1. Ecuaciones sencillas:

a)
$$x + 4 = 5$$
$$x = 5 - 4$$
$$x = 1$$

b)
$$x - 3 = 6$$
$$x = 6 + 3$$
$$x = 9$$

c)
$$7 + x = 10$$
$$x = 10 - 7$$
$$x = 3$$

d)
$$7 - x = 5$$
$$-x = 5 - 7$$
$$-x = -2$$
$$x = 2$$

e)
$$11 = x + 5$$
$$x + 5 = 11$$
$$x = 11 - 5$$
$$x = 6$$

f)
$$2 = x - 9$$
$$x - 9 = 2$$
$$x = 2 + 9$$
$$x = 11$$

g)
$$5 = 2 + x$$
$$x = 5 - 2$$
$$x = 3$$

h)
$$9 = 15 - x$$
$$x = 15 - 9$$
$$x = 6$$

i)
$$2 - x = 9$$
$$-x = 9 - 2$$
$$-x = 7$$
$$x = -7$$

2. Ecuaciones sencillas.

a)
$$2x + x = 5$$
$$3x = 5$$
$$x = \frac{5}{3}$$

b)
$$7x - 3x = 10 - 7$$
$$4x = 3$$
$$x = \frac{3}{4}$$

c)
$$x - 9x = 9 - 7$$
$$-8x = 2$$
$$x = \frac{2}{-8}$$
$$x = -\frac{1}{4}$$

d)
$$5x - x = 3 - 5$$
$$4x = -2$$
$$x = \frac{-2}{4}$$
$$x = -\frac{1}{2}$$

e)
$$6 = 12x - 2x$$
$$6 = 10x$$
$$x = \frac{6}{10}$$
$$x = \frac{3}{5}$$

f)
$$2 - 8 = x + 2x$$
$$-6 = 3x$$
$$x = \frac{-6}{3}$$
$$x = -2$$

g)

$$5x - 13x = 6 - 10$$

$$-8x = -4$$

$$x = \frac{-4}{-8}$$

$$x = \frac{1}{2}$$

h)

$$2x + 4 + 5x = 18$$

$$7x = 18 - 4$$

$$7x = 14$$

$$x = \frac{14}{2}$$

$$x = 7$$

i)

$$11x + 17 - 6x = 2$$

$$5x = -15$$

$$x = \frac{-15}{5}$$

$$x = -3$$

j)

$$9 = 12x - 6 - 7x$$

$$9 = 5x - 6$$

$$5x = 9 + 6$$

$$x = \frac{15}{5}$$

$$x = 3$$

k)

$$2x - 5 + 3x + 1 = 3x - 2$$

$$5x - 3x = -2 + 5 - 1$$

$$2x = 2$$

$$x = \frac{2}{2}$$

$$x = 1$$

l)

$$x + 7 = 12x - 3 - 8x + 1$$

$$x - 12x + 8x = -2 - 7$$

$$-3x = -9$$

$$x = \frac{-9}{-3}$$

$$x = 3$$

m)

$$6x - 1 + x = 4 - 5x + 3$$

$$7x + 5x = 7 + 1$$

$$12x = 8$$

$$x = \frac{8}{12}$$

$$x = \frac{2}{3}$$

n)

$$x + 2x + 3x - 5 = 4x - 9$$

$$6x - 4x = -9 + 5$$

$$2x = -4$$

$$x = \frac{-4}{2}$$

$$x = -2$$

ñ)

$$5x + 4 - 6x = 7 - x - 3$$

$$-x + x = 4 - 4$$

$$0 = 0$$

Identidad

o)

$$4x + 2 + 7x = 10x + 3 + x$$

$$11x - 10x - x = 3 - 2$$

$$0 = 1 \ !$$

No tiene solución

Nota 1: Cuando una expresión algebraica como la del apartado ñ, nos da una expresión como 0=0, ó 5=5 ó cualquier otra igualdad sin término literal, significa que dicha expresión no es una ecuación sino una *identidad*. Esto significa que en dicha expresión algebraica no existe un resultado para la *x*, sino infinitos resultados.

