

ACTIVIDADES: SISTEMAS DE ECUACIONES. INECUACIONES.

1. Resuelve los siguientes sistemas:

$$a) \left\{ \begin{array}{l} \frac{2}{x} + \frac{3}{y} = 7 \\ \frac{3}{x} + \frac{2}{y} = \frac{11}{2} \end{array} \right\} \quad b) \left\{ \begin{array}{l} 2y - x = 4 \\ x^2 - y^2 = -5 \end{array} \right\} \quad c) \left\{ \begin{array}{l} x^2 - y^2 = 9 \\ x \cdot y = 20 \end{array} \right\} \quad d) \left\{ \begin{array}{l} x + y = 41 \\ \sqrt{x} + \sqrt{y} = 9 \end{array} \right\}$$

$$e) \left\{ \begin{array}{l} 2^x \cdot 2^y = 256 \\ x - y = 4 \end{array} \right\} \quad f) \left\{ \begin{array}{l} 5 \cdot 2^{x+y} = 80 \\ 9^x = 3^{y-1} \end{array} \right\} \quad g) \left\{ \begin{array}{l} 2^{x+1} + 3^y = 35 \\ 5 \cdot 2^x - 2 \cdot 3^y = -34 \end{array} \right\}$$

$$h) \left\{ \begin{array}{l} 2^x + 2^y = 24 \\ 2^{x+y} = 128 \end{array} \right\} \quad i) \left\{ \begin{array}{l} 5^x = 5^y \cdot 625 \\ 2^x \cdot 2^y = 256 \end{array} \right\} \quad j) \left\{ \begin{array}{l} x + y = 22 \\ \log x - \log y = 1 \end{array} \right\}$$

$$k) \left\{ \begin{array}{l} 2 \log x - 3 \log y = 7 \\ \log x + \log y = 1 \end{array} \right\} \quad l) \left\{ \begin{array}{l} \log(x+y) - \log(x-y) = \log 33 \\ 2^x \cdot 2^y = 2^{11} \end{array} \right\}$$

$$m) \left\{ \begin{array}{l} \log_2 x + 3 \cdot \log_2 y = 5 \\ \log_2 x^2 - \log_2 y = 3 \end{array} \right\} \quad n) \left\{ \begin{array}{l} \log x - \log y = 5 \\ \log x^3 + \log y^2 = 10 \end{array} \right\}$$

2. Aplica el método de Gauss para discutir y resolver los sistemas siguientes:

$$a) \left\{ \begin{array}{l} x + y + z = 6 \\ 2x - y + 2z = 3 \\ 3x + 2y - 3z = 3 \end{array} \right. \quad SCD (1, 3, 2)$$

$$b) \left\{ \begin{array}{l} x - y - z = -2 \\ 2x + 3y - z = 2 \\ 4x + y - 3z = -2 \end{array} \right. \quad SCI (4 - 4t, t, 6 - 5t)$$

$$c) \left\{ \begin{array}{l} x - 9y + 5z = 33 \\ x + 3y - z = -9 \\ x - y + z = 5 \end{array} \right. \quad SCI (-t - 2, t, 2t + 7)$$

$$d) \left\{ \begin{array}{l} x + 3y = 2 \\ x + 2y + z = 1 \\ y - z = -1 \end{array} \right. \quad SI \text{ (Sin solución)}$$

$$e) \left\{ \begin{array}{l} x - 2y + 3z = 4 \\ 2x - y - z = 1 \\ x + y - 4z = 0 \end{array} \right. \quad SI \text{ (no tiene solución)}$$

$$f) \left\{ \begin{array}{l} x + y + z = 2 \\ 2x + 3y + 5z = 11 \\ x - 5y + 6z = 29 \end{array} \right. \quad SCD (1, -2, 3)$$

3. Resuelve los siguientes sistemas de inecuaciones lineales:

$$a) \left\{ \begin{array}{l} \frac{7-3x}{2} < x+1 \\ \frac{x+4}{3} \geq x \end{array} \right\} \quad (1, 2]$$

$$b) \left\{ \begin{array}{l} -3 \cdot (x-3) \leq x+1 \\ \frac{-x+2}{3} < 1 - \frac{x+6}{9} \end{array} \right\} \quad [2, +\infty)$$

$$c) \left\{ \begin{array}{l} 3x-1 \leq 2x-6 \\ 5x+4 > x \end{array} \right\} \quad (\emptyset)$$

