CALCULA LAS INTEGRALES:

1º) $\int (x^2+3).e^{3x}.dx$ (1p)

2º) $\int \frac{x^4+x+4}{x^3-4x}\,.dx$ (1p)

3º) $\int \frac{cosx}{1+sen^2x}\,.dx$ (1p)

4º) $\int \frac{-x+3}{4x^2+9}\,.dx$ (1p)

5º) Halla una primitiva G(x) de $g(x) = x.\sqrt{x^2+1}$ tal que G(0)=1 (1p)

6º) Halla el área de la región limitada por la gráfica de la función $f(x) = x^3 - 16x$ y el eje de abcisas. (1´5p)

7º) Halla el área de la región plana limitada por las gráficas de las funciones:

$$f(x) = \frac{2}{x} \qquad y \qquad g(x) = 3 - x \qquad (1´5p)$$

8º) Halla el área de la región plana limitada por la gráfica de la función $y = x^2 - 9$ y las rectas tangentes a ella en x=3 y en x=-3. (2p)

<u>EN LOS EJERCICIOS 6,7 Y 8 HAY QUE HACER UN ESBOZO DE LAS GRÁFICAS</u>

1º $\int (x^2+3)e^{3x}dx = (x^2+3)\cdot\frac{e^{3x}}{3} - \int \frac{e^{3x}}{3}\cdot 2x\,dx = \frac{(x^2+3)e^{3x}}{3} - \frac{2}{3}\int xe^{3x}dx$

$\left.\begin{array}{l} u = x^2+3 \rightarrow du = 2x\,dx \\ dv = e^{3x}dx \rightarrow v = \int e^{3x}dx = \frac{e^{3x}}{3} \end{array}\right\}$ $\quad = \frac{(x^2+3)e^{3x}}{3} - \frac{2}{3}\left[\frac{xe^{3x}}{3} - \int \frac{e^{3x}}{3}dx\right]$

$u = x \rightarrow du = dx$

$dv = e^{3x}dx \rightarrow v = \frac{e^{3x}}{3}$ $\quad = \frac{(x^2+3)e^{3x}}{3} - \frac{2xe^{3x}}{9} - \frac{2}{9}\cdot\frac{e^{3x}}{3} + C$

2º $\int \frac{x^4+x+4}{x^3-4x}dx = \int\left(x + \frac{4x^2+x+4}{x^3-4x}\right)dx = \frac{x^2}{2} + \int\frac{4x^2+x+4}{x^3-4x}dx$

$$\begin{array}{r|l} x^4+x+4 & x^3-4x \\ -x^4+4x^2 & x \\ \hline 4x^2+x+4 & \end{array}$$

Resolvemos $\int \frac{4x^2+x+4}{x^3-4x}dx = \int\frac{4x^2+x+4}{x(x+2)(x-2)}dx$

$= \int\left(\frac{A}{x} + \frac{B}{(x+2)} + \frac{C}{(x-2)}\right)dx \overset{(*)}{=} \int\left(-\frac{1}{x} + \frac{9/4}{(x+2)} + \frac{11/4}{x-2}\right)dx =$

(*) Obtenemos A, B, C: $4x^2+x+4 = A(x^2-4) + Bx(x-2) + Cx(x+2)$

Si $x = 0 \Rightarrow 4 = -4A \Rightarrow A = -1$

Si $x = 2 \Rightarrow 22 = 8C \Rightarrow C = 11/4$

Si $x = -2 \Rightarrow 18 = 8B \Rightarrow B = \frac{9}{4}$

$= -\ln x + \frac{9}{4}\ln|x+2| + \frac{11}{4}\ln|x-2| \Rightarrow$

$\int\frac{x^4+x+4}{x^3-4x}dx = \frac{x^2}{2} - \ln|x| + \frac{9}{4}\ln|x+2| + \frac{11}{4}\ln|x-2| + C$

3º $\displaystyle\int \frac{\cos x}{1+\operatorname{sen}^2 x}\,dx = \int \frac{dt}{1+t^2} = \arctan|t| = \underline{\arctan(\operatorname{sen} x) + C}$

$\operatorname{sen} x = t \to \cos x \cdot dx = dt$

4º $\displaystyle\int \frac{-x+3}{4x^2+9}\,dx = \int \frac{-x}{4x^2+9}\,dx + \int \frac{3}{4x^2+9}\,dx = (1)$

Calculamos a) $\displaystyle\int \frac{-x}{4x^2+9}\,dx = -\frac{1}{8}\int \frac{8x}{4x^2+9}\,dx = -\frac{1}{8}\ln|4x^2+9|$

b) $\displaystyle\int \frac{3}{4x^2+9}\,dx = \int \frac{3/9}{\frac{4x^2}{9}+1}\,dx = \frac{3}{9}\int \frac{1}{\left(\frac{2x}{3}\right)^2+1}\,dx = \frac{3}{9}\int \frac{\frac{3}{2}\,dt}{t^2+1} =$

