Relaciones entre Razones Trigonométricas

1. Utilizando las razones trigonométricas de 30º, 45º y 60º, calcula el valor **exacto** y racionalizado de:
a) sen 75º **b)** sen 15º **c)** tg 135º **d)** tg 285º

2. Encuentra fórmulas que nos permitan calcular cos(3x) y cos(4x) en función del cos$x$

3. a) Sabiendo que $\begin{cases} sen\alpha = \dfrac{3}{5} & (90^o < \alpha < 180^o) \\ \cos\beta = \dfrac{5}{13} & (270^o < \beta < 360^o) \end{cases}$ halla <u>sin calculadora</u> los valores exactos de $sen(\alpha+\beta)$, $tg(\alpha-\beta)$ y $\cos(2\alpha+\beta)$ dando los resultados en forma de fracción irreducible.

b) Repítelo usando la calculadora escribiendo los resultados con tres cifras significativas comprobando así los resultados del apartado anterior.

4. Sabiendo que $tg\alpha = -\dfrac{40}{9}$ con $0 \le \alpha < \pi$:

a) Halla el valor **exacto** de $sen(2\alpha)$

b) Repítelo usando ahora la calculadora escribiendo los resultados con tres cifras significativas comprobando así el resultado del apartado anterior.

5. Utilizando la fórmula de la tangente de la suma de dos ángulos, demuestra: $arctg(1/2) + arctg(1/3) = \pi/4$

6. Halla <u>todos</u> los ángulos x, $0 \le x < 2\pi$, que resuelvan cada ecuación trigonométrica:

a) $\cos 3x = \dfrac{1}{2}$ **b)** $senx \cdot \cos x = 0$ **c)** $\cos(2x-\pi) = -\dfrac{1}{2}$

d) $senx + \cos x = 0$ **e)** $\cos^2 x = \dfrac{3}{4}$ **f)** $tg2x = 1$

g) $sen\dfrac{x}{2} = \dfrac{\sqrt{2}}{2}$ **h)** $\cot x + \dfrac{senx}{1+\cos x} = 2$ **i)** $sen(\pi - 3x) = -\dfrac{\sqrt{2}}{2}$

7. Resuelve las siguientes ecuaciones trigonométricas con $0 \le x < 2\pi$:

a) $\cos^2 x - sen^2 x = \dfrac{1}{2}$ **b)** $senx - \cos x = \dfrac{\sqrt{2}}{2}$ **c)** $tgx \cdot \sec x = \sqrt{2}$

d) $3\cos x = 2\sec x - 5$ **e)** $\log_2(\cos x) + 1 = \log_2(\cos ecx)$ **f)** $tg^2 2x = 1$

g) $\cos^2\left(\dfrac{x}{2}\right) - sen^2\left(\dfrac{x}{2}\right) = senx$ **h)** $\cos(2x) + senx = 4sen^2 x$ **i)** $tg(2x) = -tgx$

8. a) Demuestre que la ecuación $4\cos(2x) - 3senx \cdot \cos ec^3 x + 6 = 0$ puede expresarse como $8t^4 - 10t^2 + 3 = 0$

b) Partiendo de aquí, resuelva dicha ecuación para $0 \le x < \pi$.

9. a) Demuestre que $\dfrac{sen(2\alpha)}{1+\cos(2\alpha)} = tg\alpha$

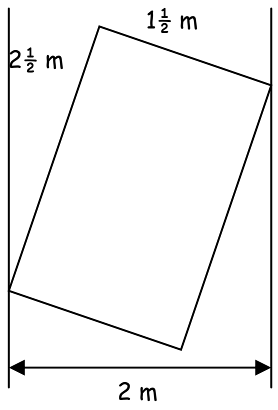
b) Partiendo de aquí, halle el valor de $ctg(\pi/8)$ en la forma $a + b\sqrt{2}$ con $a, b \in \mathbb{Z}$.

10. a) Investiga, utilizando una hoja de cálculo para ángulos positivos menores de 360º, entre qué valores oscila la resta de cinco veces su coseno menos doce veces su seno y para qué dos ángulos toma su máximo y su mínimo valor.

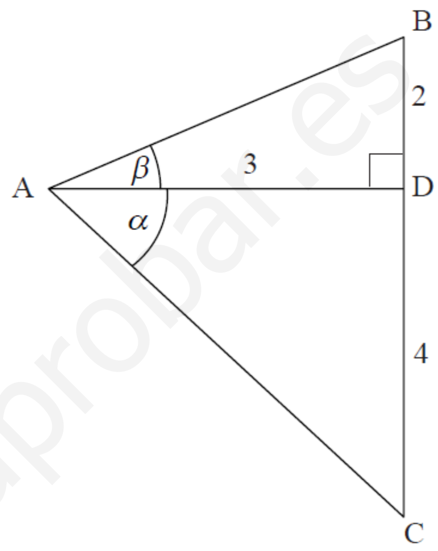
b) Halla *A* y α para que: $5\cos x - 12 senx = A \cdot \cos(x+\alpha)$ y con ello comprueba lo obtenido en el apartado anterior.

c) Partiendo del apartado anterior, resuelve $5\cos x - 12 senx = -2$ con $0 \le x < 2\pi$ comprobando las soluciones en la hoja de cálculo construida.

11. Una mesa rectangular de $1\frac{1}{2}$ m x $2\frac{1}{2}$ m se ha cruzado en un pasillo de 2m de ancho hasta tocar ambas paredes como muestra el diagrama. Calcula los ángulos determinados entre la mesa y las paredes en grados, minutos y segundos.



