## Ecuaciones trigonométricas resueltas

**1.-** Resolver las siguientes ecuaciones:

**a)** $\boldsymbol{sen2x = 0}$

$$sen2x = 0 \rightarrow \begin{cases} 2x = 0 + 2k\pi \\ 2x = \pi + 2k\pi \end{cases} \rightarrow \begin{cases} x_1 = 0 + k\pi \\ x_2 = \dfrac{\pi}{2} + k\pi \end{cases} \quad con\ k \in Z$$

**b)** $\boldsymbol{\cos\left(\frac{\pi}{3} - x\right) = 1}$

$$\cos\left(\frac{\pi}{3} - x\right) = 1 \rightarrow \frac{\pi}{3} - x = 0 + 2k\pi \rightarrow x = \frac{\pi}{3} + 2k\pi \quad con\ k \in Z$$

**c)** $\boldsymbol{sen2x - senx = 0}$

$$sen2x - senx = 0 \rightarrow 2senx\cos x - senx = 0 \rightarrow senx(2\cos x - 1) = 0 \quad por\ tanto: \begin{cases} senx = 0 \\ 2\cos x - 1 = 0 \end{cases}$$

$$\bullet\ Si\ sen\ x = 0 \rightarrow \begin{cases} x_1 = 0 + 2k\pi \\ x_2 = \pi + 2k\pi \end{cases} \quad con\ k \in Z$$

$$\bullet\ Si\ \ 2\cos x - 1 = 0 \rightarrow \cos x = \frac{1}{2} \rightarrow \begin{cases} x_3 = \pi/3 + 2k\pi \\ x_4 = 5\pi/3 + 2k\pi \end{cases} \quad con\ k \in Z$$

**d)** $\boldsymbol{\cos 2x - sen^2 x - 1 = 0}$

$$\cos 2x - sen^2 x - 1 = 0 \rightarrow \cos^2 x - sen^2 x - sen^2 x - 1 = 0 \rightarrow 1 - sen^2 x - sen^2 x - sen^2 x - 1 = 0 \rightarrow$$

$$\rightarrow -3sen^2 x = 0 \rightarrow sen^2 x = 0 \rightarrow senx = 0 \rightarrow \begin{cases} x_1 = 0 + 2k\pi \\ x_2 = \pi + 2k\pi \end{cases} \quad con\ k \in Z$$

**e)** $\boldsymbol{\cos 2x = -1/2}$

$$\cos 2x = -1/2 \rightarrow \begin{cases} 2x = \dfrac{2\pi}{3} + 2k\pi \\ 2x = \dfrac{4\pi}{3} + 2k\pi \end{cases} \rightarrow \begin{cases} x_1 = \dfrac{\pi}{3} + k\pi \\ x_2 = \dfrac{2\pi}{3} + k\pi \end{cases} \quad con k \in Z$$

**f)** $\boldsymbol{senx - \cos x = 0}$

*$senx - \cos x = 0 \rightarrow senx = \cos x$; es decir busco aquellos ángulos donde el seno y el coseno tienen el mismo valor. Esto sólo pasa en 45º y en 225º.*

*Por tanto, pasando a radianes:* $x_1 = \dfrac{\pi}{4} + 2k\pi;\ x_2 = \dfrac{5\pi}{4} + 2k\pi \quad con\ k \in Z.$

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**g)** $\boldsymbol{sen2x + cos\, x = 0}$

$$sen2x + cos\, x = 0 \rightarrow 2senx\, cos\, x + cos\, x = 0 \rightarrow cos\, x(2senx + 1) = 0 \;\; por\;\; tanto \begin{cases} cosx = 0 \\ 2senx + 1 = 0 \end{cases}$$

- $cos\, x = 0 \rightarrow \begin{cases} x_1 = \dfrac{\pi}{2} + 2k\pi \\ x_2 = \dfrac{3\pi}{2} + 2k\pi \end{cases} \quad con\; k \in Z$
- $2senx + 1 = 0 \rightarrow senx = \dfrac{-1}{2} \rightarrow \begin{cases} x_3 = \dfrac{7\pi}{6} + 2k\pi \\ x_4 = \dfrac{11\pi}{6} + 2k\pi \end{cases} \quad con\; k \in Z$

**h)** $\boldsymbol{tgx + 2senx = 0}$

$$tgx + 2senx = 0 \rightarrow \frac{senx}{cos\, x} + 2senx = 0 \rightarrow \frac{senx + 2senx\, cos\, x}{cos\, x} = 0 \rightarrow senx + 2senx\, cos\, x = 0 \rightarrow$$

$$\rightarrow senx(1 + 2\cos x) = 0 \;\; por\;\; tanto \begin{cases} senx = 0 \\ 1 + 2\cos x = 0 \end{cases}$$

