Resuelve las siguientes ecuaciones y sistemas:

1) $\text{sen } x = \frac{1}{2}$

2) $\cos x = -1$

3) $\text{tg } x = \frac{\sqrt{3}}{3}$

4) $2\cos^2 x = \cos 2x$

5) $\cos 2x = \cos x - 1$

6) $1 + \text{sen } 2x = (\text{sen } x + \cos x)$

7) $\cos 2x = 1 + 4 \text{ sen } x$

8) $\text{tg}^2 x - 3\,\text{tg}\, x + 2 = 0$

9) $\text{sen } 2x - \text{sen } x = 0$

10) $2\cos x + \cos x - 1 = 0$

11) $\sec x + \text{tg } x = 0$

12) $\text{sen}\, x + \cos x = 1$

13) $2 \cos x = 3 \text{ tg } x$

14) $\text{sen}\, x + \sqrt{3} \cos x = 2$

15) $4 \,\text{sen}\, \frac{x}{2} + 2 \cos x = 3$

SOLUCIONES

1) $\text{sen } x = \frac{1}{2} \quad \begin{cases} x_1 = 30^o + 360^o \cdot k \\ x_2 = 150^o + 360^o \cdot k \end{cases} ; k \in \mathbb{Z}$

2) $\cos x = -1 \Rightarrow x = 180^o + 360^o \cdot k \; ; k \in \mathbb{Z}$

3) $\text{tg } x = \frac{\sqrt{3}}{3} \Rightarrow \begin{cases} x_1 = 30^o + 360^o \cdot k \\ x_2 = 210^o + 360^o \cdot k \end{cases} \Rightarrow x = 30^o + 180^o \cdot k ; \; k \in \mathbb{Z}$

$\text{tg}^2 x - 3\,\text{tg}\, x + 2 = 0$

4) $2\cos^2 x = \cos 2x \Rightarrow 2\cos^2 x = \cos^2 x - \text{sen}^2 x \qquad \cos^2 x + \text{sen}^2 x = 0$!!! Sin solución.

5) $\cos 2x = \cos x - 1 \Rightarrow \cos^2 x - \text{sen}^2 x = \cos x - 1 \Rightarrow \cos^2 x - (1 - \cos^2 x) = \cos x - 1 \Rightarrow 2\cos^2 x - \cos x = 0$

$2\cos^2 x - \cos x = 0 \underset{t = \cos x}{\Rightarrow} 2t^{\;} - t = 0 \Rightarrow t \cdot (2t - 1) = 0 \begin{cases} t = 0 \rightarrow \cos x = 0 \rightarrow x = 90^o \text{ y } 270^o \\ t = \frac{1}{2} \rightarrow \cos x = - \qquad x = 60^o \text{ y } 300 \end{cases}$

Soluciones (con vueltas): $\begin{cases} x_1 = 90^{\;} + 180^o \cdot k \\ x_2 = 60^{\;} + 360^o \cdot k \\ x_3 = 300^{\;} + 360^o \cdot k \end{cases} ; k \in \mathbb{Z}$

6) $1 + \text{sen } 2x = (\text{sen } x + \cos x)$

$1 \cancel{+2\,\text{sen}\, x \cos x} = \text{sen } x + \cos^2 x + \cancel{2\,\text{sen}\, x \cos x} \Rightarrow 1 = \text{sen } x + \cos^2 x$

Es una identidad, no es una ecuación.

