

Ejercicios de derivadas

1. Halla la derivada de la función $f(x) = \frac{2}{x+1}$ en el punto $x = 3$, aplicando la definición de derivada.

$$1^{\circ} f(a) \Rightarrow f(3) = \frac{2}{3+1} = \frac{2}{4} = \frac{1}{2}$$

$$2^{\circ} f(a+h) \Rightarrow f(3+h) = \frac{2}{3+h+1} = \frac{2}{4+h}$$

$$3^{\circ} f(a+h) - f(a) \Rightarrow f(3+h) - f(3) = \frac{2}{4+h} - \frac{1}{2} = \frac{4-1 \cdot (4+h)}{2(4+h)} = \frac{-h}{2(4+h)}$$

$$4^{\circ} f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} \Rightarrow f'(3) = \lim_{h \rightarrow 0} \frac{\frac{-h}{2(4+h)}}{h} = \lim_{h \rightarrow 0} \frac{-h}{2h(4+h)} = \lim_{h \rightarrow 0} \frac{-1}{2(4+h)} = \frac{-1}{8}$$

2. Calcula la derivada de las siguientes funciones :

a) $f(x) = \sin x^2$ $f(x) = \sin x^2 \rightarrow f = x^2 \rightarrow f' = 2x \Rightarrow f'(x) = 2x \cos x^2$ (seno)

b) $f(x) = \sin^2 x$ $f(x) = (\sin x)^2 \rightarrow f = \sin x \rightarrow f' = \cos x \Rightarrow f'(x) = 2 \sin x \cdot \cos x$ (potencial)

c) $f(x) = (3x^2 - 2)^5$ $f(x) = (3x^2 - 2)^5 \rightarrow f = 3x^2 - 2 \rightarrow f' = 6x \Rightarrow f'(x) = 30x (3x^2 - 2)^4$

d) $f(x) = \sqrt[3]{x^2 - 3}$ $f(x) = \sqrt[3]{x^2 - 3} \rightarrow f = x^2 - 3 \rightarrow f' = 2x \Rightarrow f'(x) = \frac{2x}{3 \sqrt[3]{(x^2 - 3)^2}}$

e) $f(x) = e^{3x+2}$ $f(x) = e^{3x+2} \rightarrow f = 3x+2 \rightarrow f' = 3 \Rightarrow f'(x) = 3 \cdot e^{3x+2}$

f) $\log_3(4x+1)$ $\log_3(4x+1) \rightarrow f = 4x+1 \rightarrow f' = 4 \Rightarrow f'(x) = \frac{4}{(4x+1) \cdot \ln 3}$

g) $f(x) = \sqrt{x^2 - 3x}$ $f(x) = \sqrt{x^2 - 3x} \rightarrow f = x^2 - 3x \rightarrow f' = 2x - 3 \Rightarrow f'(x) = \frac{2x - 3}{2\sqrt{x^2 - 3x}}$

h) $f(x) = \sin^2(2x^3 + 2x)$ $f(x) = \left[\underbrace{\sin(2x^3 + 2x)}_f \right]^2$

1. potencial $\rightarrow f'(x) = 2 \cdot [\sin(2x^3 + 2x)] \cdot f' \dots \dots f'(x) = 2 \cdot [\sin(2x^3 + 2x)] \cdot \cos(2x^3 + 2x) \cdot (6x^2 + 2)$

2. seno $\rightarrow f'(x) = \cos(2x^3 + 2x) \cdot f' \dots \dots \dots f'(x) = \cos(2x^3 + 2x) \cdot (6x^2 + 2)$ esto es f' de 1.

3. suma $\rightarrow f'(x) = 6x^2 + 2$ esto es f' de 2. Completamos.