4. Encuentra el conjunto solución de las siguientes inecuaciones cuadráticas y con denominadores:

$$a) 6x^2 - x - 1 > 0 \quad \left(-\infty, -\frac{1}{3}\right) \cup \left(\frac{1}{2}, +\infty\right)$$

$$b) 6x^2 - x - 1 \leq 0 \quad \left[\frac{-1}{3}, \frac{1}{2}\right]$$

$$c) -x^2 + 2x + 3 \geq 0 \quad [-1, 3]$$

$$d) 9x^2 - 1 < 0 \quad \left(-\frac{1}{3}, \frac{1}{3}\right)$$

$$e) x^2 - 4x + 4 > 0 \quad \mathbb{R} - \{2\}$$

$$f) x^2 - 4x + 4 < 0 \quad (\emptyset)$$

$$g) \frac{3-x}{x+1} \geq 0 \quad (-1, 3]$$

$$h) \frac{x^2-4}{-2 \cdot x} \leq 0 \quad [-2, 0) \cup [2, +\infty)$$

$$i) \frac{x+4}{2x^2+x-3} \leq 0 \quad (-\infty, -4] \cup \left(-\frac{3}{2}, 1\right)$$

$$j) \frac{4x+1}{2x-5} \geq 0 \quad (-\infty, -1/4] \cup \left(\frac{5}{2}, +\infty\right)$$

$$k) \frac{2}{x^2+4} > 0 \quad (\mathbb{R})$$

$$l) \frac{2}{x^2+4} < 0 \quad (\emptyset)$$

$$1.- \begin{cases} \frac{2}{x} + \frac{3}{y} = 7 \\ \frac{3}{x} + \frac{2}{y} = \frac{11}{2} \end{cases} \Rightarrow 2y + 3x = 7xy \Rightarrow 3x = 7xy - 2y \Rightarrow 3x = (7x-2)y \Rightarrow \frac{3x}{7x-2} = y \quad (1)$$

$$\Downarrow$$

$$6y + 4x = 11xy \xrightarrow{(1)} 6 \cdot \left(\frac{3x}{7x-2} \right) + 4x = 11x \cdot \left(\frac{3x}{7x-2} \right)$$

$$\Rightarrow \frac{18x}{7x-2} + \frac{4x(7x-2)}{7x-2} = \frac{33x^2}{7x-2} \Rightarrow 0 = 5x^2 - 10x = x(5x-10)$$

$$x \neq 0 \quad 5x-10=0 \rightarrow \boxed{x=2}$$

$x=0$ no puede ser válida por estar en un denominador

Por tanto $x=2$, $y = \frac{3 \cdot 2}{7 \cdot 2 - 2} = \frac{6}{12} = \frac{1}{2}$

$$\boxed{\begin{matrix} x=2 \\ y=1/2 \end{matrix}}$$

OTRA FORMA: (Haciendo una reducción doble)

$$\begin{cases} \frac{2}{x} + \frac{3}{y} = 7 \\ \frac{3}{x} + \frac{2}{y} = \frac{11}{2} \end{cases} \begin{cases} \cdot (-3) \rightarrow \frac{-6}{x} - \frac{9}{y} = -21 \\ \cdot 2 \rightarrow \frac{6}{x} + \frac{4}{y} = 11 \end{cases}$$

$$\frac{-5}{y} = 10 \Rightarrow \boxed{y = \frac{-5}{-10} = \frac{1}{2}}$$

$$\begin{cases} \frac{2}{x} + \frac{3}{y} = 7 \\ \frac{3}{x} + \frac{2}{y} = \frac{11}{2} \end{cases} \begin{cases} \cdot (-2) \rightarrow \frac{-4}{x} - \frac{6}{y} = -14 \\ \cdot 3 \rightarrow \frac{9}{x} + \frac{6}{y} = \frac{33}{2} \end{cases}$$