$t = \frac{2x}{3} \to dt = \frac{2\,dx}{3}$

$\displaystyle = \frac{3}{9}\cdot\frac{3}{2}\cdot\int \frac{dt}{t^2+1} = \frac{1}{2}\arctan|t| = \frac{1}{2}\arctan\left(\frac{2x}{3}\right)$

(1) Luego: $\displaystyle\int \frac{-x+3}{4x^2+9}\,dx = \underline{-\frac{1}{8}\ln|4x^2+9| + \frac{1}{2}\arctan\left(\frac{2x}{3}\right) + C}$

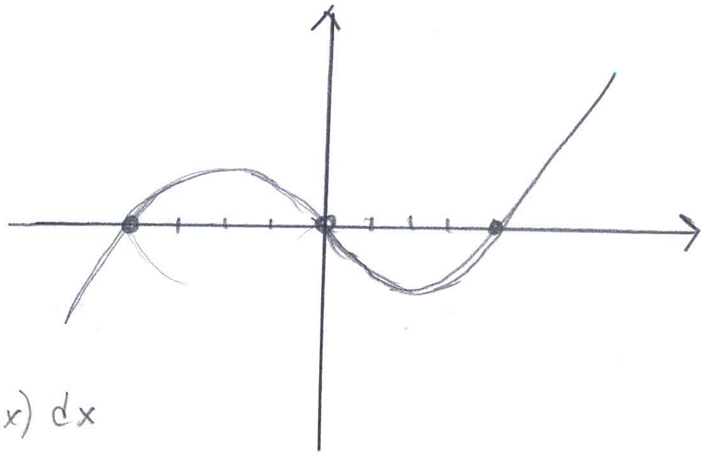
5º $\displaystyle G(x) = \int x\cdot\sqrt{x^2+1}\,dx = \int x\cdot\sqrt{t}\cdot\frac{dt}{2x} = \frac{1}{2}\int \sqrt{t}\,dt =$

$x^2+1 = t \Rightarrow 2x\,dx = dt$ $\displaystyle = \frac{1}{2}\int t^{1/2}\,dt = \frac{1}{2}\cdot\frac{t^{3/2}}{3/2} = \frac{1}{3}t^{3/2} = \frac{1}{3}\sqrt{t^3} =$

$\displaystyle = \frac{1}{3}\sqrt{(x^2+1)^3} = \underline{\frac{1}{3}\cdot(x^2+1)\cdot\sqrt{x^2+1} + C} \quad = \underline{\frac{1}{3}(x^2+1)\cdot\sqrt{x^2+1} + \frac{2}{3}}$

Como $G(0) = 1 \Rightarrow \quad 1 = \frac{1}{3} + C \Rightarrow C = \frac{2}{3}$

6º $f(x)=x^3-16x \Rightarrow$



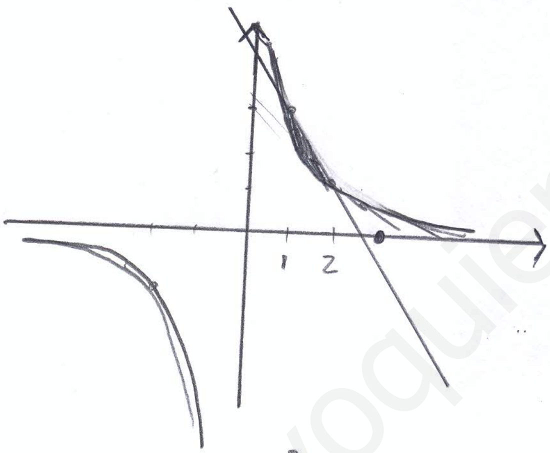
$f(x)=0 \Rightarrow x=0,4,-4$

$$\text{Área} = \int_{-4}^{0}(x^3-16x)\,dx - \int_{0}^{4}(x^3-16x)\,dx$$

$$= \left[\frac{x^4}{4}-8x^2\right]_{-4}^{0} - \left[\frac{x^4}{4}-8x^2\right]_{0}^{4} = \left[0-(64-128)\right]-\left[64-128\right] =$$