12. Halla el valor exacto de cos(α - β)



1 a) $\operatorname{sen}75^\circ = \operatorname{sen}(45^\circ+30^\circ) = \operatorname{sen}45^\circ\cos30^\circ + \cos45^\circ\operatorname{sen}30^\circ = \frac{\sqrt2}{2}\cdot\frac{\sqrt3}{2} + \frac{\sqrt2}{2}\cdot\frac12 = \boxed{\frac{\sqrt6+\sqrt2}{4}}$

b) $\operatorname{sen}15^\circ = \operatorname{sen}(45^\circ-30^\circ) = \operatorname{sen}45^\circ\cos30^\circ - \cos45^\circ\operatorname{sen}30^\circ = \frac{\sqrt2}{2}\cdot\frac{\sqrt3}{2} - \frac{\sqrt2}{2}\cdot\frac12 = \boxed{\frac{\sqrt6-\sqrt2}{4}}$

c) $\operatorname{tg}135^\circ = \operatorname{tg}(180^\circ-45^\circ) = \frac{\operatorname{tg}180^\circ-\operatorname{tg}45^\circ}{1+\operatorname{tg}180^\circ\cdot\operatorname{tg}45^\circ} = \frac{0-1}{1+0\cdot1} = \frac{-1}{1} = \boxed{-1}$

d) $\operatorname{tg}285^\circ = \operatorname{tg}(-75^\circ) = -\operatorname{tg}75^\circ = -\frac{\operatorname{sen}75^\circ}{\cos75^\circ} = -\frac{\operatorname{sen}75^\circ}{\operatorname{sen}15^\circ} = -\frac{\frac{\sqrt6+\sqrt2}{4}}{\frac{\sqrt6-\sqrt2}{4}} = -\frac{\sqrt6+\sqrt2}{\sqrt6-\sqrt2} =$

$= -\frac{(\sqrt6+\sqrt2)(\sqrt6+\sqrt2)}{(\sqrt6-\sqrt2)(\sqrt6+\sqrt2)} = -\frac{(\sqrt6)^2+2\sqrt6\sqrt2+(\sqrt2)^2}{(\sqrt6)^2-(\sqrt2)^2} = -\frac{6+2\sqrt{12}+2}{6-2} = -\frac{8+4\sqrt3}{4} = \boxed{-2-\sqrt3}$

2 a) $\cos(3x) = \cos(2x+x) = \cos2x\cos x - \operatorname{sen}2x\operatorname{sen}x = (\cos^2x-\operatorname{sen}^2x)\cos x - 2\operatorname{sen}x\cos x\cdot\operatorname{sen}x =$

$= (\cos^2x-1+\cos^2x)\cos x - 2\operatorname{sen}^2x\cos x = (2\cos^2x-1)\cos x - 2(1-\cos^2x)\cos x =$

$= (2\cos^2x-1-2+2\cos^2x)\cos x = \boxed{(4\cos^2x-3)\cos x} = \boxed{4\cos^3x-3\cos x}$

b) $\cos(4x) = \cos2(2x) = \cos^2(2x)-\operatorname{sen}^2(2x) = (\cos^2x-\operatorname{sen}^2x)^2-(2\operatorname{sen}x\cos x)^2 =$

$= (\cos^2x-1+\cos^2x)^2 - 4\operatorname{sen}^2x\cos^2x = (2\cos^2x-1)^2 - 4(1-\cos^2x)\cos^2x =$

$= 4\cos^4x-4\cos^2x+1-4\cos^2x+4\cos^4x = \boxed{8\cos^4x-8\cos^2x+1}$

3 a) $\operatorname{sen}a = \frac35 \rightarrow \cos a = -\sqrt{1-\frac{9}{25}} = -\frac45 \rightarrow \operatorname{tg}a = \frac{3/5}{-4/5} = -\frac34$

$\cos b = \frac{5}{13} \rightarrow \operatorname{sen}b = -\sqrt{1-\frac{25}{169}} = -\frac{12}{13} \rightarrow \operatorname{tg}b = \frac{-12/13}{5/13} = -\frac{12}{5}$

$\operatorname{sen}(a+b) = \operatorname{sen}a\cdot\cos b + \cos a\cdot\operatorname{sen}b = \frac35\cdot\frac{5}{13} + \frac{-4}{5}\cdot\frac{-12}{13} = \boxed{\frac{63}{65}} = 0'969$

$\operatorname{tg}(a-b) = \frac{\operatorname{tg}a-\operatorname{tg}b}{1+\operatorname{tg}a\cdot\operatorname{tg}b} = \frac{-3/4+12/5}{1+\frac{-3}{4}\cdot\frac{-12}{5}} = \boxed{\frac{33}{56}} = 0'589$

$\cos(2a+b) = \cos2a\cos b - \operatorname{sen}2a\operatorname{sen}b = (\cos^2a-\operatorname{sen}^2a)\cdot\cos b - 2\operatorname{sen}a\cos a\operatorname{sen}b =$

$= \left(\frac{16}{25}-\frac{9}{25}\right)\cdot\frac{5}{13} - 2\cdot\frac35\cdot\frac{-4}{5}\cdot\frac{-12}{13} = \boxed{-\frac{253}{325}} = -0'779$

b) $\operatorname{sen}a = \frac35 \rightarrow a = 143'13^\circ$; $\cos b = \frac{5}{13} \rightarrow b = 292'62^\circ$ $\rightarrow$

$\operatorname{sen}(a+b) = \operatorname{sen}(143'13^\circ+292'62^\circ) = 0'969$