- $senx = 0 \rightarrow \begin{cases} x_1 = 0 + 2k\pi \\ x_2 = \pi + 2k\pi \end{cases} \quad con\; k \in Z$
- $1 + 2cosx = 0 \rightarrow cos\, x = \dfrac{-1}{2} \rightarrow \begin{cases} x_3 = \dfrac{2\pi}{3} + 2k\pi \\ x_4 = \dfrac{4\pi}{3} + 2k\pi \end{cases} \quad con\; k \in Z$

**i)** $\boldsymbol{cos^2\, x = 3sen^2 x}$

$$cos^2\, x = 3sen^2 x \rightarrow 1 - sen^2 x = 3sen^2 x \rightarrow 4sen^2 x = 1 \rightarrow sen^2 x = \frac{1}{4} \rightarrow sen\, x = \pm\frac{1}{2}$$

*Por tanto :*

- $senx = \dfrac{1}{2} \rightarrow \begin{cases} x_1 = \dfrac{\pi}{6} + 2k\pi \\ x_2 = \dfrac{5\pi}{6} + 2k\pi \end{cases} \quad con\; k \in Z$
- $senx = -\dfrac{1}{2} \rightarrow \begin{cases} x_3 = \dfrac{7\pi}{6} + 2k\pi \\ x_4 = \dfrac{11\pi}{6} + 2k\pi \end{cases} \quad con\; k \in Z$

**j)** $\boldsymbol{cos\, 2x = 1 + 4senx}$

$$cos\, 2x = 1 + 4sen\, x = 0 \rightarrow cos^2\, x - sen^2 x = 1 + 4senx \rightarrow 1 - sen^2 x - sen^2 x = 1 + 4senx \rightarrow$$

$$\rightarrow 2\, sen^2 x + 4\, sen\, x = 0 \rightarrow sen^2 x + 2\, sen\, x = 0 \rightarrow sen\, x(sen\, x + 2) = 0 \rightarrow \begin{cases} sen\, x = 0 \\ sen\, x + 2 = 0 \end{cases}$$

- $senx = 0 \rightarrow \begin{cases} x_1 = 0 + 2k\pi \\ x_2 = \pi + 2k\pi \end{cases} \quad con\; k \in Z$
- $senx + 2 = 0 \rightarrow senx = -2 \rightarrow$ *No tiene solución pues el seno solo puede tomar valores comprendidos entre $-1$ y $1$.*

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**k)** $\boldsymbol{\cos 5x - \cos x = 0}$

$$\cos 5x - \cos x = 0 \to -2sen\frac{5x+x}{2}\cdot sen\frac{5x-x}{2} = 0 \to -2sen3x\cdot sen2x = 0 \to \begin{cases} sen3x = 0 \\ sen2x = 0 \end{cases}$$

- $sen\,3x = 0 \to \begin{cases} 3x_1 = 0 + 2k\pi \\ 3x_2 = \pi + 2k\pi \end{cases}$ $con\ k \in Z \to \begin{cases} x_1 = 0 + \dfrac{2k\pi}{3} \\ x_2 = \dfrac{\pi}{3} + \dfrac{2k\pi}{3} \end{cases}$ $con\ k \in Z$
- $sen\,2x = 0 \to \begin{cases} 2x_3 = 0 + 2k\pi \\ 2x_4 = \pi + 2k\pi \end{cases}$ $con\ k \in Z \to \begin{cases} x_3 = 0 + k\pi \\ x_2 = \dfrac{\pi}{2} + k\pi \end{cases}$ $con\ k \in Z$

**l)** $\boldsymbol{2tg^2 x - 3\,tg\,x + 1 = 0}$

$2tg^2 x - 3\,tg\,x + 1 = 0 \to$ *Podemos resolver la ecuación de segundo grado* $\to$

$$tg\,x = \frac{3 \pm \sqrt{9-8}}{4} \to tg\,x = \frac{3 \pm 1}{4} \to \begin{cases} tg\,x = 1 \\ tg\,x = \dfrac{1}{2} \end{cases}$$

- $tg\,x = 1 \to \begin{cases} x_1 = \dfrac{\pi}{4} + 2k\pi \\ x_2 = \dfrac{5\pi}{4} + 2k\pi \end{cases}$ $con\ k \in Z$
- $tg\,x = \dfrac{1}{2} \to \begin{cases} x_3 = 26.565^o + 360k^o \\ x_4 = 180^o + 26.565^o + 360k^o \end{cases} \to \begin{cases} x_3 = 26.565^o + 360k^o \\ x_4 = 206.565^o + 360k^o \end{cases}$ $con\ k \in Z$