$$\frac{5}{x} = -14 + \frac{33}{2} \Rightarrow \frac{5}{x} = \frac{-28+33}{2} = \frac{5}{2} \Rightarrow \boxed{x=2}$$

$$2.- \begin{cases} 2y-x=4 \\ x^2-y^2=-5 \end{cases} \begin{cases} x=2y-4 \\ (2y-4)^2 - y^2 = -5 \Rightarrow 4y^2 + 16 - 16y - y^2 + 5 = 0 \end{cases}$$

$$\Rightarrow 3y^2 - 16y + 21 = 0 \Rightarrow y = \frac{16 \pm \sqrt{16^2 - 4 \cdot 3 \cdot 21}}{6} = \frac{16 \pm 2}{6} \begin{cases} 3 \\ \frac{14}{6} = \frac{7}{3} \end{cases}$$

$$\boxed{\begin{matrix} y=3 \Rightarrow x=2 \\ y=\frac{7}{3} \Rightarrow x=\frac{14}{3}-4=\frac{2}{3} \end{matrix}}$$

$$c) \begin{cases} x^2 - y^2 = 9 \\ x \cdot y = 20 \end{cases} \rightarrow y = \frac{20}{x}$$

$$\text{Si } x^2 = 25 \Rightarrow x = \pm 5$$

$$x^2 = -16 \text{ no tiene solución}$$

$$x^2 - \frac{20^2}{x^2} = 9$$

$$x^4 - 9x^2 + 400 = 0 \quad x^2 = t$$

$$t^2 - 9t - 400 = 0 \Rightarrow t = \frac{9 \pm \sqrt{81 + 1600}}{2}$$

$$t = \frac{9 \pm 41}{2} \begin{matrix} < 25 \\ < -16 \end{matrix}$$

$$\text{Soluciones: } \boxed{x=5, y=4} ; \boxed{x=-5, y=-4}$$

$$d) \begin{cases} x+y=41 \\ \sqrt{x}+\sqrt{y}=9 \end{cases} \Rightarrow y=41-x$$

$$\sqrt{x} + \sqrt{41-x} = 9 \Rightarrow \sqrt{41-x} = 9 - \sqrt{x} \Rightarrow$$

$$\Rightarrow (\sqrt{41-x})^2 = (9 - \sqrt{x})^2 \Rightarrow 41-x = 81+x-18\sqrt{x} \Rightarrow$$

$$\Rightarrow 18\sqrt{x} = 2x+40 \xrightarrow{12} 9\sqrt{x} = x+20$$

$$(9\sqrt{x})^2 = (x+20)^2 \Rightarrow 81x = x^2 + 40x + 400 \Rightarrow 0 = x^2 - 41x + 400$$

$$\boxed{x=25; y=16} \text{ válida, pues } \sqrt{25} + \sqrt{16} = 9$$

$$\boxed{x=16; y=25} \text{ válida.}$$

$$x < \begin{matrix} 25 \\ 16 \end{matrix}$$

$$e) \begin{cases} 2^x + 2^y = 256 \\ x-y=4 \end{cases} \Rightarrow \begin{cases} 2^{x+y} = 2^8 \\ x-y=4 \end{cases}$$

$$x+y=8$$

$$x-y=4$$

$$2x=12 \Rightarrow \boxed{x=6; y=2}$$

$$f) \begin{cases} 5 \cdot 2^{x+y} = 80 \\ 9^x = 3^{y-1} \end{cases}$$

$$\begin{cases} 5 \cdot 2^{x+y} = 5 \cdot 2^4 \\ 3^{2x} = 3^{y-1} \end{cases}$$

$$x+y=4$$

$$2x=y-1$$

$$\begin{cases} x+y=4 \\ 2x-y=-1 \end{cases}$$

$$3x=3$$

$$\boxed{x=1; y=3}$$

$$g) \begin{cases} 2^{x+1} + 3^y = 35 \\ 5 \cdot 2^x - 2 \cdot 3^y = -34 \end{cases}$$

