

Apellidos:

Nombre:

Grupo:

- 1.- Sabiendo que $\operatorname{tg} \alpha = -12$ y $\frac{3\pi}{4} < \alpha < \pi$ calcular el **verdadero valor** de $\operatorname{sen}(\pi + \alpha)$ y $\operatorname{tg}(2\alpha)$

$$1 + \operatorname{tg}^2 \alpha = \sec^2 \alpha \Leftrightarrow 1 + 144 = 1 / \cos^2 \alpha \Rightarrow \cos \alpha = -1 / \sqrt{145}$$

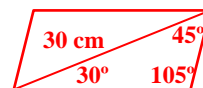
$$\operatorname{sen} \alpha = \operatorname{tg} \alpha \cdot \cos \alpha = (-12) \left(-1 / \sqrt{145} \right) = 12 / \sqrt{145}$$

$$\operatorname{sen}(\pi + \alpha) = -\operatorname{sen} \alpha = -12 / \sqrt{145} \quad \operatorname{tg} 2\alpha = \frac{2(-12)}{1 - (-12)^2} = \frac{24}{143} \quad \text{como } \frac{3\pi}{2} < 2\alpha < 2\pi \Rightarrow \operatorname{tg} 2\alpha = -\frac{24}{143}$$

- 2.- La diagonal de un paralelogramo mide 30 cm, sabiendo que forma con los lados 30° y 45° . Calcular su área y perímetro.

$$\text{Trma.Seno } \frac{30}{\operatorname{sen} 105} = \frac{x}{\operatorname{sen} 30} = \frac{y}{\operatorname{sen} 45} \Rightarrow x = \frac{30 \cdot \operatorname{sen} 30}{\operatorname{sen} 105} = 15,53 \quad y = \frac{30 \cdot \operatorname{sen} 45}{\operatorname{sen} 105} = 21,96$$

$$\text{Área} = 21,96 \cdot 30 \cdot \operatorname{sen} 30^\circ = 329,4 \quad \text{Perímetro} = 2 \cdot (15,53 + 21,96) = 74,98$$



- 3.- Demostrar si es cierta o no, la igualdad: $\frac{\operatorname{sen} 2a \cdot \cos a}{(1 + \cos 2a) \cdot (1 - \cos a)} = \cot g \left(\frac{a}{2} \right)$.

$$\begin{aligned} \frac{\operatorname{sen} 2a \cdot \cos a}{(1 + \cos 2a) \cdot (1 - \cos a)} &= \frac{2 \operatorname{sen} a \cdot \cos^2 a}{(1 + \cos^2 a - \operatorname{sen} a)(1 - \cos a)} \stackrel{1. \operatorname{sen}^2 a = \cos^2 a}{=} \frac{\cancel{2 \operatorname{sen} a} \cos^2 a}{2 \cos^2 a (1 - \cos a)} = \\ &= \frac{\pm \sqrt{1 - \cos^2 a}}{1 - \cos a} = \pm \sqrt{\frac{(1 - \cos a)(1 + \cos a)}{(1 - \cos a)^2}} = \cot g \left(\frac{a}{2} \right) \end{aligned}$$

- 4.- Resolver la ecuación: $3 \operatorname{sen}^2 x + \cos^2 x + \cos x = 0$

$$3 \operatorname{sen}^2 x + \cos^2 x + \cos x = 3(1 - \cos^2 x) + \cos^2 x + \cos x = -2 \cos^2 x + \cos x + 3 = 0$$

$$\cos x = \frac{-1 \pm \sqrt{1 + 24}}{-4} \quad \left\{ \begin{array}{l} 6/4 > 1 \text{ no sol} \\ -1 \Rightarrow \cos x = -1 \Rightarrow x = 180 + 360k \quad k \in \mathbb{Z} \end{array} \right.$$

- 5.- Resolver la ecuación: $z^3 - 2z^2 + 4z - 8 = 0$

$$z^3 - 2z^2 + 4z - 8 = 0 \Leftrightarrow (z - 2)(z^2 + 4) = 0 \Rightarrow z = 2; \quad z = \pm 2i$$

- 6.- Sabiendo que el punto A(5,12) es un vértice de un cuadrado con centro en el origen de coordenadas, calcular los restantes vértices.

$$z_1 = \sqrt[4]{z} \Rightarrow z = (5 + 12i)^4 = \left(13_{67,38^\circ} \right)^4 = 13^4_{269,53}$$

$$\begin{aligned} \text{Los vértices son } \sqrt[4]{z} &= \sqrt[4]{13^4_{269,53}} = 13_{\frac{269,53 + 360k}{4}} \quad k = 0, 1, 2, 3 = \\ z_1 &= 13_{67,38^\circ} = (5, 12) \\ z_2 &= 13_{157,38^\circ} = (-12, 5) \\ z_3 &= 13_{247,38^\circ} = (-5, -12) \\ z_4 &= 13_{337,38^\circ} = (12, -5) \end{aligned}$$

7.- Sea $z = 3_{30^\circ}(3 - ki)$. Calcula el valor de k para que z sea un número imaginario puro.

$$z = 3_{30^\circ}(3 - ki) = 3(\cos 30^\circ + i \operatorname{sen} 30^\circ)(3 - ki) = 3 \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i \right) (3 - ki) = \frac{3}{2} \left((3\sqrt{3} + k) + (3 - \sqrt{3})i \right)$$

$$\text{Imaginario puro} \Leftrightarrow \operatorname{Re}(z) = 0 \Leftrightarrow 3\sqrt{3} + k = 0 \Rightarrow k = -3\sqrt{3}$$

8.- Resolver $\frac{(7-i)z + (4+3i)}{2-i} = -1+i$

$$\frac{(7-i)z + (4+3i)}{2-i} = -1+i$$

$$(7-i)z + (4+3i) = (-1+i)(2-i)$$

$$(7-i)z + (4+3i) = (-1+3i)$$

$$(7-i)z = (-1+3i) - (4+3i)$$

$$(7-i)z = -5$$

$$= \frac{-5}{7-i} = \frac{-35-5i}{50} = -\frac{7}{10} - \frac{1}{10}i$$