3. Halla las funciones derivadas de las siguientes funciones :

$$\text{a) } f(x) = \frac{1}{7x+1} \Rightarrow f'(x) = \frac{0 \cdot (7x+1) - 1 \cdot 7}{(7x+1)^2} \Rightarrow f'(x) = -\frac{7}{(7x+1)^2}$$

$$\text{b) } f(x) = x^{2/3} \Rightarrow f'(x) = \frac{2}{3} x^{\frac{2}{3}-1} \Rightarrow f'(x) = \frac{2}{3} x^{-\frac{1}{3}} \Rightarrow f'(x) = \frac{2}{3\sqrt[3]{x}}$$

$$\text{c) } f(x) = x^2 \cdot x^{\frac{1}{3}} \quad f(x) = x^{\frac{7}{3}} \Rightarrow f'(x) = \frac{7}{3} x^{\frac{7}{3}-1} \Rightarrow f'(x) = \frac{7}{3} x^{\frac{4}{3}} \Rightarrow f'(x) = \frac{7\sqrt[3]{x^4}}{3}$$

$$\text{d) } f(x) = (x - \sqrt{1-x^2})^2 \Rightarrow f'(x) = 2(x - \sqrt{1-x^2}) \left(1 + \frac{2x}{2\sqrt{1-x^2}} \right) \Rightarrow f'(x) = 2(x - \sqrt{1-x^2}) \left(1 + \frac{x}{\sqrt{1-x^2}} \right)$$

$$\text{e) } f(x) = \frac{e^{2x}}{x^2} \Rightarrow f'(x) = \frac{(2 \cdot e^{2x}) \cdot x^2 - (e^{2x} \cdot 2x)}{x^4} \Rightarrow f'(x) = \frac{2x \cdot e^{2x} \cdot (x-1)}{x^4} \Rightarrow f'(x) = \frac{2 \cdot e^{2x} \cdot (x-1)}{x^3}$$

$$\text{f) } f(x) = x \cos 2x \Rightarrow f'(x) = 1 \cdot \cos 2x + x(-\sin 2x) \cdot 2 \Rightarrow f'(x) = \cos 2x - 2x \sin 2x$$

$$\text{g) } f(x) = \ln \cos x \Rightarrow f'(x) = \frac{-\sin x}{\cos x} \Rightarrow f'(x) = -\tan x$$

$$\text{h) } f(x) = \sqrt{\frac{1-x}{1+x}} \Rightarrow 1. f'(x) = \frac{\frac{-2}{(1+x)^2}}{2\sqrt{\frac{1-x}{1+x}}} \Rightarrow f'(x) = -\frac{1}{(1+x)^2 \sqrt{\frac{1-x}{1+x}}}$$

Derivamos: $\frac{1-x}{1+x} = \frac{-1(1+x) - (1-x) \cdot 1}{(1+x)^2} \Rightarrow \frac{-1-x-1+x}{(1+x)^2} = \frac{-2}{(1+x)^2}$ lo colocamos en f' de 1.

$$\text{i) } f(x) = e^{x^2} \cdot \tan x \Rightarrow f'(x) = 2x \cdot e^{x^2} \cdot \tan x + e^{x^2} \cdot \frac{1}{\cos^2 x} \Rightarrow$$

$$f'(x) = e^{x^2} \left(2x \cdot \tan x + \frac{1}{\cos^2 x} \right) \Rightarrow f'(x) = e^{x^2} (2x \cdot \tan x + 1 + \tan^2 x)$$

$$f(x) = (1 + 3x^4)^5 \Rightarrow f'(x) = 60x^3(1 + 3x^4)^4$$

$$f(x) = (1 + x + x^2)^3 \Rightarrow f'(x) = 3(2x + 1)(1 + x + x^2)^2$$

$$f(x) = \frac{1}{(x^2 - 1)^4} \Rightarrow f'(x) = -8x(x^2 - 1)^{-5}$$

$$f(x) = \frac{1}{x-1} + \frac{2}{(x-1)^2} + \frac{3}{(x-1)^3} \Rightarrow f'(x) = -(x-1)^{-2} - 4(x-1)^{-3} - 9(x-1)^{-4}$$