$$2 \cdot 2^x + 3^y = 35$$

$$5 \cdot 2^x - 2 \cdot 3^y = -34$$

$$\begin{cases} 2^x = t \\ 3^y = z \end{cases}$$

Cambio de variable

$$2t + z = 35$$

$$5t - 2z = -34$$

$$\Rightarrow \begin{cases} t=4 \\ z=27 \end{cases}$$

$$t = 2^x = 4 = 2^2 \Rightarrow x=2$$

$$z = 27 = 3^3 = 3^y \Rightarrow y=3$$

$$h) \begin{cases} 2^x + 2^y = 24 \\ 2^{x+y} = 128 \end{cases}$$

$$\rightarrow 2^{x+y} = 2^7 \Rightarrow x+y=7 \Rightarrow y=7-x$$

$$\text{Entonces } 2^x + 2^{7-x} = 24 \Rightarrow 2^x + \frac{2^7}{2^x} = 24 \xrightarrow{2^x=t}$$

$$t + \frac{128}{t} = 24 \Rightarrow t^2 - 24t + 128 = 0 \Rightarrow t = \frac{24 \pm \sqrt{64}}{2}$$

$$t = \frac{24 \pm 8}{2} < \frac{16}{8}$$

$$\text{Si } t = 2^x = 16 = 2^4 \rightarrow x = 4$$

$$\text{Si } t = 2^x = 8 = 2^3 \rightarrow x = 3$$

$$\boxed{\begin{array}{l} x=4 \Rightarrow y=3 \\ x=3 \Rightarrow y=4 \end{array}} \text{ Soluciones.}$$

$$i) \quad \left. \begin{array}{l} 5^x = 5^y \cdot 625 \\ 2^x \cdot 2^y = 256 \end{array} \right\} \begin{array}{l} 5^x = 5^y \cdot 5^4 \Rightarrow x = y + 4 \\ 2^{x+y} = 2^8 \Rightarrow x + y = 8 \end{array} \Rightarrow \boxed{\begin{array}{l} x = 6 \\ y = 2 \end{array}}$$

$$j) \quad \left. \begin{array}{l} x + y = 22 \\ \log x - \log y = 1 \end{array} \right\} \begin{array}{l} y = 22 - x \\ \log\left(\frac{x}{y}\right) = \log 10 \Rightarrow \frac{x}{22-x} = 10 \end{array}$$

$$\Rightarrow x = 220 - 10x \Rightarrow 11x = 220 \Rightarrow \boxed{x = 20 \quad y = 2} \text{ son válidas}$$

$$k) \quad \left. \begin{array}{l} 2 \log x - 3 \log y = 7 \\ \log x + \log y = 1 \end{array} \right\} \rightarrow \begin{array}{l} 2 \log x - 3 \log y = 7 \\ -2 \log x + 2 \log y = -2 \end{array}$$

$$-5 \log y = 5$$

$$\log y = -1$$

$$\boxed{y = 10^{-1}}$$

$$\Downarrow$$

$$\begin{array}{l} 2 \log x - 3 \log y = 7 \\ + 3 \log x + 3 \log y = 3 \end{array}$$

$$\underline{5 \log x = 10} \Rightarrow \log x = 2 \Rightarrow \boxed{x = 10^2}$$

$$\boxed{x = 100; y = \frac{1}{10}}$$

$$l) \quad \log(x+y) - \log(x-y) = \log 33 \quad \left. \begin{array}{l} \frac{x+y}{x-y} = 33 \\ x+y=11 \rightarrow y=11-x \end{array} \right\}$$

$$2^x \cdot 2^y = 2^{11}$$

$$11 = 33(x - 11 + x)$$

$$11 = 33x - 363 + 33x$$

$$\Rightarrow 374 = 66x \Rightarrow \boxed{x = \frac{17}{3} \quad y = \frac{16}{3}}$$

válida

$$m) \begin{cases} \log_2 x + 3 \log_2 y = 5 \\ \log_2 x^2 - \log_2 y = 3 \end{cases} \rightarrow \begin{cases} \log_2 x + 3 \log_2 y = 5 \\ 2 \log_2 x - \log_2 y = 3 \end{cases} \cdot \underline{(+3)}$$

