \\ du=-\dfrac{\operatorname{sen}(\ell\eta x)}{x}dx & v=x \end{array}$$

$\int \cos(\ell\eta x)dx = x\cos(\ell\eta x)+\int \operatorname{sen}(\ell\eta x)dx$, aprovechando el ejercicio:4.43

$\int \operatorname{sen}(\ell\eta x)dx = \frac{x}{2}\left[\operatorname{sen}(\ell\eta x)-\cos(\ell\eta x)\right]+c$, Luego:

$$= x\cos(\ell\eta x)+\frac{x}{2}\left[\operatorname{sen}(\ell\eta x)-\cos(\ell\eta x)\right]+c = x\cos(\ell\eta x)+\frac{x}{2}\operatorname{sen}(\ell\eta x)-\frac{x}{2}\cos(\ell\eta x)+c$$

$$=\frac{x}{2}\left[\cos(\ell\eta x)+\operatorname{sen}(\ell\eta x)\right]+c$$

4.78.- $\int \ell\eta(9+x^2)dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u=\ell\eta(9+x^2) & dv=dx \\ du=\dfrac{2xdx}{9+x^2} & v=x \end{array}$$

$$\int \ell\eta(9+x^2)dx = x\ell\eta(9+x^2)-2\int \frac{x^2dx}{9+x^2} = x\ell\eta(9+x^2)-2\int\left(1-\frac{9}{9+x^2}\right)dx$$

$$= x\ell\eta(9+x^2)-2\int dx+18\int\frac{dx}{9+x^2} = x\ell\eta(9+x^2)-2x+6\operatorname{arc}\tau g\, {}^{x}\!/_{3}+c$$

4.79.- $\int x\cos(2x+1)dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u=x & dv=\cos(2x+1)dx \\ du=dx & v=\dfrac{1}{2}\operatorname{sen}(2x+1) \end{array}$$

$$\int x\cos(2x+1)dx = \frac{x}{2}\operatorname{sen}(2x+1)-\frac{1}{2}\int \operatorname{sen}(2x+1)dx$$

$$=\frac{x}{2}\operatorname{sen}(2x+1)+\frac{1}{4}\cos(2x+1)+c$$

4.80.- $\int x\operatorname{arcsec} x dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u=\operatorname{arcsec} x & dv=xdx \\ du=\dfrac{dx}{x\sqrt{x^2-1}} & v=\dfrac{x^2}{2} \end{array}$$

$$\int x\operatorname{arcsec} x dx = \frac{x^2}{2}\operatorname{arcsec} x-\frac{1}{2}\int\frac{xdx}{\sqrt{x^2-1}} = \frac{x^2}{2}\operatorname{arcsec} x-\frac{1}{2}\sqrt{x^2-1}+c$$

4.81.- $\int \operatorname{arcsec}\sqrt{x}dx$

Solución.-

$$\therefore \quad u = \operatorname{arc}\sec x \qquad dv = dx$$
$$du = \frac{1}{2}\frac{dx}{x\sqrt{x-1}} \qquad v = x$$

$$\int \operatorname{arc}\sec\sqrt{x}dx = x\operatorname{arc}\sec x - \frac{1}{2}\int\frac{dx}{\sqrt{x-1}} = x\operatorname{arc}\sec x - \sqrt{x-1} + c$$

4.82.- $\int\sqrt{a^2-x^2}dx = \int\frac{a^2-x^2}{\sqrt{a^2-x^2}}dx = a^2\int\frac{dx}{\sqrt{a^2-x^2}} - \int\frac{x^2dx}{\sqrt{a^2-x^2}}$

$= a^2 \operatorname{arcs}e\operatorname{n}\frac{x}{a} - \int x\frac{xdx}{\sqrt{a^2-x^2}} *$, integral que se desarrolla por partes:

Solución.-

$$\therefore \quad u = x \qquad dv = \frac{xdx}{\sqrt{a^2-x^2}}$$
$$du = dx \qquad v = -\sqrt{a^2-x^2}$$

$* = a^2 \operatorname{arcs}e\operatorname{n}\frac{x}{a} - \left(-x\sqrt{a^2-x^2} + \int\sqrt{a^2-x^2}dx\right)$, Se tiene que:

$\int\sqrt{a^2-x^2}dx = a^2 \operatorname{arcs}e\operatorname{n}\frac{x}{a} + x\sqrt{a^2-x^2} - \int\sqrt{a^2-x^2}dx$, De donde:

$$2\int\sqrt{a^2-x^2}dx = a^2 \operatorname{arcs}e\operatorname{n}\frac{x}{a} + x\sqrt{a^2-x^2} + c$$

$$\int\sqrt{a^2-x^2}dx = \frac{a^2}{2}\operatorname{arcs}e\operatorname{n}\frac{x}{a} + \frac{x}{2}\sqrt{a^2-x^2} + c$$