7) $\cos 2x = 1 + 4\,\text{sen}\, x$

$\cos^2 x - \text{sen}^2 x = 1 + 4\,\text{sen}\, x \Rightarrow 1 - \text{sen}^2 x - \text{sen}^2 x = 1 + 4\,\text{sen}\, x \Rightarrow 2\,\text{sen}^2 x + 4\,\text{sen}\, x = 0 \Rightarrow$

$\Rightarrow 2\,\text{sen}\, x \cdot (\text{sen}\, x + 2) = 0 \begin{cases} \text{sen}\, x = 0 \rightarrow x = 0 \text{ y } 180^o \\ \cancel{\text{sen}\, x = -2} \end{cases}$

Solución: $x = 0^o + 180^o \cdot k \; ; k \in \mathbb{Z}$

8)

$\text{tg}^2 x - 3\,\text{tg}\, x + 2 = 0 \underset{t = \text{tg}\, x}{\Rightarrow} t^2 - 3t + 2 = 0 \Rightarrow t = \frac{3 \pm \sqrt{(-3)^2 - 4 \cdot 2}}{2} = \frac{3 \pm 1}{} \begin{cases} 2 \Rightarrow \text{tg}\, x = 2 \Rightarrow x = 63^o\ 26'\ 6'' \\ 1 \Rightarrow \text{tg}\, x = 1 \Rightarrow x = 45^o \end{cases}$

La tangente es positiva en los cuadrantes I y III.

Soluciones (con vueltas): $\begin{cases} x_1 = 45^o + 180^{\;} \cdot k \\ x_2 = 63^o\ 26'\ 6'' + 180^o \cdot k \end{cases} ; k \in \mathbb{Z}$

9) $\operatorname{sen} 2x - \operatorname{sen} x = 0$

$$2\operatorname{sen} x \cdot \cos x - \operatorname{sen} x = 0 \Rightarrow \operatorname{sen} x \cdot (2\cos x - 1) = 0 \begin{cases} \operatorname{sen} x = 0 \rightarrow x = 0^o \text{ y } 180 \\ \cos x = \dfrac{1}{2} \rightarrow x = 60^o \text{ y } 300 \end{cases}$$

$$\operatorname{sen} x + \cos x = 1$$

$$\text{Soluciones: } \begin{cases} x_1 = 0^o + 180^o \cdot k \\ x_2 = 60^o + 360^o \cdot k \\ x_3 = 300^o + 360^o \cdot k \end{cases} ; k \in \mathbb{Z}$$

10) $2\cos^2 x + \cos x - 1 = 0$

$$2\cos^2 x + \cos x - 1 = 0 \underset{t=\cos x}{\Rightarrow} 2t^2 + t - 1 = 0 \Rightarrow t = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 2 \cdot (-1)}}{2 \cdot 2} = \frac{-1 \pm \sqrt{1+8}}{4} =$$

$$= \frac{-1 \pm 3}{4} \begin{cases} \dfrac{1}{2} \Rightarrow \cos x = - \quad x = 60^o \text{ y } 300^o \\ -1 \Rightarrow \cos x = -1 \Rightarrow x = 135^o \text{ y } 225^o \end{cases}$$

$$\text{Soluciones: } \begin{cases} x_1 = 60^o + 360^o \cdot k \\ x_2 = 300^o + 360^o \cdot k \\ x_3 = 135^o + 360^o \cdot k \\ x_4 = 225^o + 360^o \cdot k \end{cases} ; \; k \in \mathbb{Z}$$

11) $\sec x + \operatorname{tg} x = 0$

$$\frac{1}{\cos x} + \frac{\operatorname{sen} x}{\cos x} = 0 \Rightarrow \frac{1 + \operatorname{sen} x}{\cos x} = 0 \Rightarrow \operatorname{sen} x = -1 \rightarrow x = 270^o + 360^o \cdot k; \; k \in \mathbb{Z}$$

12)

$$(\cos x + \operatorname{sen} x)^2 = 1^2 \rightarrow \underbrace{\cos^2 x + \operatorname{sen}^2 x}_{1} + 2\operatorname{sen} x \cos x = 1 \rightarrow 2\operatorname{sen} x \cos x = 0 \begin{cases} \operatorname{sen} x = 0 \\ \cos x = 0 \end{cases}$$

$$\left.\begin{array}{l} \operatorname{sen} x = 0 \Rightarrow \begin{cases} x = 0^o + 360^o \cdot k \\ x = 180^o + 360^o \cdot k \end{cases} \\ \cos x = 0 \Rightarrow \begin{cases} x = 90^o + 360^o \cdot k \\ x = 270^o + 360^o \cdot k \end{cases} \end{array}\right\} \Rightarrow x = 0^o + 90^o \cdot k, k \in \mathbb{Z}$$