$$\begin{array}{r} \log_2 x + 3 \log_2 y = 5 \\ + 6 \log_2 x + 3 \log_2 y = 9 \\ \hline 7 \log_2 x = 14 \end{array}$$

$$\log_2 x = 2 \Rightarrow x = 2^2 = 4$$

$$2 \log_2 x - \log_2 y = 3 \Rightarrow 2 \cdot 2 - \log_2 y = 3 \Rightarrow -\log_2 y = 3 - 4$$

$$\Rightarrow \log_2 y = 1 \Rightarrow y = 2^1 = 2$$

$$\boxed{\begin{array}{l} x = 4 \\ y = 2 \end{array}}$$

$$n) \begin{cases} \log x - \log y = 5 \\ \log x^3 + \log y^2 = 10 \end{cases} \rightarrow \begin{cases} \log x - \log y = 5 \\ 3 \log x + 2 \log y = 10 \end{cases}$$

$$\cdot (2) \rightarrow \begin{array}{r} 2 \log x - 2 \log y = 10 \\ 3 \log x + 2 \log y = 10 \\ \hline 5 \log x = 20 \end{array}$$

$$\Rightarrow \log x = 4 \quad \boxed{x = 10^4}$$

$$\log 10^4 - \log y = 5 \Rightarrow 4 - 5 = \log y \Rightarrow -1 = \log y \Rightarrow \boxed{y = 10^{-1}}$$

$$2.- a) \begin{cases} x + y + z = 6 \\ 2x - y + 2z = 3 \\ 3x + 2y - 3z = 3 \end{cases} \rightarrow \begin{array}{l} -2E_1 + E_2 \rightarrow E_2 \\ -3E_1 + E_3 \rightarrow E_3 \end{array} \rightarrow \begin{cases} x + y + z = 6 \\ -3y = -9 \\ -y - 6z = -15 \end{cases}$$

$$\text{Despejando: } y = 3; \quad z = \frac{-y + 15}{-6} = \frac{-3 + 15}{-6} = 2; \quad x = 6 - y - z = 6 - 3 - 2 = 1.$$

$$\text{Sistema Compatible Determinado: } \boxed{x=1; y=3; z=2}$$

$$b) \begin{cases} x - y - z = -2 \\ 2x + 3y - z = 2 \\ 4x + y - 3z = -2 \end{cases} \rightarrow \begin{array}{l} -2E_1 + E_2 \rightarrow E_2 \\ -4E_1 + E_3 \rightarrow E_3 \end{array} \rightarrow \begin{cases} x - y - z = -2 \\ 5y + z = 6 \\ 5y + z = 6 \end{cases} \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{son iguales.}$$

Por tanto el sistema es COMPATIBLE INDETERMINADO, tendrá infinitas soluciones que dependen de una incógnita:

$$\text{Consideremos } \begin{cases} x - y - z = -2 \\ 5y + z = 6 \end{cases} \rightarrow \begin{array}{l} \text{Sea } y = t \\ \text{parámetro} \end{array} \rightarrow \begin{cases} x - z = -2 + t \\ z = 6 - 5t \end{cases}$$

Entonces: $x - (6 - 5t) = -2 + t \Rightarrow x - 6 + 5t = -2 + t \Rightarrow x = 4 - 4t$

Con ello, la solución tiene la forma:

$$\begin{cases} x = 4 - 4t \\ y = t \\ z = 6 - 5t \end{cases} ; t \in \mathbb{R}$$

c)
$$\begin{cases} x - 9y + 5z = 33 \\ x + 3y - z = -9 \\ x - y + z = 5 \end{cases} \begin{cases} -E_1 + E_2 \rightarrow E_2 \\ -E_1 + E_3 \rightarrow E_3 \end{cases}$$

$$\rightarrow \begin{cases} x - 9y + 5z = 33 \\ 2y - z = -7 \\ 2y - z = -7 \end{cases} \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{iguales}$$

$$\begin{cases} x - 9y + 5z = 33 \\ 12y - 6z = -42 \\ 8y - 4z = -28 \end{cases} \begin{cases} E_2/6 \\ E_3/4 \end{cases}$$

Sistema Compatible Indeterminado.