Nota 2: Cuando una expresión algebraica como la del apartado o, nos da una expresión incongruente como 0=1 ó cualquier otra incongruencia, significa que la ecuación *no tiene solución*, es decir, no existe ningún valor para x que confirme la ecuación.

3. Ecuaciones con paréntesis.

a)
$$6(x+1)-4x=5x-9$$
$$6x+6-4x=5x-9$$
$$2x-5x=-9-6$$
$$-3x=-15$$
$$x=\frac{-15}{-3}$$
$$x=5$$

b)
$$18x-13=8-4(3x-1)$$
$$18x-13=8-12x+4$$
$$18x+12x=12+13$$
$$30x=25$$
$$x=\frac{25}{30}$$
$$x=\frac{5}{6}$$

c)
$$3x+5(2x-1)=8-3(4-5x)$$
$$3x+10x-5=8-12+15x$$
$$13x-15x=-4+5$$
$$-2x=1$$
$$x=-\frac{1}{2}$$

d)
$$5-(4x+6)=3x+(7-4x)$$
$$5-4x-6=3x+7-4x$$
$$-4x-3x+4x=7-5+6$$
$$-3x=8$$
$$x=-\frac{8}{3}$$

e)
$$x-7(2x+1)=2(6-5x)-13$$
$$x-14x-7=12-10x-13$$
$$-13x+10x=-1+7$$
$$-3x=6$$
$$x=\frac{6}{-3}$$
$$x=-2$$

f)
$$11-5(3x+2)+7x=1-8x$$
$$11-15x-10+7x=1-8x$$
$$-8x+8x=1-11+10$$
$$0=0$$
Identidad

g)
$$13x-5(x+2)=4(2x-1)+7$$
$$13x-5x-10=8x-4+7$$
$$8x-8x=3+10$$
$$0=13 \ !$$
No tiene solución

4. Ecuaciones con denominadores.

a)

$$x + \frac{1}{3} = \frac{x}{3}$$

$$\frac{3x}{3} + \frac{1}{3} = \frac{x}{3}$$

$$\frac{3x+1}{3} = \frac{x}{3}$$

$$3x + 1 = x$$

$$3x - x = -1$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

b)

$$\frac{5x}{3} + 1 = \frac{5}{6} + x$$

$$\frac{10x}{6} + \frac{6}{6} = \frac{5}{6} + \frac{6x}{6}$$

$$\frac{10x+6}{6} = \frac{5+6x}{6}$$

$$10x + 6 = 5 + 6x$$

$$10x - 6x = 5 - 6$$

$$4x = -1$$

$$x = -\frac{1}{4}$$

c)

$$\frac{3x}{5} - \frac{1}{4} = x - \frac{7x}{10} - \frac{1}{5}$$

$$\frac{12x}{20} - \frac{5}{20} = \frac{20x}{20} - \frac{14x}{20} - \frac{4}{20}$$

$$\frac{12x-5}{20} = \frac{20x-14x-4}{20}$$

$$12x - 5 = 20x - 14x - 4$$

$$12x - 20x + 14x = -4 + 5$$

$$6x = 1$$

$$x = \frac{1}{6}$$

d)

$$\frac{x}{3} + \frac{4}{15} - x = \frac{1}{6} - \frac{7x}{10}$$

$$\frac{10x}{30} + \frac{8}{30} - \frac{30x}{30} = \frac{5}{30} - \frac{21x}{30}$$

$$\frac{10x+8-30x}{30} = \frac{5-21x}{30}$$

$$10x + 8 - 30x = 5 - 21x$$

$$-20x + 21x = 5 - 8$$

$$x = -3$$

e)

$$\frac{7x}{4} - 1 - \frac{x}{8} = x + \frac{5x}{8} + 1$$

$$\frac{14x}{8} - \frac{8}{8} - \frac{x}{8} = \frac{8x}{8} + \frac{5x}{8} + \frac{8}{8}$$

