MATEMÁTICAS 1º BACHILLERATO
Ejercicios de exámenes de complejos

1. Calcular: $\sqrt{-2+2\sqrt{3}i} =$

2. Calcular: $a)\left(1-\frac{\sqrt{3}-i}{2}\right)^{12}$ $b)\left(\frac{1+\sqrt{3}i}{1-i}\right)^{\frac{1}{4}}$

3. Expresar en forma binómica el resultado de: $a)\left(\frac{3\sqrt{3}}{2}+\frac{3}{2}i\right)^{5}$ $b)(-1+i)^{10}$

4. Calcular: $\sqrt[5]{\frac{-8-8\sqrt{3}i}{\left(-2\sqrt{3}\right)^2}}+2i$

5. Resolver:

$$\left.\begin{array}{l} z^4 + 27z = 0 \\ z^5 + 125\, z^2 = 0 \end{array}\right\}$$

6. Resolver, expresando el resultado de forma binómica y trigonométrica: $\sqrt[3]{\frac{5+i}{2+3i}}$

7. Resolver: $a)\sqrt[3]{\frac{1-i}{1+i}}$ $b)\sqrt[3]{\frac{-1+i}{1+\sqrt{3}i}}$ $c)\sqrt[4]{81(\cos 120^{\circ}+i sen 120^{\circ})}$

8. Siendo $z=1-3i$ y $w=\sqrt{8}_{45^{\circ}}$, calcular: *z + w, z·w, 2z,* $\frac{z^3}{w}$

9. Calcula: $\frac{i^{35}-i^{5}}{2\cdot i^{10}}$

10. Opera: $\frac{(1-2i)(-2+i)}{3i\cdot(1-i)}$

11. Resuelve la siguiente ecuación y expresa el resultado en forma binómica: $\frac{3-zi+2i}{2}=z+i$

12. Siendo $z=4\sqrt{3}-4i$, se pide:
   a) z^5
   b) Las raíces cúbicas de z.

13. Hallar el valor de x para que el cociente $\frac{1+3xi}{3-4i}$:

a) Sea un número imaginario puro.
b) Sea un número real.
c) Tenga módulo 1.

14. Encontrar a y b para que $\frac{a-6i}{3+bi}=\sqrt{2}_{315^{\circ}}$

① $\sqrt{-2+2\sqrt{3}\,i} = \sqrt{4_{120^\circ}} = \sqrt{4}\left\{\begin{array}{l}\frac{120+k360}{2} \\ k=0,1\end{array}\right. =$

$= 2_{60},\ 2_{240^\circ}$

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② a) $\left(1-\frac{\sqrt{3}-i}{2}\right)^{12} = \left(\frac{2-\sqrt{3}+i}{2}\right)^{12} = \left(\frac{2-\sqrt{3}}{2}+\frac{1}{2}i\right)^{12} =$

$= \left(\sqrt{2-\sqrt{3}}_{\,75^\circ}\right)^{12} = \left(\sqrt{2-\sqrt{3}}\right)^{12}_{75\cdot 12} = (2-\sqrt{3})^{6}_{900^\circ} =$

$= (2-\sqrt{3})^{6}_{180^\circ}$

b) $\left(\frac{1+\sqrt{3}i}{1-i}\right)^{1/4} = \sqrt[4]{\frac{2_{60^\circ}}{\sqrt{2}_{315}}} = \sqrt[4]{\frac{2}{\sqrt{2}}_{-255^\circ}} = \sqrt[4]{\sqrt{2}_{105^\circ}} =$

$= \sqrt[8]{2}\left\{\begin{array}{l}\frac{105+k360}{4} \\ k=0,1,2,3\end{array}\right.$ $\quad \sqrt[8]{2}_{26'25^\circ},\ \sqrt[8]{2}_{116'25^\circ}$