Sea
$$\begin{cases} x - 9y + 5z = 33 \\ 2y - z = -7 \end{cases} \begin{cases} \rightarrow x + 5z = 33 + 9y \\ \rightarrow -z = -7 - 2y \end{cases}$$
 Sea $y = t$ parámetro.

$$\begin{cases} x + 5z = 33 + 9t \\ z = 7 + 2t \end{cases} \begin{cases} x + 5(7 + 2t) = 33 + 9t \\ x + 35 + 10t = 33 + 9t \end{cases} \Rightarrow x = -5 - t$$

$$\begin{cases} x = -5 - t \\ y = t \\ z = 7 + 2t \end{cases} ; t \in \mathbb{R}$$

d)
$$\begin{cases} x + 3y = 2 \\ x + 2y + z = 1 \\ y - z = -1 \end{cases} \begin{cases} -E_1 + E_2 \rightarrow E_2 \end{cases}$$

$$\begin{cases} x + 3y = 2 \\ -y + z = -1 \\ y - z = -1 \end{cases} \begin{cases} y - z = 1 \\ y - z = -1 \end{cases}$$

las dos últimas ecuaciones son incompatibles; luego el sistema es incompatible.

e)
$$\begin{cases} x - 2y + 3z = 4 \\ 2x - y - z = 1 \\ x + y - 4z = 0 \end{cases} \begin{cases} -2E_1 + E_2 \\ -E_1 + E_3 \end{cases}$$

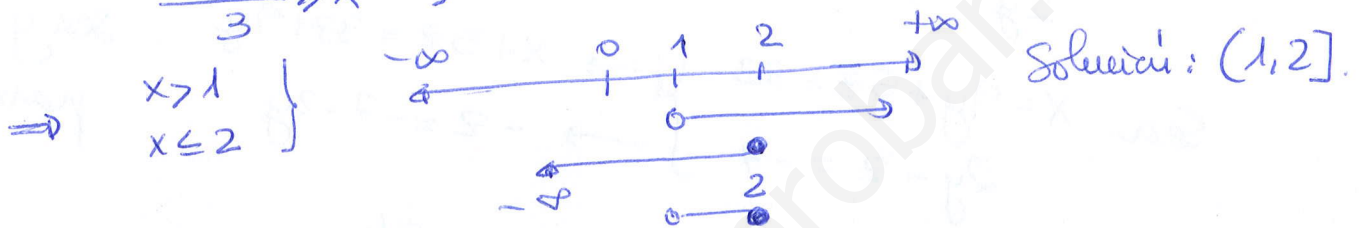
$$\begin{cases} x - 2y + 3z = 4 \\ 3y - 7z = -7 \\ 3y - 7z = -4 \end{cases} \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Incompatible.}$$

sistema Incompatible. No hay solución.

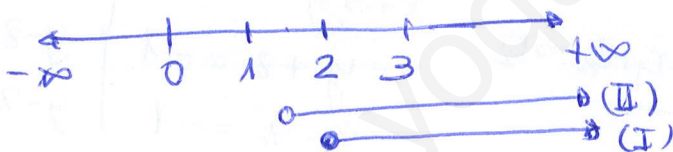
$$f) \begin{cases} x+y+z=2 \\ 2x+3y+5z=11 \\ x-5y+6z=29 \end{cases} \left\{ \begin{array}{l} -2E_1+E_2 \rightarrow E_2 \\ -E_1+E_3 \rightarrow E_3 \end{array} \right. \quad \begin{cases} x+y+z=2 \\ y+3z=7 \\ -6y+5z=27 \end{cases} \left\{ \begin{array}{l} 6E_2+E_3 \end{array} \right.$$

$$\begin{cases} x+y+z=2 \\ y+3z=7 \\ 23z=69 \end{cases} \begin{cases} x=1 \\ y=-2 \\ z=3 \end{cases} \quad S \text{ Compatible Determin.}$$

$$3. a) \begin{cases} \frac{7-3x}{2} < x+1 \\ \frac{x+4}{3} \geq x \end{cases} \left\{ \begin{array}{l} \Rightarrow 7-3x < 2x+2 \Rightarrow -5x < -5 \Rightarrow 5x > 5 \\ \Rightarrow x+4 \geq 3x \Rightarrow -2x \geq -4 \Rightarrow 2x \leq 4 \end{array} \right.$$