$$\frac{14x-8-x}{8} = \frac{8x+5x+8}{8}$$

$$14x - 8 - x = 8x + 5x + 8$$

$$13x - 8x - 5x = 8 - 8$$

$$0 = 0$$

Identidad

f)

$$\frac{x}{2} + \frac{1}{6} - \frac{x}{3} = \frac{x}{6} - \frac{2}{3} + \frac{5}{6}$$

$$\frac{3x}{6} + \frac{6}{6} - \frac{2x}{6} = \frac{x}{6} - \frac{4}{6} + \frac{5}{6}$$

$$\frac{3x+6-2x}{6} = \frac{x-4+5}{6}$$

$$3x + 6 - 2x = x - 4 + 5$$

$$x - x = 1 - 6$$

$$0 = -5 \ !$$

No tiene solución

5. Ecuaciones con denominadores y paréntesis.

a)

$$2x - \frac{5}{2} = \frac{1}{2}(x-3)$$

$$2x - \frac{5}{2} = \frac{x}{2} - \frac{3}{2}$$

$$\frac{4x}{2} - \frac{5}{2} = \frac{x}{2} - \frac{3}{2}$$

$$\frac{4x-5}{2} = \frac{x-3}{2}$$

$$4x - 5 = x - 3$$

$$4x - x = -3 + 5$$

$$3x = 2$$

$$x = \frac{2}{3}$$

b)

$$\frac{5}{6}(2x-1) - x = \frac{x}{6}$$

$$\frac{10x}{6} - \frac{5}{6} - x = \frac{x}{6}$$

$$\frac{10x}{6} - \frac{5}{6} - \frac{6x}{x} = \frac{x}{6}$$

$$\frac{10x-5-6x}{6} = \frac{x}{6}$$

$$10x - 5 - 6x = x$$

$$4x - x = 5$$

$$3x = 5$$

$$x = \frac{5}{3}$$

c)

$$\frac{x}{5} - 1 = 2\left(x - \frac{4}{5}\right)$$

$$\frac{x}{5} - 1 = 2x - \frac{8}{5}$$

$$\frac{x}{5} - \frac{5}{5} = \frac{10x}{5} - \frac{8}{5}$$

$$\frac{x-5}{5} = \frac{10x-8}{5}$$

$$x - 5 = 10x - 8$$

$$x - 10x = -8 + 5$$

$$-9x = -3$$

$$x = \frac{-3}{-9}$$

$$x = \frac{1}{3}$$

d)

$$x - \frac{1}{3} = \frac{1}{6}(2x-5)$$

$$x - \frac{1}{3} = \frac{2x}{6} - \frac{5}{6}$$

$$\frac{6x}{6} - \frac{2}{6} = \frac{2x}{6} - \frac{5}{6}$$

$$\frac{6x-2}{6} = \frac{2x-5}{6}$$

$$6x - 2 = 2x - 5$$

$$6x - 2x = -5 + 2$$

$$4x = -3$$

$$x = -\frac{3}{4}$$

6. Ecuaciones con denominadores y paréntesis.

a)

$$\frac{1}{5}(2+5x) = \frac{1}{2}\left(x - \frac{1}{5}\right)$$

$$\frac{2}{5} + \frac{5x}{5} = \frac{x}{2} - \frac{1}{10}$$

$$\frac{4}{10} + \frac{10x}{10} = \frac{5x}{10} - \frac{1}{10}$$

$$\frac{4+10x}{10} = \frac{5x-1}{10}$$

$$4 + 10x = 5x - 1$$

$$10x - 5x = -1 - 4$$

$$5x = -5$$

$$x = \frac{-5}{5}$$

$$x = -1$$

b)

$$2(x-3) - \frac{1}{3} = x - \frac{1}{3}(x-1)$$

$$2x - 6 - \frac{1}{3} = x - \frac{x}{3} + \frac{1}{3}$$

$$\frac{6x}{3} - \frac{18}{3} - \frac{1}{3} = \frac{3x}{3} - \frac{x}{3} + \frac{1}{3}$$