$$f(x) = \sqrt{1-x^2} \Rightarrow f'(x) = \frac{-x}{\sqrt{1-x^2}}$$

$$f(x) = \sqrt[3]{2+5x^2} \Rightarrow f'(x) = \frac{10x}{3\sqrt[3]{(2+5x^2)^2}}$$

$$f(x) = \frac{1}{\sqrt[3]{(x^3-2)^2}} \Rightarrow f'(x) = -2x^2(x^3-2)^{-5/3}$$

$$f(x) = (5x^3+1)^3 \cdot (x^2+x+1)^4 \Rightarrow$$

$$f'(x) = 45x^2(5x^3+1)^2(x^2+x+1)^4 + 4(2x+1)(5x^3+1)^3(x^2+x+1)^3$$

$$f(x) = (5 - 3\cos x)^4 \Rightarrow f'(x) = 12\sin x(5 - 3\cos x)^3$$

$$f(x) = \sin x + \sin^2 x + \sin^3 x \Rightarrow f'(x) = \cos x(1 + 2\sin x + 3\sin^2 x)$$

$$f(x) = \frac{1}{\arctan x} \Rightarrow f'(x) = \frac{-1}{(1+x^2)(\arctan x)^2}$$

$$f(x) = \sin^3 x - \cos^3 x \Rightarrow f'(x) = 3\sin x \cos x (\sin x + \cos x)$$

$$f(x) = \frac{1}{3\cos^3 x} - \frac{1}{\cos x} \Rightarrow f'(x) = \sin x \left[\frac{1}{\cos^4 x} - \frac{1}{\cos^2 x} \right]$$

$$f(x) = \sin(x^2) \Rightarrow f'(x) = 2x \cos(x^2)$$

$$f(x) = (1 + \sin 5x)^4 \Rightarrow f'(x) = 20 \cos 5x (1 + \sin 5x)^3$$

$$f(x) = \sqrt{x e^x + x} \Rightarrow f'(x) = \frac{e^x + x e^x + 1}{2\sqrt{x e^x + x}}$$

$$f(x) = \sqrt[3]{2^x + x} \Rightarrow f'(x) = \frac{2^x \ln 2 + 1}{3\sqrt[3]{(2^x + x)^2}}$$

$$f(x) = \ln(\ln x) \Rightarrow f'(x) = \frac{1}{x \ln x}$$

$$f(x) = \arccos \sqrt{x} \Rightarrow f'(x) = \frac{-1}{2\sqrt{x-x^2}}$$

$$f(x) = \arcsin\left(\frac{1}{x^2}\right) \Rightarrow f'(x) = \frac{-2}{x\sqrt{x^4-1}}$$

$$f(x) = \frac{1 + \cos 2x}{1 - \cos 2x} \Rightarrow f'(x) = \frac{-4 \sin 2x}{(1 - \cos 2x)^2}$$

$$f(x) = \operatorname{Arctg}\left(\frac{1}{x}\right) \Rightarrow f'(x) = \frac{-1}{1+x^2}$$

$$f(x) = \frac{1 + \sin^2 x}{1 + \cos^2 x} \Rightarrow f'(x) = \frac{6 \sin x \cos x}{(1 + \cos^2 x)^2}$$

$$f(x) = x^2 \cdot e^{x^6} \Rightarrow f'(x) = x \cdot e^{x^6} (2 + 3x^3)$$

$$f(x) = e^{\sqrt{x^2+1}} \Rightarrow f'(x) = \frac{x \cdot e^{\sqrt{x^2+1}}}{\sqrt{x^2+1}}$$

$$f(x) = \left[\frac{x^2 + x + 1}{x^3 - 6} \right]^5 \Rightarrow f'(x) = 5 \cdot \left(\frac{x^2 + x + 1}{x^3 - 6} \right)^4 \cdot \frac{-x^4 - 2x^3 - 3x^2 - 12x - 6}{(x^3 - 6)^2}$$