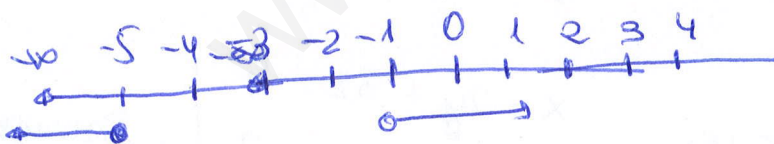


$$b) \begin{cases} -3(x-3) \leq x+1 \\ \frac{-x+2}{3} < 1 - \frac{x+6}{9} \end{cases} \left\{ \begin{array}{l} \Rightarrow -3x+9 \leq x+1 \Rightarrow -4x \leq -8 \Rightarrow 4x \geq 8 \Rightarrow x \geq 2 \text{ (I)} \\ \Rightarrow 3(-x+2) < 9-(x+6) \Rightarrow -3x+6 < 9-x-6 \Rightarrow -2x < -3 \Rightarrow 2x > 3 \Rightarrow x > 3/2 \text{ (II)} \end{array} \right.$$



$$S = [2, +\infty)$$

$$c) \begin{cases} 3x-1 \leq 2x-6 \\ 5x+4 > x \end{cases} \left\{ \begin{array}{l} \Rightarrow 3x-2x \leq -6+1 \Rightarrow x \leq -5 \\ \Rightarrow 5x-x > -4 \Rightarrow 4x > -4 \Rightarrow x > -1 \end{array} \right.$$



$$S = \emptyset$$

$$4. a) 6x^2 - x - 1 > 0 \quad \text{Calculamos sus raíces: } 6x^2 - x - 1 = 0$$

$$x = \frac{1 \pm \sqrt{1+24}}{12} = \frac{1 \pm 5}{12} \quad \begin{matrix} 1/2 \\ -1/3 \end{matrix}$$



$$S = (-\infty, -\frac{1}{3}) \cup (\frac{1}{2}, +\infty)$$

b) $6x^2 - x - 1 \leq 0$.

Utilizando el diagrama de a)

$S = [-\frac{1}{3}, \frac{1}{2}]$

* Otra forma: usando una tabla como la que sigue:

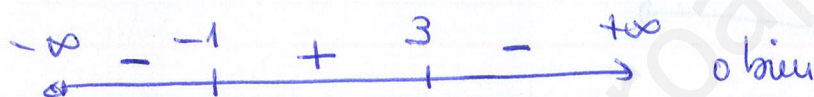
$6x^2 - x - 1 = 6(x + \frac{1}{3})(x - \frac{1}{2}) = (3x+1)(2x-1)$

	$(-\infty, -\frac{1}{3})$	$(-\frac{1}{3}, \frac{1}{2})$	$(\frac{1}{2}, +\infty)$
$3x+1$	-	+	+
$2x-1$	-	-	+
$(3x+1)(2x-1)$	+	-	+

c) $-x^2 + 2x + 3 \geq 0$
 $-(x+1)(x-3) \geq 0$

Raíces de $-x^2 + 2x + 3$:

$-x^2 + 2x + 3 = 0 \Rightarrow x = \frac{-2 \pm \sqrt{4+12}}{-2} = \frac{-2 \pm 4}{-2} = -1, 3$



$S = [-1, 3]$

	$(-\infty, -1)$	$(-1, 3)$	$(3, +\infty)$
-1	-	+	+
$x+1$	-	+	+
$x-3$	-	-	+
	-	+	-

d) $9x^2 - 1 < 0$

Raíces: $9x^2 - 1 = 0 \Leftrightarrow x^2 = \frac{1}{9} \Rightarrow x = \pm \frac{1}{3}$.