$$\frac{6x-18-1}{3} = \frac{3x-x+1}{3}$$

$$6x - 18 - 1 = 3x - x + 1$$

$$6x - 3x + x = 1 + 1 + 18$$

$$4x = 20$$

$$x = \frac{20}{4}$$

$$x = 5$$

c)

$$1 - \frac{3x}{8} = \frac{3}{4} - \frac{1}{2}(x-2)$$

$$1 - \frac{3x}{8} = \frac{3}{4} - \frac{x}{2} + \frac{2}{2}$$

$$\frac{8}{8} - \frac{3x}{8} = \frac{6}{8} - \frac{4x}{8} + \frac{8}{8}$$

$$\frac{8-3x}{8} = \frac{6-4x+8}{8}$$

$$8 - 3x = 6 - 4x + 8$$

$$-3x + 4x = 14 - 8$$

$$x = 6$$

d)

$$x - \frac{3x}{4} = \frac{1}{3}(2x-1) + \frac{x}{6}$$

$$x - \frac{3x}{4} = \frac{2x}{3} - \frac{1}{3} + \frac{x}{6}$$

$$\frac{12x}{12} - \frac{9x}{12} = \frac{8x}{12} - \frac{4}{12} + \frac{2x}{12}$$

$$\frac{12x-9x}{12} = \frac{8x-4+2x}{12}$$

$$12x - 9x = 8x - 4 + 2x$$

$$3x - 8x - 2x = -4$$

$$-7x = -4$$

$$x = \frac{4}{7}$$

e)

$$5\left(\frac{x}{4}-\frac{1}{10}\right)=\frac{1}{2}\left(3x-\frac{1}{2}\right)$$

$$\frac{5x}{4}-\frac{5}{10}=\frac{3x}{2}-\frac{1}{4}$$

$$\frac{25x}{20}-\frac{10}{20}=\frac{30x}{20}-\frac{5}{20}$$

$$\frac{25x-10}{20}=\frac{30x-5}{20}$$

$$25x-10=30x-5$$

$$25x-30x=-5+10$$

$$-5x=5$$

$$x=\frac{5}{-5}$$

$$x=-1$$

f)

$$1-\frac{3}{7}(x+1)=\frac{2x}{3}-\frac{1}{7}$$

$$1-\frac{3x}{7}-\frac{3}{7}=\frac{2x}{3}-\frac{1}{7}$$

$$\frac{21}{21}-\frac{9x}{21}-\frac{9}{21}=\frac{14x}{21}-\frac{3}{21}$$

$$\frac{21-9x-9}{21}=\frac{14x-3}{21}$$

$$21-9x-9=14x-3$$

$$-9x-14x=-3-21+9$$

$$-23x=-15$$

$$x=\frac{15}{23}$$

7. Ecuaciones con numeradores compuestos.

a)

$$x-\frac{x-3}{5}=1$$

$$\frac{5x}{5}-\frac{x-3}{5}=\frac{5}{5}$$

$$\frac{5x-x+3}{5}=\frac{5}{5}$$

$$5x-x+3=5$$

$$4x=5-3$$

$$x=\frac{2}{4}$$

$$x=\frac{1}{2}$$

b)

$$1-\frac{x+1}{3}=2x-\frac{1}{3}$$

$$\frac{3}{3}-\frac{x+1}{3}=\frac{6x}{3}-\frac{1}{3}$$

$$\frac{3-x-1}{3}=\frac{6x-1}{3}$$

$$3-x-1=6x-1$$

$$-x-6x=-1+1-3$$

$$-7x=-3$$

$$x=\frac{3}{7}$$

c)

$$1 - \frac{1-x}{3} = x + \frac{1}{2}$$

$$\frac{6}{6} - \frac{2-2x}{6} = \frac{6x}{6} + \frac{3}{6}$$

$$\frac{6-2+2x}{6} = \frac{6x+3}{6}$$

$$6 - 2 + 2x = 6x + 3$$

$$2x - 6x = 3 - 6 + 2$$

$$-4x = -1$$

$$x = \frac{1}{4}$$

d)