$\sqrt[8]{2}_{206'25^\circ},\ \sqrt[8]{2}_{296'25^\circ}$

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③ a) $\left(\frac{3\sqrt{3}}{2}+\frac{3}{2}i\right)^5 = \left(3_{30°}\right)^5 = 3^5_{150°} =$

$243_{150°} = 243(\cos 150° + i\,\text{sen}\,150°) =$

$= 243\left(\frac{-\sqrt{3}}{2}+\frac{1}{2}i\right) = \frac{-243\sqrt{3}}{2}+\frac{243}{2}i$

b) $(-1+i)^{10} = \left(\sqrt{2}_{135°}\right)^{10} = (\sqrt{2})^{10}_{1350°} =$

$= 2^5_{270} = 32_{270°} = 32(\cos 270 + \text{sen}\,270\, i) = 0-32i$

$= -32i$

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④ $x = \sqrt[5]{\frac{-8-8\sqrt{3}i}{(-2\sqrt{3})^2}} + 2i$

Calculemos a parte la raíz:

$y = \sqrt[5]{\frac{16_{240°}}{12}} = \sqrt[5]{\frac{4}{3}_{240°}} = \sqrt[5]{\frac{4}{3}} \left\{ \begin{array}{l} \frac{240+k360}{5} \\ k=0,1,2,3,4 \end{array} \right.$

$x_1 = y_1+2i = \sqrt[5]{\frac{4}{3}}_{48°}+2i = \sqrt[5]{\frac{4}{3}}(\cos 48 + i\,\text{sen}\,48)+2i$

$x_2 = \sqrt[5]{\frac{4}{3}}_{120°}+2i = \dots$

$x_3 = \sqrt[5]{\frac{4}{3}}_{192°}+2i = \dots$

$x_4 = \sqrt[5]{\frac{4}{3}}_{264°}+2i = \dots$

$x_5 = \sqrt[5]{\frac{4}{3}}_{336°}+2i = \dots$

Calculadora y decimales

¡¡Quién se inventa estos problemas para un examen?!! ④

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⑤ $\begin{cases} z^4+27z=0 \\ z^5+125z^2=0 \end{cases}$

• $z^4+27z=0 \quad z(z^3+27)=0 \quad \begin{matrix} z=0 \\ z^3+27=0 \end{matrix}$

$$z=\sqrt[3]{-27}=\sqrt[3]{27_{180^\circ}}=\sqrt[3]{27}\left\{\begin{matrix}\frac{180+k360}{3} \\ k=0,1,2\end{matrix}\right. =$$

$$=\begin{cases} 3_{60^\circ} \\ 3_{180^\circ} \\ 3_{300^\circ} \end{cases}$$

• $z^5+125z^2=0 \qquad z^2=0$

$z^2(z^3+125)=0 \qquad z^3=-125 \quad z=\sqrt[3]{-125}=\sqrt[3]{125_{180^\circ}}$

$$=5\left\{\begin{matrix}\frac{180+k360}{3} \\ k=0,1,2\end{matrix}\right. =\begin{cases} 5_{60^\circ} \\ 5_{180^\circ} \\ 5_{300^\circ} \end{cases}$$

Solución del sistema: $z=0$

—o—

⑥ • $\sqrt[3]{\dfrac{5+i}{2+3i}}=\sqrt[3]{\sqrt{26}}=\sqrt[3]{\sqrt{2}_{315^\circ}}=\sqrt[6]{2}\left\{\begin{matrix}\frac{315^\circ+k360^\circ}{3} \\ k=0,1,2\end{matrix}\right.=$

$=\sqrt[6]{2}_{105^\circ},\ \sqrt[6]{2}_{225^\circ},\ \sqrt[6]{2}_{345^\circ}$

—o—

⑦ a) $\sqrt[3]{\dfrac{1-i}{1+i}} = \sqrt[3]{-i} = \sqrt[3]{1_{270^\circ}} = \sqrt[3]{1}\left\{\begin{array}{l}\dfrac{270+k360}{3}\\ k=0,1,2\end{array}\right. =$

