1 a) Resuelve la ecuación: $x^2-(2+2i)x+2i-1=0$.
b) Calcula $(i^{20}+i^{35}-i^{462}).i^7$

2 Eleva al cubo $\sqrt{3}-i$ y expresa el resultado de todos los modos posibles.

3 Escribe una ecuación de segundo grado cuyas raíces sean $2_{60°}$ y $2_{300°}$.

4 Siendo $z=4\sqrt{2}+4\sqrt{2}\,i$. Se pide calcular $\sqrt[3]{z}$

5 Calcular: $\sqrt[4]{8+8\sqrt{3}\,i}$

(1) $$x=\frac{(2+2i)\pm\sqrt{(2+2i)^2-4.(2i-1)}}{2}=\frac{2+2i\pm\sqrt{\cancel{4}+4i^2+\cancel{8i}-\cancel{8i}+4}}{2}=$$

$$=\frac{2+2i\pm\sqrt{4}}{2}=1+i\pm 1=\begin{cases}2+i\\ i\end{cases}$$

b) $i^{20}=i^0=1$

$i^{35}=-i$

$i^{462}=i^2=-1$

$i^7=i^3=-i$

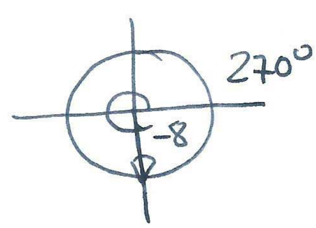
$$[1-i-(-1)].(-i)=$$
$$=[1-i+1](-i)=$$
$$=1-i-i=1-2i$$

$$\begin{array}{r|l} 35 & 4 \\ \hline 3 & 8 \end{array}\qquad \begin{array}{r|l} 462 & 4 \\ \hline 06 & 115 \\ 22 & \\ 2 & \end{array}$$

(2) $$(\sqrt{3}-i)^3=1(\sqrt{3})^3-3(\sqrt{3})^2 i+3\sqrt{3}i^2-1i^3=$$

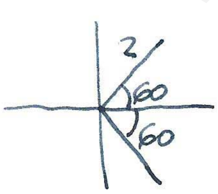
$$=3\sqrt{3}-9i-3\sqrt{3}+i=0-8i=-8i$$

$$=(0,-8)=8_{270°}=8(\cos 270°+i\,\text{sen}\,270°)$$



(3) $2_{60°}$ $2_{300°}$

$$2_{60°}:\begin{cases}a=2\cos 60=2\cdot\frac{1}{2}=1\\ b=2\,\text{sen}\,60=2\frac{\sqrt{3}}{2}=\sqrt{3}\end{cases}\quad 1+\sqrt{3}i$$



$$2_{300°}\begin{cases}a=2\cos 300°=2.\frac{1}{2}=1\\ b=2\,\text{sen}\,300°=-\frac{2\sqrt{3}}{2}=-\sqrt{3}\end{cases}\quad 1-\sqrt{3}i$$

La ecuación será

$$[x-(1+\sqrt{3}i)].[x-(1-\sqrt{3}i]=0\quad=(x-1-\sqrt{3}i)(x-1+\sqrt{3}i)=0$$

$$=(x-1)^2-(\sqrt{3}i)^2=x^2+1-2x-3(-1)=x^2-2x+4=0$$

(4) $z = 4\sqrt{2} + 4\sqrt{2}i \qquad r = \sqrt{16 \cdot 2 + 16 \cdot 2} = \sqrt{64} = 8$

$\alpha = \operatorname{arctg} \frac{b}{a} = \operatorname{arctg} 1 = 45^\circ \qquad 8_{45^\circ}$

$\sqrt[3]{8_{45^\circ}} = \quad 2_{15^\circ}, \quad 2_{135^\circ}, \quad 2_{255^\circ}$

(5) $\sqrt[4]{8 + 8\sqrt{3}i} \qquad 8 + 8\sqrt{3}i \Rightarrow r = \sqrt{64 + 192} = \sqrt{16} = 4$

$\sqrt[4]{4_{60^\circ}} \qquad \alpha = \operatorname{arctg} \frac{8\sqrt{3}}{8} = \operatorname{arctg} \sqrt{3} = 60^\circ$

Solucions $\quad \sqrt{2}_{15^\circ} \quad \sqrt{2}_{105^\circ} \quad \sqrt{2}_{195^\circ} \quad \sqrt{2}_{285^\circ}$