$$= 64-(-64) = \boxed{128\ u^2}$$

7º



$f(x)=g(x) \Rightarrow \frac{2}{x}=3-x \Rightarrow$

$2=3x-x^2 \rightarrow x^2-3x+2=0$

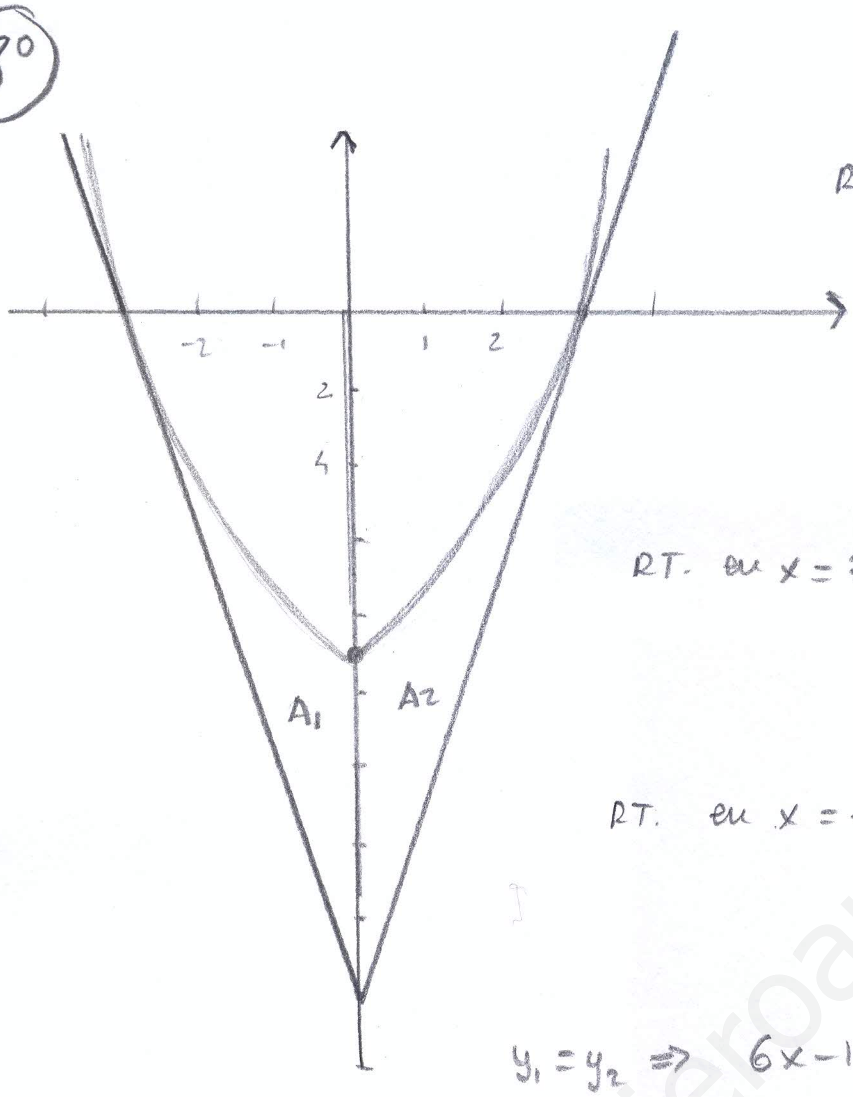
$x=1,2$

$$\int_{1}^{2}\left((3-x)-\frac{2}{x}\right)dx = \int_{1}^{2}\left(3-x-\frac{2}{x}\right)dx = \left[3x-\frac{x^2}{2}-2\ln|x|\right]_{1}^{2} =$$

$$= \left(6-2-2\ln 2\right)-\left(3-\frac{1}{2}-\cancel{2\ln 1}\right) = 4-2\ln 2-3+\frac{1}{2} =$$

$$= \frac{3}{2}-2\ln 2 = \boxed{\frac{3}{2}-\ln 4} \approx 0'137\ u^2$$

8º



RT. $y - f(a) = f'(a)(x-a)$

$f(x) = x^2 - 9$

$f'(x) = 2x$

RT. en $x = 3$: $y - 0 = 6(x-3)$

$\boxed{y_1 = 6x - 18}$

RT. en $x = -3$: $y - 0 = -6(x+3)$

$\boxed{y_2 = -6x - 18}$

$y_1 = y_2 \Rightarrow 6x - 18 = -6x - 18 \rightarrow x = 0$

$f(x) = y_1 \rightarrow x = 3$

$f(x) = y_2 \rightarrow x = -3$

$$A_T = A_1 + A_2 = \int_{-3}^{0} \left((x^2-9) - (-6x-18)\right) dx + \int_{0}^{3} \left((x^2-9) - (6x-18)\right) dx$$

$$= \int_{-3}^{0} (x^2+6x+9)\, dx + \int_{0}^{3} (x^2-6x+9)\, dx = \left[\frac{x^3}{3} + 3x^2 + 9x\right]_{-3}^{0} + \left[\frac{x^3}{3} - 3x^2 + 9x\right]_{0}^{3} =$$

$$= \left[0 - (-9+27-27)\right] + \left[(9-27+27) - 0\right] = 9 + 9 = 18\ u^2$$