$= 1_{90^\circ}, 1_{210^\circ}, 1_{330^\circ}$

b) $\sqrt[3]{\dfrac{-1+i}{1+\sqrt{3}i}} = \sqrt[3]{\dfrac{\sqrt{2}_{135^\circ}}{2_{60^\circ}}} = \sqrt[3]{\dfrac{\sqrt{2}}{2}_{75^\circ}} = \sqrt[3]{\dfrac{\sqrt{2}}{2}}\left\{\begin{array}{l}\dfrac{75+k360}{3}\\ k=0,1,2\end{array}\right. =$

$= \sqrt[3]{\dfrac{\sqrt{2}}{2}}\left\{\begin{array}{l}25^\circ\\ 145^\circ\\ 265^\circ\end{array}\right.$

c) $\sqrt[4]{81(\cos 120^\circ + i\,\mathrm{sen}\,120^\circ)} = \sqrt[4]{81_{120^\circ}} = \sqrt[4]{81}\left\{\begin{array}{l}\dfrac{120+k360}{4}\\ k=0,1,2,3\end{array}\right. =$

$= 3_{30^\circ}, 3_{120^\circ}, 3_{210^\circ}, 3_{300^\circ}$

———o———

⑧ $z = 1-3i$, $w = \sqrt{8}_{45^\circ} = \sqrt{8}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = 2+2i$

$z+w = (1-3i)+(2+2i) = 3-i$

$z\cdot w = 8-4i$

$2\cdot z = 2-6i$

$\dfrac{z^3}{w} = \dfrac{(1-3i)^3}{2+2i} = \dfrac{-26+18i}{2+2i} = -2+11i$

———o———

⑨ $\dfrac{i^{35}-i^{5}}{2\cdot i^{10}} = \dfrac{(-i)-i}{2(-1)} = \dfrac{-2i}{-2} = i$

———o———

⑩ $\dfrac{(1-2i)(-2+i)}{3i(1-i)} = \dfrac{5i}{3+3i} = \dfrac{5}{6} + \dfrac{5}{6}i$

———o———

(11) $\frac{3-2i+2i}{2}=z+i \rightarrow 3-2i+2i=2z+2i \rightarrow$

$2z+2i=3+2i-2i \rightarrow (2+i)z=3 \rightarrow z=\frac{3}{2+i} \rightarrow$

$z=\frac{3(2-i)}{(2+i)(2-i)}=\frac{6-3i}{5}=\frac{6}{5}-\frac{3}{5}i$

(12) $z=4\sqrt{3}-4i= \quad 8_{330^\circ}$

a) $z^5=(8_{330^\circ})^5=8^5{}_{330\cdot 5}=32768_{1650^\circ}=$

$=32768_{210^\circ}$

b) $\sqrt[3]{4\sqrt{3}-4i}=\sqrt[3]{8_{330^\circ}}=\sqrt[3]{8}\begin{cases}\frac{330^\circ+k360}{2}\\ k=0,1,2\end{cases}=$

$=2_{165^\circ}, \ 2_{285^\circ}, \ 2_{405^\circ}$

(13) $\frac{1+3xi}{3-4i}\cdot\frac{(3+4i)}{(3+4i)}=\frac{3+9xi+4i+12xi^2}{9+16}=\frac{(3-12x)}{25}+\frac{(9x+4)i}{25}$

a) $\frac{3-12x}{25}=0 \rightarrow x=\frac{1}{4}$; b) $\frac{9x+4}{25}=0 \quad x=\frac{-4}{9}$

c) $\sqrt{\left(\frac{3-12x}{25}\right)^2+\left(\frac{9x+4}{25}\right)^2}=1 \rightarrow x=\pm\sqrt{\frac{8}{3}}$

(14) Igual que el 10 de la otra hoja.