$\operatorname{tg}(a-b) = \operatorname{tg}(143'13^\circ-292'62^\circ) = 0'589$

$\cos(2a+b) = \cos(2\cdot143'13^\circ+292'62^\circ) = -0'779$

4 a) $\operatorname{tg}\alpha = -\frac{40}{9} \rightarrow 1+\left(\frac{-40}{9}\right)^2 = \sec^2\alpha$; $1+\frac{1600}{81} = \sec^2\alpha$; $\sec^2\alpha = \frac{1681}{81}$;

$\sec\alpha = -\sqrt{\frac{1681}{81}} = -\frac{41}{9} \Rightarrow \cos\alpha = -\frac{9}{41} \rightarrow \operatorname{sen}\alpha = \operatorname{tg}\alpha\cdot\cos\alpha = \frac{-40}{9}\cdot\frac{-9}{41} = \frac{40}{41}$

$\operatorname{sen}(2\alpha) = 2\operatorname{sen}\alpha\cos\alpha = 2\cdot\frac{40}{41}\cdot\frac{-9}{41} = \boxed{\frac{-720}{1681}} \simeq -0'428$

b) $\operatorname{tg}\alpha = -\frac{40}{9} \rightarrow \alpha = -77'32+180 = 102'68^\circ \longrightarrow \operatorname{sen}(2\cdot102'68^\circ) = \boxed{-0'428}$

(5) $\operatorname{arcTg}(1/2) + \operatorname{arcTg}(1/3) = \pi/4$

$$\operatorname{Tg}\left(\operatorname{arcTg}(1/2) + \operatorname{arcTg}(1/3)\right) = \frac{\operatorname{Tg}(\operatorname{arcTg}(1/2)) + \operatorname{Tg}(\operatorname{arcTg}(1/3))}{1 - \operatorname{Tg}(\operatorname{arcTg}(1/2)) \cdot \operatorname{Tg}(\operatorname{arcTg}(1/3))} = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2}\cdot\frac{1}{3}} = \frac{\frac{5}{6}}{1 - 1/6} = \frac{5/6}{5/6} = 1$$

Cualquier $\operatorname{arcTg} x$ toma valores entre $-\pi/2$ y $\pi/2$, por lo tanto, la suma $\operatorname{arcTg}(1/2) + \operatorname{arcTg}(1/3)$ tomará algún valor entre $-\pi$ y π. En ese intervalo, el único ángulo con tangente igual a 1 es $45°$.

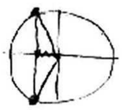
Por lo tanto: $\boxed{\operatorname{arcTg}(1/2) + \operatorname{arcTg}(1/3) = \pi/4}$ ✓

(6)

a) $\cos 3X = \frac{1}{2} \Rightarrow 3X = \begin{cases} 60° + N\cdot 360° \\ 300° + N\cdot 360° \end{cases}$; $X = \begin{cases} 20° + N\cdot 120° \\ 100° + N\cdot 120° \end{cases}$ $\boxed{X = \begin{cases} 20° \\ 140° \\ 260° \end{cases} \quad X = \begin{cases} 100° \\ 220° \\ 340° \end{cases}}$

b) $\operatorname{sen} X \cdot \cos X = 0 \Rightarrow$ $\operatorname{sen} X = 0$; $\cos X = 0$ $\boxed{X = \begin{cases} 0° \\ 180° \end{cases} \quad X = \begin{cases} 90° \\ 270° \end{cases}}$

c) $\cos(2X - \pi) = -\frac{1}{2} \Rightarrow 2X - 180° = \begin{cases} 120° + N\cdot 360° \\ 240° + N\cdot 360° \end{cases}$; $2X = \begin{cases} 300° + N\cdot 360° \\ 420° + N\cdot 360° \end{cases}$; $X = \begin{cases} 150° + N\cdot 180° \\ 210° + N\cdot 180° \end{cases}$ $\boxed{X = \begin{cases} 150° \\ 330° \end{cases} \quad X = \begin{cases} 210° \\ 30° \end{cases}}$

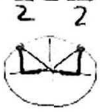


d) $\operatorname{sen} X + \cos X = 0$; $\operatorname{sen} X = -\cos X$; $\frac{\operatorname{sen} X}{\cos X} = -1$; $\operatorname{Tg} X = -1 \Rightarrow \boxed{X = \begin{cases} 135° \\ 315° \end{cases}}$

e) $\cos^2 X = \frac{3}{4}$; $\cos X = \pm\frac{\sqrt{3}}{2}$ $X = \begin{cases} 30° \\ 150° \\ 210° \\ 330° \end{cases}$

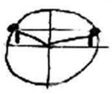
f) $\operatorname{Tg} 2X = 1 \Rightarrow 2X = \begin{cases} 45° + N\cdot 360° \\ 225° + N\cdot 360° \end{cases}$; $2X = 45 + N\cdot 180°$; $X = 22{,}5° + N\cdot 90°$; $\boxed{X = \begin{cases} 22{,}5° \\ 112{,}5° \\ 202{,}5° \\ 292{,}5° \end{cases}}$

g) $\operatorname{sen}\frac{X}{2} = \frac{\sqrt{2}}{2} \Rightarrow \frac{X}{2} = \begin{cases} 45° + N\cdot 360° \\ 135° + N\cdot 360° \end{cases}$; $X = \begin{cases} 90° + N\cdot 720° \\ 270° + N\cdot 720° \end{cases}$; $\boxed{X = \begin{cases} 90° \\ 270° \end{cases}}$



h) $\operatorname{ctg} X + \frac{\operatorname{sen} X}{1 + \cos X} = 2$; $\frac{\cos X}{\operatorname{sen} X} + \frac{\operatorname{sen} X}{1 + \cos X} = 2$; $\frac{\cos X + \cos^2 X + \operatorname{sen}^2 X}{\operatorname{sen} X(1 + \cos X)} = 2$; $\frac{1 + \cancel{\cos X}}{\operatorname{sen} X(1 + \cancel{\cos X})} = 2$;