$$f(x) = \left[\frac{\sin x + \cos x}{\sin x - \cos x} \right]^3 \Rightarrow f'(x) = \frac{-6}{1 - 2 \sin x \cos x} \cdot \left(\frac{\sin x + \cos x}{\sin x - \cos x} \right)^2$$

$$f(x) = x \cdot e^{-1/x^2} \Rightarrow f'(x) = e^{-1/x^2} \left(1 + \frac{2}{x^2} \right)$$

$$f(x) = \sec(x^2) + \operatorname{cosec}(x^2) \Rightarrow f'(x) = 2x \left(\operatorname{tg}(x^2) \sec(x^2) - \cot(x^2) \operatorname{cosec}(x^2) \right)$$

$$f(x) = \frac{1 - \cos(x^2)}{1 + \cos(x^2)} \Rightarrow f'(x) = \frac{4x \sin x^2}{(1 + \cos(x^2))^2}$$

$$f(x) = (1 + e^{\sin x})^3 \Rightarrow f'(x) = 3 e^{\sin x} \cos x (1 + e^{\sin x})^2$$

$$f(x) = \operatorname{Ln}(\sqrt{1+e^x} - 1) - \operatorname{Ln}(\sqrt{1+e^x} + 1) \Rightarrow f'(x) = \frac{1}{\sqrt{1+e^x}}$$

$$f(x) = \operatorname{Ln} \frac{x}{\sqrt{x^2+9}} \Rightarrow f'(x) = \frac{9}{x \cdot (x^2+9)}$$

$$f(x) = \operatorname{Ln} \sqrt{\frac{1 - \cos x}{1 + \cos x}} \Rightarrow f'(x) = \operatorname{cosec} x$$

$$f(x) = \operatorname{Arctg}(x^2 - 1) \Rightarrow f'(x) = \frac{2x}{x^4 - 2x^2 + 2}$$

$$f(x) = \operatorname{Arctg} \left(\frac{1 - x^2}{1 + x^2} \right) \Rightarrow f'(x) = \frac{-2x}{1 + x^4}$$

$$f(x) = \operatorname{Arcsen}(1+x) + \sqrt{2x-x^2} \Rightarrow f'(x) = \frac{-x}{\sqrt{2x-x^2}}$$

$$f(x) = \frac{\operatorname{Arc} \cos x}{\sqrt{1-x^2}} \Rightarrow f'(x) = \frac{x \operatorname{Arc} \cos x - \sqrt{1-x^2}}{(1-x^2) \sqrt{1-x^2}}$$

$$f(x) = \operatorname{Ln} \left(\cos \frac{x-1}{x} \right) \Rightarrow f'(x) = \frac{1}{x^2} \cdot \operatorname{tg} \left(\frac{x-1}{x} \right)$$

$$f(x) = \sqrt{x^2+1} - \operatorname{Ln} \frac{1 + \sqrt{x^2+1}}{x} \Rightarrow f'(x) = \frac{x^2 + 1 + \sqrt{x^2+1}}{x(1 + \sqrt{x^2+1})} = \frac{\sqrt{1+x^2}}{x}$$

$$f(x) = \operatorname{Ln} \frac{1 + \sqrt{\sin x}}{1 - \sqrt{\sin x}} \Rightarrow f'(x) = \frac{\cos x}{\sqrt{\sin x} (1 - \sin x)}$$

$$f(x) = \operatorname{Ln}(\operatorname{Ln}(\operatorname{Ln}(\operatorname{Ln} x))) \Rightarrow f'(x) = \frac{1}{(\operatorname{Ln}(\operatorname{Ln}(\operatorname{Ln} x))) (\operatorname{Ln}(\operatorname{Ln} x)) (\operatorname{Ln} x) x}$$

$$f(x) = (x^2)^x \Rightarrow f(x) = x^{2x} \Rightarrow f'(x) = 2 \cdot x^{2x} (\operatorname{Ln} x + 1)$$