Examen de Matemáticas 1º de Bachillerato
Enero 2016

Problema 1 Dados los números complejos $z_1 = 3 - 7i$ y $z_2 = -1 + 2i$. Se pide calcular:

a) $z_1 + z_2$ y $z_1 - z_2$

b) $z_1 \cdot z_2$

c) $\dfrac{z_1}{z_2}$

Solución:

a) $z_1 + z_2 = 2 - 5i$ y $z_1 - z_2 = 4 - 9i$

b) $z_1 \cdot z_2 = 11 + 13i$

c) $\dfrac{z_1}{z_2} = -\dfrac{17}{5} + \dfrac{1}{5}i$

Problema 2 Resolver la siguiente ecuación de segundo grado:

$$z^2 + 2z + 7 = 0$$

Solución:

$$z^2 + 2z + 7 = 0 \Longrightarrow z = \begin{cases} -1 + \sqrt{6}i \\ -1 - \sqrt{6}i \end{cases}$$

Problema 3 Si $z = 2 - 7i$ calcular z^{10}.

Solución:

$$z = 2 - 7i = \sqrt{53}_{285^{\circ}56'44''} = \sqrt{53}(\cos 285^{\circ}56'44'' + i \sin 285^{\circ}56'44'')$$

$$z^{10} = (2 - 7i)^{10} = 53^5_{10\cdot 285^{\circ}56'44''} = 53^5_{2859^{\circ}27'20''} = 53^5_{339^{\circ}27'20''} =$$
$$53^5(\cos 339^{\circ}27'20'' + i \sin 339^{\circ}27'20'')$$

Problema 4 Resolver la ecuación $z^3 - 2i = 7$.

Solución:

$$z^3 = 7 + 2i \Longrightarrow z = \sqrt[3]{7 + 2i}$$

$$7 + 2i = \sqrt{53}_{15^{\circ}56'44''} = \sqrt{53}(\cos 15^{\circ}56'44'' + i \sin 15^{\circ}56'44'')$$

$$z = \sqrt[3]{7 + i} = \begin{cases} \sqrt[6]{53}_{5^{\circ}18'55''} = \sqrt[6]{53}(\cos 5^{\circ}18'55'' + i \sin 5^{\circ}18'55'') \\ \sqrt[6]{53}_{125^{\circ}18'55''} = \sqrt[6]{53}(\cos 125^{\circ}18'55'' + i \sin 125^{\circ}18'55'') \\ \sqrt[6]{53}_{245^{\circ}18'55''} = \sqrt[6]{53}(\cos 245^{\circ}18'55'' + i \sin 245^{\circ}18'55'') \end{cases}$$

Problema 5 Calcular las raices de $\sqrt[3]{-2+3i}$

Solución:

$$z = -2+3i = \sqrt{13}_{123^{o}41'24''} = \sqrt{13}(\cos 123^{o}41'24'' + i\sin 123^{o}41'24'')$$

$$\sqrt[3]{z} = \left\{ \begin{array}{l} \sqrt[6]{13}_{41^{o}13'48''} = \sqrt[6]{13}(\cos 41^{o}13'48'' + i\sin 41^{o}13'48'') \\ \sqrt[6]{13}_{161^{o}13'48''} = \sqrt[6]{13}(\cos 161^{o}13'48'' + i\sin 161^{o}13'48'') \\ \sqrt[6]{13}_{281^{o}13'48''} = \sqrt[6]{13}(\cos 281^{o}13'48'' + i\sin 281^{o}13'48'') \end{array} \right.$$