$\frac{1}{\operatorname{sen} X} = 2$; $\operatorname{sen} X = \frac{1}{2} \Rightarrow \boxed{X = \begin{cases} 30° \\ 150° \end{cases}}$



i) $\sin(\pi - 3x) = -\frac{\sqrt{2}}{2} \Rightarrow 180° - 3x = \begin{cases} 225° + N \cdot 360° \\ 315° + N \cdot 360° \end{cases}$; $-3x = \begin{cases} 45° + N \cdot 360° \\ 135° + N \cdot 360° \end{cases}$;

$x = \begin{cases} -15° + N \cdot 120° \\ -45° + N \cdot 120° \end{cases}$ $\boxed{x = \begin{cases} 105° \\ 225° \\ 345° \end{cases} \quad x = \begin{cases} 75° \\ 195° \\ 315° \end{cases}}$

7

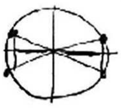
a) $\cos^2 x - \sin^2 x = \frac{1}{2}$

$\cos 2x = \frac{1}{2} \Rightarrow 2x = \begin{cases} 60° + N \cdot 360° \\ 300° + N \cdot 360° \end{cases}$; $x = \begin{cases} 30° + N \cdot 180° \\ 150° + N \cdot 180° \end{cases}$; $\boxed{x = \begin{cases} 30° \\ 210° \\ 150° \\ 330° \end{cases}}$; $\boxed{x = \begin{cases} \pi/6 \\ 7\pi/6 \\ 5\pi/6 \\ 11\pi/6 \end{cases}}$

También:

$\cos^2 x - (1 - \cos^2 x) = \frac{1}{2}$; $2\cos^2 x - 1 = \frac{1}{2}$; $2\cos^2 x = \frac{3}{2}$; $\cos^2 x = \frac{3}{4}$;

$\cos x = \pm \frac{\sqrt{3}}{2} \Rightarrow x = \begin{cases} 30° + N \cdot 360° \\ 150° + N \cdot 360° \\ 210° + N \cdot 360° \\ 330° + N \cdot 360° \end{cases}$; $x = \begin{cases} 30° + N \cdot 180° \\ 150° + N \cdot 180° \end{cases}$ ✓



b) $\sin x - \cos x = \frac{\sqrt{2}}{2}$

$(\sin x - \cos x)^2 = \left(\frac{\sqrt{2}}{2}\right)^2$; $\sin^2 x - 2\sin x \cos x + \cos^2 x = \frac{1}{2}$; $1 - 2\sin x \cos x = \frac{1}{2}$;

$2\sin x \cos x = \frac{1}{2}$; $\sin 2x = \frac{1}{2} \Rightarrow 2x = \begin{cases} 30° + N \cdot 360° \\ 150° + N \cdot 360° \end{cases}$; $x = \begin{cases} 15° + N \cdot 180° \\ 75° + N \cdot 180° \end{cases}$; $x = \begin{cases} 15° \\ 195° \\ 75° \\ 255° \end{cases}$

Como elevamos al cuadrado, hay que comprobar:

$\sin 15° - \cos 15° = -\frac{\sqrt{2}}{2} \neq \frac{\sqrt{2}}{2}$

$\sin 195° - \cos 195° = \frac{\sqrt{2}}{2}$ ✓

$\sin 75° - \cos 75° = \frac{\sqrt{2}}{2}$ ✓

$\sin 255° - \cos 255° = -\frac{\sqrt{2}}{2} \neq \frac{\sqrt{2}}{2}$

$\boxed{x = \begin{cases} 75° \\ 195° \end{cases}}$; $\boxed{x = \begin{cases} 5\pi/12 \\ 13\pi/12 \end{cases}}$

c) $\operatorname{tg} x \cdot \sec x = \sqrt{2}$

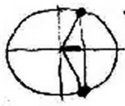
$\frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \sqrt{2}$; $\sin x = \sqrt{2}\cos^2 x$; $\sin x = \sqrt{2}(1 - \sin^2 x)$; $\sqrt{2}\sin^2 x + \sin x - \sqrt{2} = 0$

$\sin x = \frac{-1 \pm \sqrt{1 + 4(\sqrt{2})^2}}{2\sqrt{2}} = \frac{-1 \pm 3}{2\sqrt{2}} = \begin{cases} \frac{-4}{2\sqrt{2}} = \cancel{-\frac{2}{\sqrt{2}}} & \text{Porque es menor que } -1. \\ \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \Rightarrow \end{cases}$ $\boxed{x = \begin{cases} 225° \\ 135° \end{cases}}$; $\boxed{x = \begin{cases} 5\pi/4 \\ 3\pi/4 \end{cases}}$

d) $3\cos X = 2\sec X - 5$

$3\cos X = \dfrac{2}{\cos X} - 5$; $3\cos^2 X = 2 - 5\cos X$; $3\cos^2 X + 5\cos X - 2 = 0$

