

1º.- Dados Los números complejos $z_1 = 3+i$ y $z_2 = 1-i$ calcular:

- a) $z_1 + z_2$
- b) $z_1 \cdot z_2$
- c) z_1 / z_2
- d) $z_1^2 \cdot z_2^2$

2º.- Escribir en forma binómica los siguientes números complejos:

- a) 2_{270°
- b) 1_{180°
- c) 3_{-270°
- d) 5_{0°
- e) 6_{2700°

3º.- Calcular:

$$\left(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)^4 \left(-\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\right)^4 (1+i)^{10}$$

4º.- Hallar a y b para que se cumpla la siguiente igualdad:

$$\frac{a+19i}{-5+bi} = 3-2i$$

5º.- Calcular dando los resultados en forma binómica:

$$\sqrt[4]{-\frac{1}{2} - i\frac{\sqrt{3}}{2}}$$

1° Dadas los nros complejos $z_1 = 3+i$ y $z_2 = 1-i$ calcular:

a) $z_1 + z_2$ b) $z_1 \cdot z_2$ c) $\frac{z_1}{z_2}$ d) $z_1^2 \cdot z_2^2$

a) $z_1 + z_2 = (3+i) + (1-i) = (3+1) + (i-i) = 4+0 = 4$

b) ~~$(3+i)(1-i) = 3 - 3i + i - i^2 = 3 - 2i + 1 = 4 - 2i$~~

$(3+i)(1-i) = 3 - 3i + i - i^2 = 3 - 2i + 1 = 4 - 2i$

c) $\frac{3+i}{1-i} = \frac{(3+i)(1+i)}{(1-i)(1+i)} = \frac{3 + 3i + i + i^2}{1^2 - i^2} = \frac{2 + 4i}{2} = 1 + 2i$

d) $z_1^2 = (3+i)^2 = 3^2 + i^2 + 6i = 9 - 1 + 6i = 8 + 6i$

$z_2^2 = (1-i)^2 = 1 + i^2 - 2i = 1 - 1 - 2i = -2i$

$z_1^2 \cdot z_2^2 = (8+6i)(-2i) = -16i - 12i^2 = 12 - 16i$

2° a) $z_{220^\circ} = -2i$ b) $1_{30^\circ} = -1$ c) $z_{-720^\circ} = z_{50^\circ} = 3i$ d) $5_0^\circ = 5$

3° Calcular $\left(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)^4 \left(-\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\right)^4 (1+i)^{10} = ?$

$z_1 = \left(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)^4 = 1_{135^\circ}$

$z_2 = \left(-\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\right)^4 = 1_{225^\circ}$

$z_3 = (1+i)^{10} = 1_{45^\circ}$

$r = \sqrt{\left(-\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2} = \sqrt{\frac{2}{4} + \frac{2}{4}} = \sqrt{\frac{4}{4}} = 1$

$\alpha = \arctan\left(\frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}\right) = -1 = 135^\circ$

$\beta = \sqrt{\left(-\frac{\sqrt{2}}{2}\right)^2 + \left(-\frac{\sqrt{2}}{2}\right)^2} = 1 \quad \beta = \arctan\left(\frac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}\right) = 225^\circ$

$\gamma = \sqrt{1^2 + 1^2} = \sqrt{2} \quad \alpha = \arctan\left(\frac{1}{1}\right) = 45^\circ$

$\rightarrow (1_{135^\circ})^4 \cdot (1_{225^\circ})^4 \cdot \left[\sqrt{2}\right]_{45^\circ}^{10} = 1_{540^\circ} \cdot 1_{900^\circ} \cdot 2_{45^\circ}^5 = (1 \cdot 1 \cdot 2^5)_{540^\circ + 900^\circ + 45^\circ} =$

$= 32_{45^\circ} = 32(\cos 45^\circ + i \sin 45^\circ) = \boxed{16\sqrt{2} + 16i}$

$\begin{array}{r} 540 \\ 900 \\ 45 \\ \hline 1485 \\ 45 \end{array} \quad \begin{array}{r} 360 \\ 4 \\ \hline 1440 \end{array}$

6° Hallar a y b por que se cumple la igualdad: $\frac{a+19i}{-5+6i} = 3-2i$

$$a+19i = (3-2i)(-5+6i) = -15 + 36i + 10i - 26 = (-15-26) + i(10+36)$$

$$\left. \begin{array}{l} a = -15-26 \\ 19 = 10+36 \end{array} \right\} \rightarrow 36 = 9 \quad \boxed{b=3} \rightarrow \boxed{a = -15-7 \cdot 3 = -15-6 = -21}$$

5° Calcular donde los resultados en forma binómica: $\sqrt[4]{-\frac{1}{2} - i \frac{\sqrt{3}}{2}}$

$$z = -\frac{1}{2} - i \frac{\sqrt{3}}{2} \quad (\text{II}^{\text{o}} \text{ cuadrante}) \quad r = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{\frac{4}{4}} = 1$$

$$\alpha = \arctan\left(\frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}}\right) = \arctan \sqrt{3} = 240^\circ$$

$$\sqrt[4]{1_{240^\circ}} = \left(\sqrt[4]{1}\right)_{\frac{240^\circ}{4} + k \cdot \frac{360^\circ}{4}} = 1_{60^\circ + k \cdot 90^\circ}$$

$$= \left\{ \begin{array}{l} 1_{60^\circ} = 1 \cdot (\cos 60^\circ + i \sin 60^\circ) = \frac{1}{2} + i \frac{\sqrt{3}}{2} \\ 1_{150^\circ} = 1 \cdot (\cos 150^\circ + i \sin 150^\circ) = -\frac{\sqrt{3}}{2} + i \frac{1}{2} \\ 1_{240^\circ} = 1 \cdot (\cos 240^\circ + i \sin 240^\circ) = -\frac{1}{2} - i \frac{\sqrt{3}}{2} \\ 1_{330^\circ} = 1 \cdot (\cos 330^\circ + i \sin 330^\circ) = \frac{\sqrt{3}}{2} - i \frac{1}{2} \end{array} \right.$$

$$\begin{array}{l} 150^\circ = 180^\circ - 30^\circ \\ 240^\circ = 180^\circ + 60^\circ \\ 330^\circ = 360^\circ - 30^\circ \end{array}$$