4.83.- $\int \ell\eta|1-x|dx$

Solución.-

$$\therefore \quad u = \ell\eta|1-x| \qquad dv = dx$$
$$du = -\frac{dx}{1-x} \qquad v = x$$

$$\int \ell\eta|1-x|dx = x\ell\eta|1-x| - \int\frac{xdx}{x-1} = x\ell\eta|1-x| - \int\left(1+\frac{1}{x-1}\right)dx$$

$$= x\ell\eta|1-x| - \int dx - \int\frac{dx}{x-1} = x\ell\eta|1-x| - x - \ell\eta|x-1| + c$$

4.84.- $\int \ell\eta(x^2+1)dx$

Solución.-

$$\therefore \quad u = \ell\eta(x^2+1) \qquad dv = dx$$
$$du = \frac{2xdx}{x^2+1} \qquad v = x$$

$$\int \ell\eta(x^2+1)dx = x\ell\eta(x^2+1) - 2\int\frac{x^2dx}{x^2+1} = x\ell\eta(x^2+1) - 2\int\left(1-\frac{1}{x^2+1}\right)dx$$

$$= x\ell\eta(x^2+1) - 2x + 2\operatorname{arc}\tau gx + c$$

4.85.- $\int \operatorname{arc}\tau g\sqrt{x}dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u=\operatorname{arc}\tau g\sqrt{x} & dv=dx \\ du=\dfrac{dx}{1+x}\dfrac{1}{2\sqrt{x}} & v=x \end{array}$$

$\int \operatorname{arc}\tau g\sqrt{x}dx = x\operatorname{arc}\tau g\sqrt{x}-\frac{1}{2}\int\frac{\sqrt{x}dx}{1+x}*$ En la integral resultante, se recomienda la sustitución: $\sqrt{x}=t$, esto es $x=t^2, dx=2tdt$

$$=x\operatorname{arc}\tau g\sqrt{x}-\frac{1}{\cancel{2}}\int\frac{t\cancel{2}tdt}{1+t^2}=x\operatorname{arc}\tau g\sqrt{x}-\int\frac{t^2dt}{1+t^2}=x\operatorname{arc}\tau g\sqrt{x}-\int\left(1-\frac{1}{1+t^2}\right)dt$$

$$=x\operatorname{arc}\tau g\sqrt{x}-\int dt+\int\frac{dt}{1+t^2}=x\operatorname{arc}\tau g\sqrt{x}-t+\operatorname{arc}\tau gt+c$$

$$=x\operatorname{arc}\tau g\sqrt{x}-\sqrt{x}+\operatorname{arc}\tau g\sqrt{x}+c$$

4.86.- $\int\frac{x\operatorname{arcsen}x}{\sqrt{1-x^2}}dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u=\operatorname{arcsen}x & dv=\dfrac{xdx}{\sqrt{1-x^2}} \\ du=\dfrac{dx}{\sqrt{1-x^2}} & v=-\sqrt{1-x^2} \end{array}$$

$$\int\frac{x\operatorname{arcsen}x}{\sqrt{1-x^2}}dx=-\sqrt{1-x^2}\operatorname{arcsen}x+\int dx=-\sqrt{1-x^2}\operatorname{arcsen}x+x+c$$

4.87.- $\int x\operatorname{arc}\tau g\sqrt{x^2-1}dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u=\operatorname{arc}\tau g\sqrt{x^2-1} & dv=xdx \\ du=\dfrac{dx}{x\sqrt{x^2-1}} & v=\dfrac{x^2}{2} \end{array}$$

$$\int x\operatorname{arc}\tau g\sqrt{x^2-1}dx=\frac{x^2}{2}\operatorname{arc}\tau g\sqrt{x^2-1}-\frac{1}{2}\int\frac{xdx}{\sqrt{x^2-1}}=\frac{x^2}{2}\operatorname{arc}\tau g\sqrt{x^2-1}-\frac{1}{2}\sqrt{x^2-1}+c$$

4.88.- $\int\frac{x\operatorname{arc}\tau gx}{(x^2+1)^2}dx$

Solución.-

$$\therefore \quad \begin{array}{ll} u=\operatorname{arc}\tau gx & dv=\dfrac{xdx}{(x^2+1)^2} \\ du=\dfrac{dx}{x^2+1} & v=\dfrac{-1}{2(x^2+1)} \end{array}$$