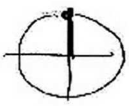
$\cos X = \dfrac{-5 \pm \sqrt{25+24}}{6} = \dfrac{-5 \pm 7}{6}$ $\begin{cases} \frac{2}{6} = \frac{1}{3} \Rightarrow \boxed{x = \begin{cases} 71^\circ \\ 289^\circ \end{cases}} ; \boxed{x = \begin{cases} 1{,}23 \text{ rad} \\ 5{,}05 \text{ rad} \end{cases}} \\ \cancel{-2} \end{cases}$



e) $\log_2 \cos x + 1 = \log_2 \operatorname{cosec} x$

$\log_2 \cos x - \log_2 \operatorname{cosec} X = -1$; $\log_2 \dfrac{\cos X}{\operatorname{cosec} X} = -1$; $\dfrac{\cos X}{\operatorname{cosec} X} = 2^{-1}$; $\dfrac{\cos X}{\operatorname{cosec} X} = \dfrac{1}{2}$;

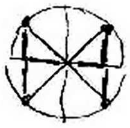
$2\cos X = \operatorname{cosec} X$; $2\cos X = \dfrac{1}{\sin X}$; $2\sin X \cos X = 1$; $\sin 2X = 1 \Rightarrow$



$\Rightarrow 2X = 90^\circ + N\cdot 360^\circ$; $x = 45^\circ + N\cdot 180^\circ$; $\boxed{x = \begin{cases} 45^\circ \\ \cancel{225^\circ} \end{cases}}$ No sirve ; $\boxed{x = \pi/4 \text{ rad}}$

f) $\operatorname{tg}^2 2x = 1$

$\operatorname{tg} 2x = \pm 1$; $2X = \begin{cases} 45^\circ + N\cdot 360^\circ \\ 225^\circ + N\cdot 360^\circ \\ 135^\circ + N\cdot 360^\circ \\ 315^\circ + N\cdot 360^\circ \end{cases}$; $x = \begin{cases} 22{,}5^\circ + N\cdot 180^\circ \\ 112{,}5^\circ + N\cdot 180^\circ \\ 67{,}5^\circ + N\cdot 180^\circ \\ 157{,}5^\circ + N\cdot 180^\circ \end{cases}$; $\boxed{x = \begin{cases} 22{,}5^\circ \\ 202{,}5^\circ \\ 112{,}5^\circ \\ 292{,}5^\circ \\ 67{,}5^\circ \\ 247{,}5^\circ \\ 157{,}5^\circ \\ 337{,}5^\circ \end{cases}}$; $\boxed{x = \begin{cases} \pi/8 \\ 9\pi/8 \\ 5\pi/8 \\ 13\pi/8 \\ 3\pi/8 \\ 11\pi/8 \\ 7\pi/8 \\ 15\pi/8 \end{cases}}$ rad



g) $\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} = \sin x$; $\cos 2\frac{x}{2} = \sin x$; $\cos X = \sin X$; $1 = \operatorname{tg} X \Rightarrow x = \begin{cases} 45^\circ = \boxed{\pi/4} \\ 225^\circ = \boxed{5\pi/4} \end{cases}$ rad

h) $\cos(2x) + \sin x = 4\sin^2 x$

$\cos^2 x - \sin^2 x + \sin x = 4\sin^2 x$; $1 - \sin^2 x - \sin^2 x + \sin x = 4\sin^2 x$; $-6\sin^2 x + \sin x + 1 = 0$;

$\sin x = \dfrac{-1 \pm \sqrt{1+24}}{-12} = \dfrac{-1 \pm 5}{-12}$ $\begin{cases} \frac{-4}{12} = -\frac{1}{3} \Rightarrow x = \begin{cases} 340'53^\circ = 1'89\pi \\ 199'47^\circ = 1'11\pi \end{cases} \\ \frac{-6}{-12} = \frac{1}{2} \Rightarrow x = \begin{cases} 30^\circ = \pi/6 \\ 150^\circ = 5\pi/6 \end{cases} \end{cases}$

i) $\operatorname{tg}(2x) = -\operatorname{tg} x$

$\dfrac{2\operatorname{tg} x}{1-\operatorname{tg}^2 x} = -\operatorname{tg} x$; $2\operatorname{tg} x = -\operatorname{tg} x + \operatorname{tg}^3 x$; $3\operatorname{tg} x - \operatorname{tg}^3 x = 0$; $\operatorname{tg} x\cdot(3-\operatorname{tg}^2 x) = 0$

$\operatorname{tg} x\cdot(3-\operatorname{tg}^2 x) = 0$ $\begin{cases} \operatorname{tg} x = 0 \rightarrow x = \boxed{\begin{cases} 0 \\ \pi \end{cases}} \\ \operatorname{tg}^2 x = 3 ; \operatorname{tg} x = \begin{cases} \sqrt{3} \rightarrow x = \begin{cases} 60^\circ = \pi/3 \\ 240^\circ = 4\pi/3 \end{cases} \\ -\sqrt{3} \rightarrow x = \begin{cases} 120^\circ = 2\pi/3 \\ 300^\circ = 5\pi/3 \end{cases} \end{cases} \end{cases}$