*(Ayudándonos de la calculadora)*

**2.-** Hallar las soluciones de las siguientes ecuaciones en el intervalo $[0,2\pi)$.

**a)** $\boldsymbol{\cos 2x = 1 + 4senx}$

$$\cos 2x = 1 + 4senx \to \cos^2 x - sen^2 x = 1 + 4senx \to 1 - sen^2 x - sen^2 x = 1 + 4senx \to$$

$$\to 2sen^2 x + 4senx = 0 \to 2senx(senx + 2) = 0 \to \begin{cases} 2senx = 0 \\ senx + 2 = 0 \end{cases}$$

- $2senx = 0 \to senx = 0 \to \begin{cases} x_1 = 0 \\ x_2 = \pi \end{cases}$
- $senx + 2 = 0 \to senx = -2 \to$ *No tiene solución pues el valor del seno está comprendido entre -1 y 1.*

**b)** $\boldsymbol{4sen^2 x = 3}$

$$4sen^2 x = 3 \to sen^2 x = \frac{3}{4} \to senx = \pm\frac{\sqrt{3}}{2}$$

- $senx = \dfrac{\sqrt{3}}{2} \to \begin{cases} x_1 = \dfrac{\pi}{3} \\ x_2 = \dfrac{2\pi}{3} \end{cases}$
- $senx = -\dfrac{\sqrt{3}}{2} \to \begin{cases} x_3 = \dfrac{4\pi}{3} \\ x_4 = \dfrac{5\pi}{3} \end{cases}$

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**c)** $\boldsymbol{sen^2 x + cos\,x - 1 = 0}$

$$sen^2 x + cos\,x - 1 = 0 \rightarrow 1 - cos^2 x + cos\,x - 1 = 0 \rightarrow cos^2 x - cos\,x = 0 \rightarrow cos\,x(cos\,x - 1) = 0$$

- $senx = 0 \rightarrow \begin{cases} x_1 = \dfrac{\pi}{2} \\ x_2 = \dfrac{3\pi}{2} \end{cases}$
- $cos\,x - 1 = 0 \rightarrow cos\,x = 1 \rightarrow x_3 = 0$

**d)** $\boldsymbol{sen^2 x + senx - 6 = 0}$

$sen^2 x + senx - 6 = 0 \rightarrow$ *La resuelvo como una ecuación de segundo grado donde* $senx = z$

*Por tanto* $z^2 + z - 6 = 0 \rightarrow z = \dfrac{-1 \pm 5}{2} \rightarrow \begin{cases} z_1 = 2 \\ z_2 = -3 \end{cases} \rightarrow \begin{cases} senx = 2 \\ senx = -3 \end{cases} \rightarrow$

$\rightarrow$ *ambas ecuaciones no tienen solución.*

**e)** $\boldsymbol{2(cos^2 x - sen^2 x) = 1}$

$$2(cos^2 x - sen^2 x) = 1 \rightarrow 2(1 - sen^2 x - sen^2 x) = 1 \rightarrow 2(1 - 2sen^2 x) = 1 \rightarrow 2 - 4sen^2 x = 1 \rightarrow$$

$$\rightarrow 4sen^2 x = 1 \rightarrow sen^2 x = \frac{1}{4} \rightarrow senx = \pm\frac{1}{2}$$

- $senx = \dfrac{1}{2} \rightarrow \begin{cases} x_1 = \dfrac{\pi}{6} \\ x_2 = \dfrac{5\pi}{6} \end{cases}$
- $senx = -\dfrac{1}{2} \rightarrow \begin{cases} x_1 = \dfrac{7\pi}{6} \\ x_2 = \dfrac{11\pi}{6} \end{cases}$

**f)** $\boldsymbol{cos\,x + cos\,3x = 0}$

$cos\,x + cos\,3x = 0$; *aplico que* $cos\,A + cos\,B = 2\,cos\dfrac{A+B}{2}cos\dfrac{A-B}{2}$ *y obtenemos :*

$$2cos2xcosx = 0 \rightarrow \begin{cases} cosx = 0 \\ cos\,2x = 0 \end{cases}$$

- $cosx = 0 \rightarrow \begin{cases} x_1 = \dfrac{\pi}{2} \\ x_2 = \dfrac{3\pi}{2} \end{cases}$
- $cos2x = 0 \rightarrow \begin{cases} 2x = \dfrac{\pi}{2} + 2k\pi \\ 2x = \dfrac{3\pi}{2} + 2k\pi \end{cases} \rightarrow \begin{cases} x = \dfrac{\pi}{4} + k\pi \\ x = \dfrac{3\pi}{4} + k\pi \end{cases}$

*con lo que restringiéndonos a ángulos del intervalo* $[0,2\pi)$ *obtenemos las soluciones :*

$$x_3 = \frac{\pi}{4};\, x_4 = \frac{5\pi}{4};\, x_5 = \frac{3\pi}{4};\, x_6 = \frac{7\pi}{4}$$

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