Demostrar las siguientes identidades:

(a) $$sen^4\alpha + 2\,sen^2\alpha(1-\frac{1}{\cos ec^2\alpha}) = 1-\cos^4\alpha$$

(b) $$\frac{1+tg^2\alpha}{1+\cot^2\alpha} = (\frac{1-tg\,\alpha}{1-\cot\alpha})^2$$

(c) $$\frac{1-sen\,\alpha\cos\alpha}{(\sec\alpha-\cos ec\,\alpha)\cos\alpha}\cdot\frac{sen^2\alpha-\cos^2\alpha}{sen^3\alpha+\cos^3\alpha} = sen\,\alpha$$

(d) $$\cos ec\,\alpha(\sec\alpha-1)-\cot\alpha(1-\cos\alpha) = tg\,\alpha - sen\,\alpha$$

(e) $$(\sec a-1)^2 - tg\,\alpha)^2 = (1-\cos\alpha)^2$$

(f) $$\frac{tg^3\alpha}{1+tg^2\alpha}+\frac{\cot^3\alpha}{1+\cot^2\alpha} = \frac{1-2sen^2\alpha\cos^2\alpha}{sen\,\alpha\ \cos\alpha}$$

(g) $$a=\cot\alpha \Rightarrow a+\frac{1}{a} = \sec\alpha\,con\sec\alpha$$

(h) $$sen^4\alpha(3-2sen^2\alpha)+\cos^4\alpha(3-2\cos^2\alpha) = 1$$

(i) $$\cot^2\alpha\frac{\sec\alpha-1}{1+sen\alpha}+\sec^2\alpha\frac{sen\,\alpha-1}{1+\sec\alpha} = 0$$

(j) $$\frac{2sen\,\alpha\cos\alpha-\cos\alpha}{1-sen\,\alpha+sen^2\alpha-\cos^2\alpha} = \cot\alpha$$

(k) $$tg\,\beta = \frac{nsen\,\alpha\cos\alpha}{1-nsen^2\alpha} \Rightarrow tg(\alpha-\beta) = (1-n)tg\,\alpha$$

(l) $$\frac{1+\cos\alpha+\cos\frac{\alpha}{2}}{sen\,\alpha+sen\frac{\alpha}{2}} = \cot\frac{\alpha}{2}$$

(m) $$tg\frac{\alpha}{2} = \cos ec\,\alpha - sen\,\alpha \Rightarrow \cos^2\frac{\alpha}{2} = \cos 36^{\circ}$$

Demostrar las identidades:

(a) $tg\,\alpha + \cot\alpha = \sec\alpha \cos ec\,\alpha$

(b) $1 - 2sen^2\alpha = 2\cos^2\alpha - 1$

(c) $sen\,\alpha \cos\alpha \cos ec\,\alpha = 1$

(d) $\dfrac{1}{1+sen\,\alpha} + \dfrac{1}{1-sen\,\alpha} = 2\sec^2\alpha$

(e) $tg\,\alpha + \dfrac{\cos\alpha}{1+sen\,\alpha} = \sec\alpha$

(f) $\cos^4\alpha - sen^4\alpha = \cos^2\alpha - sen^2\alpha$

(g) $\dfrac{tg\alpha - \cot\alpha}{tg\alpha + \cot\alpha} = 2sen^2\alpha - 1$

(h) $\dfrac{1-tg^2\alpha}{1+tg^2\alpha} = \cos^2\alpha - sen^2\alpha$

(i) $sen\alpha \sec\alpha \cot\alpha\alpha = 1$

(j) $\cos\alpha\alpha + tg\alpha sen\alpha = \sec\alpha$

Comprobar que:

(a) $tg15^{\circ} = 2 - \sqrt{3}$

(b) $\cot 15^{\circ} = 2 + \sqrt{3}$

(c) $sen9^{\circ} + \cos 9^{\circ} = \sqrt{1 + sen18^{\circ}}$

(d) $sen9^{\circ} - \cos 9^{\circ} = -\sqrt{1 + sen18^{\circ}}$

(e) $sen9^{\circ} = \dfrac{1}{4}(\sqrt{3+\sqrt{5}} + \sqrt{5-\sqrt{5}})$

(f) $\cos 9^{\circ} = \dfrac{1}{4}(\sqrt{3+\sqrt{5}} + \sqrt{5-\sqrt{5}}$

Calcular:

(a) $sen\dfrac{\pi}{10}$

(b) $\cos\dfrac{\pi}{10}$

(c) $sen\dfrac{\pi}{12}$

(d) $\cos\dfrac{\pi}{12}$

(e) Sabiendo que:

$$\frac{\pi}{10} - \frac{\pi}{12} = \frac{\pi}{60}$$

Calcular:

$$\cos\frac{\pi}{60}, sen\frac{\pi}{60}$$

$Si\; sen\alpha = \dfrac{15}{17}, sen\beta 0 - \dfrac{5}{13}, \dfrac{\pi}{2} \le \alpha \le \pi\; y\, \pi \le \beta \le \dfrac{3\pi}{2}$, calcular:

$$sen(\alpha \pm \beta), \cos(\alpha \pm \beta), \ tg(\alpha \pm \beta).$$

Si $sen\alpha = -\frac{24}{25}, sen\beta = \frac{3}{5}, \pi \leq \alpha \leq \frac{3\pi}{2}$ y $\frac{\pi}{2} \leq \beta \leq \pi$, calcular:

$$sen(\alpha \pm \beta), \cos(\alpha \pm \beta), \ tg(\alpha \pm \beta).$$

Demostrar las identidades:

(a) $(\cos\frac{\alpha}{2} - sen\frac{\alpha}{2})^2 = 1 - sen\alpha$

(b) $tg\frac{a}{2} = \cos ec\alpha - \cot\alpha$

(c) $\frac{sen2\alpha}{sen\alpha} - \frac{\cos 2\alpha}{\cos\partial} = \sec\alpha$

(d) $sen^4\alpha = \frac{3}{8} - \frac{1}{2}\cos 2\alpha + \frac{1}{8}\cos 4\alpha$

(e) $1 - \frac{1}{2}sen2\alpha = \frac{sen^3\alpha + \cos^3\alpha}{sen\alpha + \cos\alpha}$

(f) $\cos 4\alpha = 8\cos^4\alpha - 8\cos^2 a + 1$

(g) $\frac{1 - \cos 2\alpha}{sen2\alpha} = tg\alpha$

(h) $\cos ec2\alpha + \cot 2\alpha = \cot\alpha$

(i) $tg3\alpha = \frac{3tg\alpha - tg^3\alpha}{1 - 3tg^2\alpha}$

(j) $\frac{tg\alpha + \cot\alpha}{\cot\alpha - tg\alpha} = \sec 2\alpha$

(k) $\frac{sen3\alpha}{sen\alpha} - \frac{\cos 3\alpha}{\cos\alpha} = 2$

(l) $\frac{1 + \cos 2\alpha}{sen2\alpha} = \cot\alpha$

(m) $\frac{\cot^2\alpha - 1}{\cos ec^2\alpha} = \cos 2\alpha$

(n) $sen\alpha + sen3\alpha + sen5\alpha + sen7\alpha = 4\cos\alpha\cos 2\alpha\, sen4\alpha$

Desde cada extremo de una base de longitud $2a$ la elevación angular de un monte es θ y desde el punto medio de tal base es ϕ. Demostrar que el monte mide:

$$asen\theta sen\phi\sqrt{\cos ec(\phi + \theta)\cos ec(\phi - \theta)}.$$

Una torre dista 40 metros desde la orilla más cercana de un río, cuyo ancho es de 100 metros. Calcular la altura de la torre si desde la cúspide se observa el río bajo un ángulo de 30º.

Demostrar que:

$$sen^2 1'^{o} + \cos^2 20^{o} - sen10^{o}\cos 20^{o} = \frac{3}{4}$$

Demostrar que:

$$\frac{1}{sen10^{o}} - \frac{\sqrt{3}}{\cos 10^{o}} = 4$$

Demostrar que:

(a) $Cos10^{o} + sen\,40^{o} = \sqrt{3}sen70^{o}$

(b) $4\cos 10^{o} - 2\sec 18^{o} = 2tg18^{o} = 2tg18^{o}$

(c) $\cot 15^{o} + \cot 75^{o} + \cot 135^{o} - \cos ec30^{o} = 1$

(d) $\cos 6^{o}\cos 66^{o}\cos 42^{o}\cos 78^{o} = \dfrac{1}{16}$

(e) $sen18^{o} + \cos 18^{-circ} = \sqrt{2}\cos 27^{o}$

(f) $sen33^{o} + \cos 63^{o} = \cos 3^{o}$

(g) $tg\,20^{o}\,tg\,40^{o}\,tg\,80^{o} = tg\,60^{o}$

Demostrar que:

(a) $\cot(\alpha + 15^{o}) - tg(a - 15^{o}) = \dfrac{4\cos 2\alpha}{2sen2\alpha + 1}$

(b) $\dfrac{\cos\alpha - \cos 3\alpha}{sen3\alpha - sen\alpha} = tg\,2\alpha$

(c) $\dfrac{sen(\alpha + \beta) - sen4\beta}{\cos(\alpha + \beta) + \cos 4\beta} = tg\,\dfrac{\alpha - 3\beta}{2}$

(d) $\dfrac{3 - 4\cos 2\alpha + \cos 4\alpha}{3 + 4\cos 2\alpha + \cos 4\alpha} = tg^{4}\alpha$