8 a)

$$4\cos(2x) - 3\,\text{sen}\,x\,\text{cosec}^3x + 6 = 0$$

$$4(\cos^2x - \text{sen}^2x) - 3\,\text{sen}\,x\,\frac{1}{\text{sen}^3x} + 6 = 0 \quad ; \quad 4(1-2\text{sen}^2x) - \frac{3\cancel{\text{sen}\,x}}{\text{sen}^{\cancel{3}\,2}x} + 6 = 0$$

$$4 - 8\text{sen}^2x - \frac{3}{\text{sen}^2x} + 6 = 0 \quad ; \quad 10 - 8\text{sen}^2x - \frac{3}{\text{sen}^2x} = 0$$

$$t = \text{sen}\,x \qquad 10 - 8t^2 - \frac{3}{t^2} = 0 \quad ; \quad 10t^2 - 8t^4 - 3 = 0 \quad ; \quad 8t^4 - 10t^2 + 3 = 0 \quad ✓$$

b)

$$t^2 = \frac{10 \pm \sqrt{100-96}}{16} = \frac{10\pm 2}{16} \begin{cases} \frac{12}{16} = \frac{3}{4} \Rightarrow t = \frac{\pm\sqrt{3}}{2} \rightarrow \text{sen}\,x = \frac{\pm\sqrt{3}}{2} \\ \frac{8}{16} = \frac{1}{2} \Rightarrow t = \pm\frac{\sqrt{2}}{2} \rightarrow \text{sen}\,x = \frac{\pm\sqrt{2}}{2} \end{cases}$$

$$\boxed{x = \begin{cases} \pi/3 \\ 2\pi/3 \\ \pi/4 \\ 3\pi/4 \end{cases}} \quad x \in [0,\pi)$$

9 a)

$$\frac{\text{sen}(2\alpha)}{1+\cos(2\alpha)} = \frac{2\,\text{sen}\,\alpha\cos\alpha}{1+\cos^2\alpha - \text{sen}^2\alpha} = \frac{2\,\text{sen}\,\alpha\cos\alpha}{\cancel{1}+\cos^2\alpha - \cancel{1}+\cos^2\alpha} = \frac{\cancel{2}\,\text{sen}\,\alpha\,\cancel{\cos\alpha}}{\cancel{2}\cos^2\alpha} = \text{tg}\,\alpha \quad ✓$$

b)

$$\text{ctg}(\pi/8) = \frac{1}{\text{tg}(\pi/8)} = \frac{1}{\frac{\text{sen}(2\pi/8)}{1+\cos(2\pi/8)}} = \frac{1+\cos(\pi/4)}{\text{sen}(\pi/4)} = \frac{1+\sqrt{2}/2}{\sqrt{2}/2} = \frac{2+\sqrt{2}}{\sqrt{2}} =$$

$$= \frac{(2+\sqrt{2})\sqrt{2}}{(\sqrt{2})^2} = \frac{2\sqrt{2}+2}{2} = \boxed{1+\sqrt{2}} \qquad \begin{matrix} a=1 \\ b=1 \end{matrix}$$

10 a) He listado en una hoja de cálculo ángulos, de grado en grado, de 0° a 360° poniendo a su derecha el valor de $5\cos\alpha - 12\,\text{sen}\,\alpha$. El menor valor resulta ser $-12'9997$ para un ángulo de 113° mientras que el mayor valor es $12'9997$ para 293°.
Con el mismo programa de hoja de cálculo he representado estos valores, resultando puntos que perfilan una sinusoide.

b)

$$5\cos x - 12\,\text{sen}\,x = A\cos(x+\alpha)$$

$$5\cos x - 12\,\text{sen}\,x = A\cos x\cos\alpha - A\,\text{sen}\,x\,\text{sen}\,\alpha$$

$$\begin{matrix} 5 = A\cos\alpha \\ 12 = A\,\text{sen}\,\alpha \end{matrix} \qquad A = \frac{5}{\cos\alpha} \qquad 12 = \frac{5}{\cos\alpha}\cdot\text{sen}\,\alpha \quad ; \quad \frac{12}{5} = \text{tg}\,\alpha \Rightarrow \boxed{\alpha = 67'38^\circ}$$

$$\begin{aligned} 25 &= A^2\cos^2\alpha \\ +\;144 &= A^2\,\text{sen}^2\alpha \\ \hline 169 &= A^2 \end{aligned} \quad \Rightarrow \quad \boxed{A = 13} \qquad \boxed{5\cos x - 12\,\text{sen}\,x = 13\cdot\cos(x+67'38^\circ)}$$

El mayor valor de $5\cos x - 12\,\text{sen}\,x$ será 13, para $x + 67'38^\circ = 0^\circ \Rightarrow x = -67'38^\circ \equiv \boxed{293^\circ}$

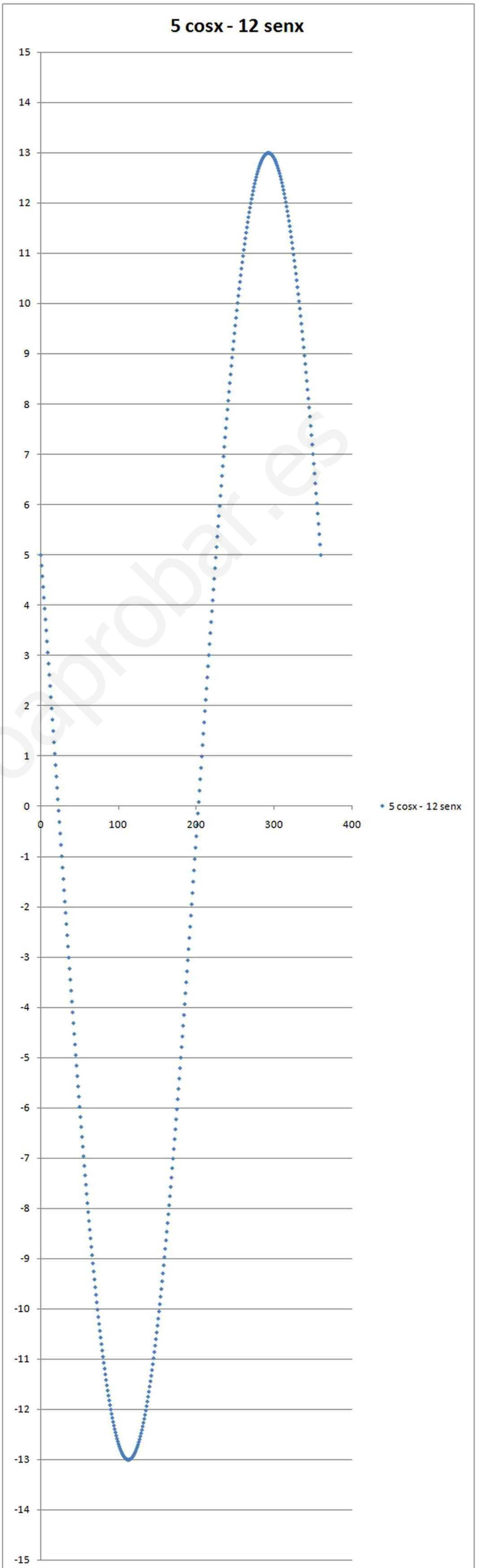
El menor valor de $5\cos x - 12\,\text{sen}\,x$ será -13, para $x + 67'38^\circ = 180^\circ \Rightarrow \boxed{x = 113^\circ}$

