EJERCICIOS PROPUESTOS

1. Demostrar las siguientes igualdades:

a) $\sec x + \operatorname{cosec} x = \operatorname{tg} x \cdot \operatorname{cosec} x + \operatorname{ctg} x \cdot \sec x$

b) $\operatorname{tg} x + \operatorname{cotg} x = \sec x \cdot \operatorname{cosec} x$

c) $\dfrac{1-\operatorname{sen} x}{\cos x} = \dfrac{\operatorname{sen}(90^\circ - x)}{1+\operatorname{sen} x}$

d) $\operatorname{sen}^4 x - \cos^4 x = \operatorname{sen}^2 x - \cos^2 x$

e) $\dfrac{\operatorname{ctg} x + 1}{\sec x + \operatorname{cosec} x} = \cos x$

f) $\dfrac{\operatorname{sen}\left(\frac{\pi}{2} - x\right) - \cos(\pi - x)}{\cos\left(\frac{\pi}{2} + x\right) + \operatorname{sen}(\pi + x)} = -\operatorname{ctg} x$

g) $\dfrac{\operatorname{tg} x}{\operatorname{tg} 2x - \operatorname{tg} x} = \cos 2x$

h) $\sec^2 x + \operatorname{cosec}^2 x = \sec^2 x \cdot \operatorname{cosec}^2 x$

$\sec^4 x - \sec^2 x = \operatorname{tg}^4 x + \operatorname{tg}^2 x$

Solución:

a) sec x + cosec x = tg x · cosec x + ctg x · sec x

$$\text{tg } x \cdot \text{cosec } x + \text{ctg } x \cdot \sec x = \frac{\text{sen } x}{\cos x}\cdot\frac{1}{\text{sen } x}+\frac{\cos x}{\text{sen } x}\cdot\frac{1}{\cos x}=\frac{1}{\cos x}+\frac{1}{\text{sen } x}=\sec x+\text{cosec } x$$

b) tg x + cotg x = sec x · cosec x

$$\text{tg } x+\text{cotg } x=\frac{\text{sen} x}{\cos x}+\frac{\cos x}{\text{sen } x}=\frac{\text{sen}^2 x+\cos^2 x}{\cos x\cdot\text{sen } x}=\frac{1}{\cos x\cdot\text{sen } x}=\frac{1}{\cos x}\cdot\frac{1}{\text{sen } x}=\sec x\cdot\text{cosec } x$$

c) $\frac{1-\text{sen}\,x}{\cos x}=\frac{\text{sen}(90^\circ-x)}{1+\text{sen}\,x}$

$$\frac{1-\text{sen}\,x}{\cos x}=\frac{\text{sen}(90^\circ-x)}{1+\text{sen}\,x}\rightarrow (1-\text{sen } x)(1+\text{sen } x)=\cos x\cdot\text{sen }(90^\circ-x)\rightarrow 1-\text{sen}^2 x=\cos x\cdot\cos x\rightarrow$$

$$\cos^2 x=\cos^2 x$$

d) $\text{sen}^4 x-\cos^4 x=\text{sen}^2 x-\cos^2 x$

$$\text{sen}^4 x-\cos^4 x=(\text{sen}^2 x-\cos^2 x)(\text{sen}^2 x+\cos^2 x)=\text{sen}^2 x-\cos^2 x$$

e) $\frac{\text{ctg}x+1}{\sec x+\text{cosec}\,x}=\cos x$

$$\frac{\text{ctg}x+1}{\sec x+\text{cosec}\,x}=\frac{\frac{\cos x}{\text{sen}\,x}+1}{\frac{1}{\cos x}+\frac{1}{\text{sen}\,x}}=\frac{\frac{\cos x+\text{sen}\,x}{\text{sen}\,x}}{\frac{\cos x+\text{sen}\,x}{\text{sen}\,x\cdot\cos x}}=\frac{\text{sen}\,x\cdot\cos x}{\text{sen}\,x}=\cos x$$

f) $\frac{\text{sen}\left(\frac{\pi}{2}-x\right)-\cos(\pi-x)}{\cos\left(\frac{\pi}{2}+x\right)+\text{sen}(\pi+x)}=-\text{ctg } x$

Teniendo en cuenta:

a) $\text{sen}\left(\frac{\pi}{2}-x\right)=\cos x$ b) $\cos(\pi-x)=-\cos x$ c) $\text{sen}(\pi+x)=-\text{sen}\,x$ d) $\cos\left(\frac{\pi}{2}+x\right)=-\text{sen}\,x$

$$\frac{\text{sen}\left(\frac{\pi}{2}-x\right)-\cos(\pi-x)}{\cos\left(\frac{\pi}{2}+x\right)+\text{sen}(\pi+x)}=\frac{\cos x+\cos x}{-\text{sen}\,x-\text{sen}\,x}=\frac{2\cos x}{-2\text{sen}\,x}=-\text{ctg } x$$

g) $\mathbf{\dfrac{tg\,x}{tg\,2x-tg\,x}=\cos 2x}$

$$\frac{tg\,x}{tg\,2x-tg\,x}=\frac{tg\,x}{\dfrac{2tg\,x}{1-tg^2x}-tg\,x}=\frac{tg\,x}{\dfrac{tg\,x+tg^3x}{1-tg^2x}}=\frac{tg\,x\left(1-tg^2x\right)}{tg\,x\left(1+tg^2x\right)}=\frac{1-\dfrac{sen^2x}{\cos^2x}}{1+\dfrac{sen^2x}{\cos^2x}}=\frac{\cos^2x-sen^2x}{\cos^2x+sen^2x}=\cos 2x$$

h) $\mathbf{sec^2\,x + cosec^2\,x = sec^2\,x \cdot cosec^2\,x}$

$$sec^2\,x + cosec^2\,x=\frac{1}{\cos^2x}+\frac{1}{sen^2x}=\frac{sen^2x+\cos^2x}{\cos^2x\cdot sen^2x}=\frac{1}{\cos^2x\cdot sen^2x}=\frac{1}{\cos^2x}\cdot\frac{1}{sen^2x}=sec^2\,x\cdot cosec^2\,x$$

i) $\mathbf{sec^4\,x - sec^2\,x = tg^4\,x + tg^2\,x}$

$$sec^4\,x-sec^2\,x=sec^2\,x\,(sec^2\,x\,-1)=sec^2\,x\left(\frac{1}{\cos^2x}-1\right)=sec^2\,x\,\frac{1-\cos^2x}{\cos^2x}=\frac{sen^2x}{\cos^4x\,\cdot}=\frac{sen^2x}{\cos^2x}\cdot\frac{1}{\cos^2x}=$$

$$=tg^2x\cdot sec^2x=tg^2x\cdot\left(tg^2x+1\right)=tg^4x+tg^2x$$