$\int\frac{x\operatorname{arc}\tau gx}{(x^2+1)^2}dx=\frac{-\operatorname{arc}\tau gx}{2(x^2+1)}+\frac{1}{2}\int\frac{dx}{(x^2+1)^2}*$, Se recomienda la siguiente sustitución:

$x = \tau g\theta$, de donde: $dx = \sec^2\theta d\theta$; $x^2 + 1 = \sec^2\theta$

$$* = \frac{-\operatorname{arc}\tau gx}{2(x^2+1)} + \frac{1}{2}\int\frac{\sec^2\theta d\theta}{\sec^4\theta} = -\frac{\operatorname{arc}\tau gx}{2(x^2+1)} + \frac{1}{2}\int\cos^2\theta d\theta = -\frac{\operatorname{arc}\tau gx}{2(x^2+1)} + \frac{1}{2}\int\frac{1+\cos 2\theta d\theta}{2}$$

$$= -\frac{\operatorname{arc}\tau gx}{2(x^2+1)} + \frac{1}{4}\theta + \frac{1}{8}\operatorname{sen}2\theta + c = -\frac{\operatorname{arc}\tau gx}{2(x^2+1)} + \frac{1}{4}\operatorname{arc}\tau gx + \frac{1}{4}\operatorname{sen}\theta\cos\theta + c$$

$$= -\frac{\operatorname{arc}\tau gx}{2(x^2+1)} + \frac{1}{4}\operatorname{arc}\tau gx + \frac{1}{4}\frac{x}{\sqrt{x^2+1}}\frac{1}{\sqrt{x^2+1}} + c$$

$$= -\frac{\operatorname{arc}\tau gx}{2(x^2+1)} + \frac{1}{4}\operatorname{arc}\tau gx + \frac{x}{4(x^2+1)} + c$$

4.89.- $\int \operatorname{arcsen} x \frac{xdx}{\sqrt{(1-x^2)^3}}$

Solución.-

$u = \operatorname{arcsen} x$ $\qquad$ $dv = \dfrac{xdx}{(1-x^2)^{3/2}}$

$\therefore$ $\quad du = \dfrac{dx}{\sqrt{1-x^2}}$ $\qquad$ $v = \dfrac{1}{\sqrt{1-x^2}}$

$$\int \operatorname{arcsen} x \frac{xdx}{\sqrt{(1-x^2)^3}} = \frac{\operatorname{arcsen} x}{\sqrt{1-x^2}} - \int\frac{dx}{1-x^2} = \frac{\operatorname{arcsen} x}{\sqrt{1-x^2}} + \frac{1}{2}\ell\eta\left|\frac{1-x}{1+x}\right| + c$$

4.90.- $\int x^2\sqrt{1-x}dx$

Solución.-

$u = \sqrt{1-x}$ $\qquad$ $dv = x^2dx$

$\therefore$ $\quad du = -\dfrac{dx}{2\sqrt{1-x}}$ $\qquad$ $v = \dfrac{x^3}{3}$

$\int x^2\sqrt{1-x}dx = \dfrac{x^3}{3}\sqrt{1-x} + \dfrac{1}{6}\int\dfrac{x^3dx}{\sqrt{1-x}}*$, Se recomienda usar la siguiente sustitución: $\sqrt{1-x} = t$, o sea: $x = 1-t^2$, De donde: $dx = -2tdt$

$$= \frac{x^3}{3}\sqrt{1-x} + \frac{1}{\cancel{6}}\int\frac{(1-t^2)^3(-\cancel{2}\cancel{t}dt)}{\cancel{t}} = \frac{x^3}{3}\sqrt{1-x} - \frac{1}{3}\int(1-t^2)^3dt$$

$$= \frac{x^3}{3}\sqrt{1-x} - \frac{1}{3}\int(1-3t^2+3t^4-t^6)dt = \frac{x^3}{3}\sqrt{1-x} - \frac{1}{3}(t - t^3 + \frac{3t^5}{5} - \frac{t^7}{7}) + c$$

$$= \frac{x^3}{3}\sqrt{1-x} - \frac{1}{3}\left[\sqrt{1-x} - (1-x)\sqrt{1-x} + \frac{3}{5}(1-x)^2\sqrt{1-x} - \frac{3}{7}(1-x)^3\sqrt{1-x}\right] + c$$

$$= \frac{\sqrt{1-x}}{3}\left[x^3 - 1 - (1-x) + \frac{3}{5}(1-x)^2 - \frac{1}{7}(1-x)^3\right] + c$$