c)

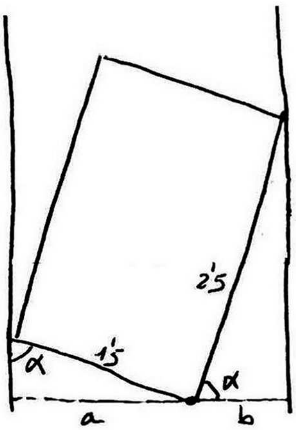
$$5\cos x - 12\,\text{sen}\,x = -2 \Rightarrow 13\cdot\cos(x+67'38^\circ) = -2 \Rightarrow \cos(x+67'38^\circ) = -\frac{2}{13} \Rightarrow$$

$$\Rightarrow x + 67'38^\circ = \begin{cases} 98'85^\circ \\ 261'15^\circ \end{cases} \Rightarrow \boxed{x = \begin{cases} 31^\circ \\ 194^\circ \end{cases}}$$

x	5 cosx - 12 senx	x	5 cosx - 12 senx	x	5 cosx - 12 senx	x	5 cosx - 12 senx
0	5,0000	90	-12,0000	180	-5,0000	270	12,0000
1	4,7898	91	-12,0854	181	-4,7898	271	12,0854
2	4,5782	92	-12,1672	182	-4,5782	272	12,1672
3	4,3651	93	-12,2452	183	-4,3651	273	12,2452
4	4,1507	94	-12,3196	184	-4,1507	274	12,3196
5	3,9351	95	-12,3901	185	-3,9351	275	12,3901
6	3,7183	96	-12,4569	186	-3,7183	276	12,4569
7	3,5003	97	-12,5199	187	-3,5003	277	12,5199
8	3,2813	98	-12,5791	188	-3,2813	278	12,5791
9	3,0612	99	-12,6344	189	-3,0612	279	12,6344
10	2,8403	100	-12,6859	190	-2,8403	280	12,6859
11	2,6184	101	-12,7336	191	-2,6184	281	12,7336
12	2,3958	102	-12,7773	192	-2,3958	282	12,7773
13	2,1724	103	-12,8172	193	-2,1724	283	12,8172
14	1,9484	104	-12,8532	194	-1,9484	284	12,8532
15	1,7238	105	-12,8852	195	-1,7238	285	12,8852
16	1,4987	106	-12,9133	196	-1,4987	286	12,9133
17	1,2731	107	-12,9375	197	-1,2731	287	12,9375
18	1,0471	108	-12,9578	198	-1,0471	288	12,9578
19	0,8208	109	-12,9741	199	-0,8208	289	12,9741
20	0,5942	110	-12,9864	200	-0,5942	290	12,9864
21	0,3675	111	-12,9948	201	-0,3675	291	12,9948
22	0,1406	112	-12,9992	202	-0,1406	292	12,9992
23	-0,0862	113	-12,9997	203	0,0862	293	12,9997
24	-0,3131	114	-12,9962	204	0,3131	294	12,9962
25	-0,5399	115	-12,9888	205	0,5399	295	12,9888
26	-0,7665	116	-12,9774	206	0,7665	296	12,9774
27	-0,9929	117	-12,9620	207	0,9929	297	12,9620
28	-1,2189	118	-12,9427	208	1,2189	298	12,9427
29	-1,4446	119	-12,9195	209	1,4446	299	12,9195
30	-1,6699	120	-12,8923	210	1,6699	300	12,8923
31	-1,8946	121	-12,8612	211	1,8946	301	12,8612
32	-2,1188	122	-12,8262	212	2,1188	302	12,8262
33	-2,3423	123	-12,7872	213	2,3423	303	12,7872
34	-2,5651	124	-12,7444	214	2,5651	304	12,7444
35	-2,7872	125	-12,6977	215	2,7872	305	12,6977
36	-3,0083	126	-12,6471	216	3,0083	306	12,6471
37	-3,2286	127	-12,5927	217	3,2286	307	12,5927
38	-3,4479	128	-12,5344	218	3,4479	308	12,5344
39	-3,6661	129	-12,4724	219	3,6661	309	12,4724
40	-3,8832	130	-12,4065	220	3,8832	310	12,4065
41	-4,0992	131	-12,3368	221	4,0992	311	12,3368
42	-4,3138	132	-12,2634	222	4,3138	312	12,2634
43	-4,5272	133	-12,1862	223	4,5272	313	12,1862
44	-4,7392	134	-12,1054	224	4,7392	314	12,1054
45	-4,9497	135	-12,0208	225	4,9497	315	12,0208
46	-5,1588	136	-11,9326	226	5,1588	316	11,9326
47	-5,3663	137	-11,8407	227	5,3663	317	11,8407
48	-5,5721	138	-11,7453	228	5,5721	318	11,7453
49	-5,7762	139	-11,6463	229	5,7762	319	11,6463
50	-5,9786	140	-11,5437	230	5,9786	320	11,5437
51	-6,1791	141	-11,4376	231	6,1791	321	11,4376
52	-6,3778	142	-11,3280	232	6,3778	322	11,3280
