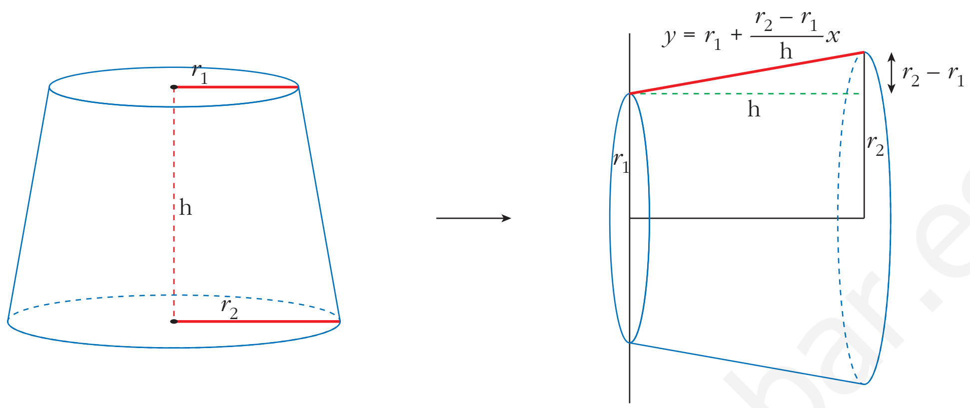
Volumen de un tronco de cono. Obtención de la fórmula

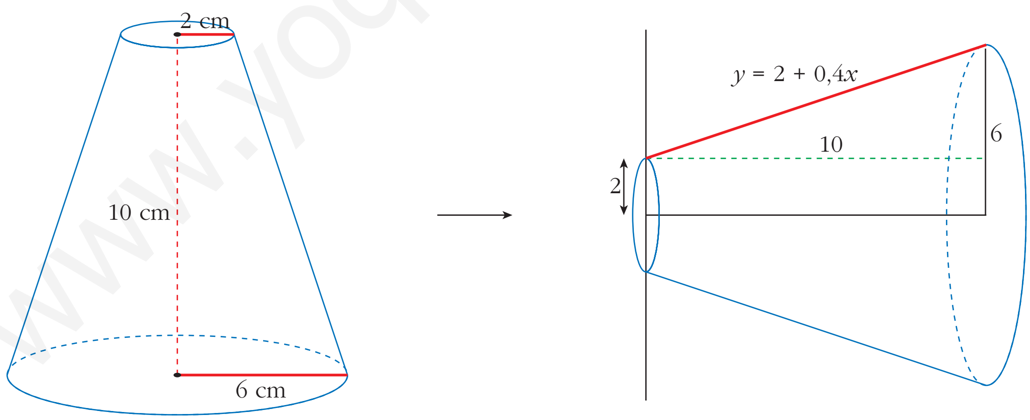


$$\text{Volumen} = \pi \int_0^h \left(r_1 + \frac{r_2 - r_1}{h}x\right)^2 dx = \pi \, \frac{h}{r_2 - r_1} \int_0^h \left(r_1 + \frac{r_2 - r_1}{h}x\right)^2 \frac{r_2 - r_1}{h}\, dx =$$

$$= \pi \, \frac{h}{r_2 - r_1} \, \frac{1}{3} \left[\left(r_1 + \frac{r_2 - r_1}{h}x\right)^3\right]_0^h = \frac{1}{3}\pi \, \frac{h}{r_2 - r_1} \left[\left(r_1 + \frac{r_2 - r_1}{h} \cdot h\right)^3 - \left(r_1 + \frac{r_2 - r_1}{h} \cdot 0\right)^3\right] =$$

$$= \frac{1}{3}\pi \, h \, \frac{1}{r_2 - r_1} [r_2^3 - r_1^3] = \frac{1}{3}\pi \, h \, \frac{r_2^3 - r_1^3}{r_2 - r_1} = \frac{1}{3}\pi \, h \, ({r_2}^2 + {r_1}^2 + r_1 \, r_2)$$

Volumen de un tronco de cono concreto

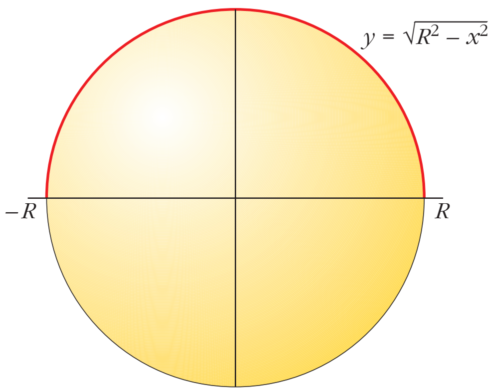


$$\text{Volumen} = \pi \int_0^{10} (2 + 0,4x)^2 \, dx = \pi \frac{1}{0,4} \int_0^{10} (2 + 0,4x)^2 \cdot 0,4 \, dx = \pi \cdot \frac{1}{0,4} \cdot \frac{1}{3} \left[(2 + 0,4x)^3\right]_0^{10} =$$

$$= \frac{\pi}{1,2} \left[(2 + 0,4 \cdot 10)^3 - (2 + 0,4 \cdot 0)^3\right] = \frac{\pi}{1,2} (6^3 - 2^3) \simeq 544,54 \text{ cm}^3$$

5. Profundización práctica: volúmenes de cuerpos conocidos, mediante integrales

Volumen de una esfera. Obtención de la fórmula

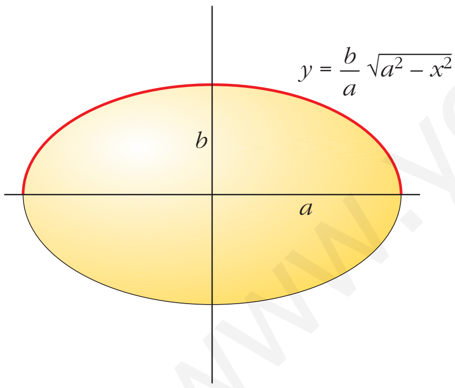


$$x^2 + y^2 = R^2 \rightarrow y = \sqrt{R^2 - x^2}$$

$$\text{Volumen} = \pi \int_{-R}^{R} \left(\sqrt{R^2 - x^2}\right)^2 dx = \pi \int_{-R}^{R} (R^2 - x^2)\, dx = \pi \left[R^2 x - \frac{x^3}{3}\right]_{-R}^{R} =$$

$$= \pi \left[\left(R^3 - \frac{R^3}{3}\right) - \left(-R^3 - \frac{-R^3}{3}\right)\right] = \pi \left[\frac{2}{3} R^3 - \left(-\frac{2}{3} R^3\right)\right] = \pi \frac{4}{3} R^3 = \frac{4}{3} \pi R^3$$

Volumen de un elipsoide. Obtención de la fórmula



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \rightarrow y^2 = \frac{b^2}{a^2}(a^2 - x^2) \rightarrow y = \frac{b}{a}\sqrt{a^2 - x^2}$$

$$\text{Volumen} = \pi \int_{-a}^{a} \left(\frac{b}{a}\sqrt{a^2 - x^2}\right)^2 dx = \pi \frac{b^2}{a^2} \int_{-a}^{a} (a^2 - x^2)\, dx = \pi \frac{b^2}{a^2} \left[a^2 x - \frac{x^3}{3}\right]_{-a}^{a} = \pi \frac{b^2}{a^2} \frac{4}{3} a^3 = \frac{4}{3} \pi\, ab^2$$