13) $2\cos x = 3\,\mathrm{tg}\,x$

$$2\cos x = \frac{3\,\mathrm{sen}\,x}{\cos x} \Rightarrow 2\cos^2 x = 3\,\mathrm{sen}\,x \Rightarrow 2(1-\mathrm{sen}^2 x) = 3\,\mathrm{sen}\,x \Rightarrow 2\,\mathrm{sen}^2 x + 3\,\mathrm{sen}\,x - 2 = 0$$

$$2t^2 + 3t - 2 = 0 \Rightarrow t = \frac{-3 \pm \sqrt{3^2 - 4\cdot 2\cdot(-2)}}{2\cdot 2} = \frac{-3 \pm \sqrt{9+16}}{4} = \frac{-3 \pm 5}{4} \begin{cases} \nearrow \ \mathrm{sen}\,x = \frac{1}{2} \rightarrow x = 30^o \text{ y } 150 \\ \searrow \ \cancel{\mathrm{sen}\,x = -2} \end{cases}$$

$$\text{Soluciones: } \begin{cases} x_1 = 30^o + 360\cdot k \\ x_2 = 150^o + 360\cdot k \end{cases};\ k \in \mathbb{Z}$$

14) $\mathrm{sen}\,x + \sqrt{3}\cos x = 2$

$$\mathrm{sen}\,x + \sqrt{3}\cos x = 2 \quad \underset{\text{divido entre 2}}{} \quad \frac{1}{2}\mathrm{sen}\,x + \frac{\sqrt{\ }}{2}\cos x = 1 \Rightarrow \mathrm{sen}(x + 60^o) = 1$$

$$\mathrm{sen}(x+60^o) = 1 \Rightarrow x + 60^o = 90^o + 360^o\cdot k \Rightarrow x = 30^o + 360^o\cdot k$$

Soluciones: $x = 30^o + 360^o\cdot k$; $k \in \mathbb{Z}$

15)

$$4\,\mathrm{sen}\frac{x}{2} + 2\left(\cos 2\frac{x}{2}\right) = 3 \Rightarrow 4\,\mathrm{sen}\frac{x}{2} + 2\left(\cos^2\frac{x}{2} - \mathrm{sen}^2\frac{x}{2}\right) = 3 \Rightarrow 4\,\mathrm{sen}\frac{x}{2} + 2\left(1 - \mathrm{sen}^2\frac{x}{2} - \mathrm{sen}^2\frac{x}{2}\right) = 3$$

$$4\,\mathrm{sen}\frac{x}{2} + 2 - 4\,\mathrm{sen}^2\frac{x}{2} = 3 \Rightarrow 4\,\mathrm{sen}^2\frac{x}{2} - 4\,\mathrm{sen}\frac{x}{2} + 1 = 0 \underset{t=\mathrm{sen}\frac{x}{2}}{\Rightarrow} 4t^2 - 4t + 1 = 0 \Rightarrow$$

$$\Rightarrow t = \frac{4 \pm \sqrt{(-4)^2 - 4\cdot 4\cdot 1}}{2\cdot 4} = \frac{4 \pm \sqrt{16-16}}{8} = \frac{4}{8} = \frac{1}{2}$$

$$\text{Entonces: } \mathrm{sen}\,\frac{x}{2} = \frac{1}{2} \Rightarrow \begin{cases} \frac{x_1}{2} = 30^o + 360^o\cdot k \\ \frac{x_2}{2} = 150^o + 360^o\cdot k \end{cases} \Rightarrow \begin{cases} x_1 = 60^o + 720\cdot k \\ x_2 = 300^o + 720\cdot k \end{cases};\ k \in \mathbb{Z}$$