53	-6,5746	143	-11,2150	233	6,5746	323	11,2150
54	-6,7693	144	-11,0985	234	6,7693	324	11,0985
55	-6,9619	145	-10,9787	235	6,9619	325	10,9787
56	-7,1525	146	-10,8555	236	7,1525	326	10,8555
57	-7,3409	147	-10,7290	237	7,3409	327	10,7290
58	-7,5270	148	-10,5993	238	7,5270	328	10,5993
59	-7,7108	149	-10,4663	239	7,7108	329	10,4663
60	-7,8923	150	-10,3301	240	7,8923	330	10,3301
61	-8,0714	151	-10,1908	241	8,0714	331	10,1908
62	-8,2480	152	-10,0484	242	8,2480	332	10,0484
63	-8,4221	153	-9,9029	243	8,4221	333	9,9029
64	-8,5937	154	-9,7544	244	8,5937	334	9,7544
65	-8,7626	155	-9,6030	245	8,7626	335	9,6030
66	-8,9289	156	-9,4486	246	8,9289	336	9,4486
67	-9,0924	157	-9,2913	247	9,0924	337	9,2913
68	-9,2532	158	-9,1312	248	9,2532	338	9,1312
69	-9,4111	159	-8,9683	249	9,4111	339	8,9683
70	-9,5662	160	-8,8027	250	9,5662	340	8,8027
71	-9,7184	161	-8,6344	251	9,7184	341	8,6344
72	-9,8676	162	-8,4635	252	9,8676	342	8,4635
73	-10,0138	163	-8,2900	253	10,0138	343	8,2900
74	-10,1570	164	-8,1140	254	10,1570	344	8,1140
75	-10,2970	165	-7,9355	255	10,2970	345	7,9355
76	-10,4339	166	-7,7545	256	10,4339	346	7,7545
77	-10,5677	167	-7,5713	257	10,5677	347	7,5713
78	-10,6982	168	-7,3857	258	10,6982	348	7,3857
79	-10,8255	169	-7,1978	259	10,8255	349	7,1978
80	-10,9495	170	-7,0078	260	10,9495	350	7,0078
81	-11,0701	171	-6,8157	261	11,0701	351	6,8157
82	-11,1874	172	-6,6214	262	11,1874	352	6,6214
83	-11,3012	173	-6,4252	263	11,3012	353	6,4252
84	-11,4116	174	-6,2270	264	11,4116	354	6,2270
85	-11,5186	175	-6,0268	265	11,5186	355	6,0268
86	-11,6220	176	-5,8249	266	11,6220	356	5,8249
87	-11,7219	177	-5,6212	267	11,7219	357	5,6212
88	-11,8182	178	-5,4157	268	11,8182	358	5,4157
89	-11,9109	179	-5,2087	269	11,9109	359	5,2087
90	-12,0000	180	-5,0000	270	12,0000	360	5,0000



(11)



$$\sin\alpha = \frac{a}{1'5} \longrightarrow a = 1'5\sin\alpha$$

$$+$$

$$\cos\alpha = \frac{b}{2'5} \longrightarrow b = 2'5\cos\alpha$$

$$2 = 1'5\sin\alpha + 2'5\cos\alpha$$

$$4 = 3\sin\alpha + 5\cos\alpha \longrightarrow \cos\alpha = \frac{4-3\sin\alpha}{5}$$

$$\sin^2\alpha + \cos^2\alpha = 1$$

$$\sin^2\alpha + \left(\frac{4-3\sin\alpha}{5}\right)^2 = 1$$

$$\sin^2\alpha + \frac{16-24\sin\alpha+9\sin^2\alpha}{25} = 1 \;;\quad 25\sin^2\alpha + 16 - 24\sin\alpha + 9\sin^2\alpha = 25 \;;$$

$$34\sin^2\alpha - 24\sin\alpha - 9 = 0 \;;\; \sin\alpha = \frac{24 \pm \sqrt{576+1224}}{68} = \begin{cases} 0'98 \Rightarrow \boxed{\alpha = 77'65^\circ} = \boxed{77^\circ 38' 59{,}64''} \;\; \cancel{102'35} \\ -0'27 \Rightarrow \alpha = \begin{cases} \cancel{344'28} \\ \cancel{164'28} \end{cases} \end{cases}$$

Los ángulos son: $\boxed{\alpha = 77'65^\circ}$ y $\boxed{90-\alpha = 12'35^\circ}$

(12) $$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta = \frac{3}{\sqrt{3^2+4^2}}\cdot\frac{3}{\sqrt{3^2+2^2}} + \frac{4}{\sqrt{3^2+4^2}}\cdot\frac{2}{\sqrt{3^2+2^2}} =$$

$$= \frac{9+8}{5\sqrt{13}} = \boxed{\frac{17}{5\sqrt{13}}}$$