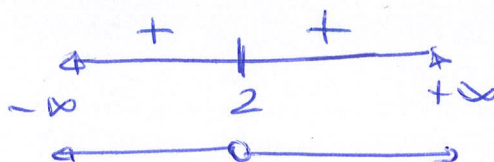


$S = (-\frac{1}{3}, \frac{1}{3})$

e) $x^2 - 4x + 4 > 0$

$(x-2)^2 > 0$

Raíces: $x^2 - 4x + 4 = 0 \Rightarrow x = 2$ doble



$S = (-\infty, 2) \cup (2, +\infty) = \mathbb{R} - \{2\}$.

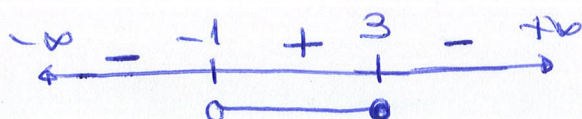
f) $x^2 - 4x + 4 < 0$

$\hookrightarrow$ Nunca es negativo, luego $S = \emptyset$.

g) $\frac{3-x}{x+1} \geq 0$

• Raíces del numerador: $3-x=0 \Leftrightarrow x=3$

• Raíces del denominador: $x+1=0 \Leftrightarrow x=-1$

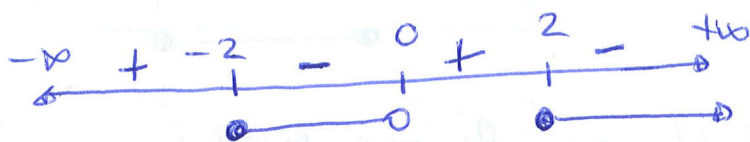


$S = [-1, 3]$.

$$h) \frac{x^2-4}{-2x} \leq 0$$

$$x^2-4=0 \Leftrightarrow x=\pm 2$$

$$-2x=0 \Leftrightarrow x=0$$

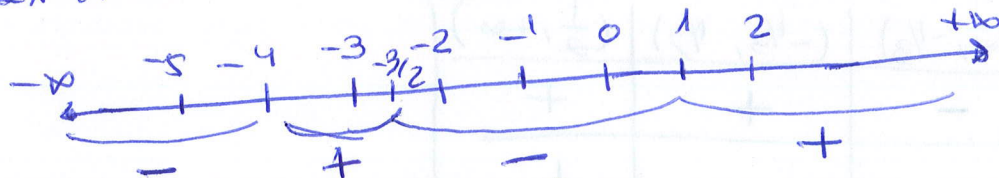


$$S = [-2, 0) \cup [2, +\infty)$$

$$i) \frac{x+4}{2x^2+x-3} \leq 0 \Rightarrow$$

$$x+4=0 \Rightarrow x=-4$$

$$2x^2+x-3=0 \Rightarrow x = \frac{-1 \pm \sqrt{1+24}}{4} = \frac{-1 \pm 5}{4} = \frac{1}{2}, -\frac{3}{2}$$

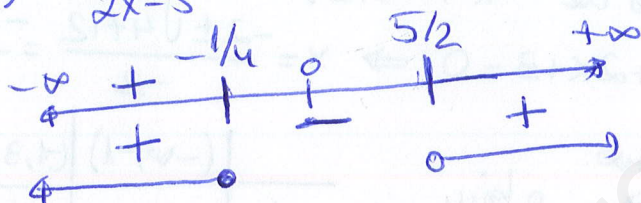


$$S = (-\infty, -4] \cup (-\frac{3}{2}, 1)$$

$$j) \frac{4x+1}{2x-5} \geq 0$$

$$4x+1=0 \rightarrow x=-\frac{1}{4}$$

$$2x-5=0 \Rightarrow x=\frac{5}{2}$$



$$S = (-\infty, -\frac{1}{4}] \cup (\frac{5}{2}, +\infty)$$

$$k) \frac{2}{x^2+4} > 0$$



No hay raíces

$$S = \mathbb{R}$$

$$l) \frac{2}{x^2+4} < 0$$

↳ Nunca es negativo

$$S = \emptyset$$