$$\frac{3x}{2} - 1 = \frac{3x+2}{4}$$

$$\frac{6x}{4} - \frac{4}{4} = \frac{3x+2}{4}$$

$$\frac{6x-4}{4} = \frac{3x+2}{4}$$

$$6x - 4 = 3x + 2$$

$$6x - 3x = 2 + 4$$

$$3x = 6$$

$$x = \frac{6}{3}$$

$$x = 2$$

e)

$$\frac{3x-1}{2} - 1 = 2x - 2$$

$$\frac{3x-1}{2} - \frac{2}{2} = \frac{4x}{2} - \frac{4}{2}$$

$$\frac{3x-1-2}{2} = \frac{4x-4}{2}$$

$$3x - 1 - 2 = 4x - 4$$

$$3x - 4x = -4 + 1 + 2$$

$$-x = -1$$

$$x = 1$$

f)

$$x + \frac{2-3x}{5} = \frac{x}{2} + 1$$

$$\frac{10x}{10} + \frac{4-6x}{10} = \frac{5x}{10} + \frac{10}{10}$$

$$\frac{10x+4-6x}{10} = \frac{5x+10}{10}$$

$$10x + 4 - 6x = 5x + 10$$

$$4x - 5x = 10 - 4$$

$$-x = 6$$

$$x = -6$$

g)

$$2x + \frac{x-3}{2} = \frac{x-3}{4}$$

$$\frac{8x}{4} + \frac{2x-6}{4} = \frac{x-3}{4}$$

$$\frac{8x+2x-6}{4} = \frac{x-3}{4}$$

$$8x + 2x - 6 = x - 3$$

$$10x - x = -3 + 6$$

$$9x = 3$$

$$x = \frac{3}{9}$$

$$x = \frac{1}{3}$$

h)

$$\frac{3x}{5} - 1 = x - \frac{x+1}{2}$$

$$\frac{6x}{10} - \frac{10}{10} = \frac{10x}{10} - \frac{5x+5}{10}$$

$$\frac{6x-10}{10} = \frac{10x-5x-5}{10}$$

$$6x - 10 = 10x - 5x - 5$$

$$6x - 10x + 5x = -5 + 10$$

$$x = 5$$

i)

$$\frac{x}{5} - \frac{x+2}{15} = \frac{x}{3}$$

$$\frac{3x}{15} - \frac{x+2}{15} = \frac{5x}{15}$$

$$\frac{3x-x-2}{15} = \frac{5x}{15}$$

$$3x-x-2 = 5x$$

$$2x-5x = 2$$

$$-3x = 2$$

$$x = -\frac{2}{3}$$

j)

$$\frac{x-5}{3} + \frac{x-2}{5} = x-2$$

$$\frac{5x-25}{15} + \frac{3x-6}{15} = \frac{15x}{15} - \frac{30}{15}$$

$$\frac{5x-25+3x-6}{15} = \frac{15x-30}{15}$$

$$5x-25+3x-6 = 15x-30$$

$$8x-15x = -30+25+6$$

$$-7x = 1$$

$$x = -\frac{1}{7}$$

k)

$$\frac{x+3}{5} - \frac{x-6}{7} = 1$$

$$\frac{7x+21}{35} - \frac{5x-30}{35} = \frac{35}{35}$$

$$\frac{7x+21-5x+30}{35} = \frac{35}{35}$$

$$7x+21-5x+30 = 35$$

$$2x = 35-21-30$$

$$2x = -16$$

$$x = \frac{-16}{2}$$

$$x = -8$$

l)

$$\frac{1-x}{3} - \frac{x-1}{12} = \frac{3x-1}{4}$$

$$\frac{4-4x}{12} - \frac{x-1}{12} = \frac{9x-3}{12}$$

$$\frac{4-4x-x+1}{12} = \frac{9x-3}{12}$$

$$4-4x-x+1 = 9x-3$$

$$-5x-9x = -3-4-1$$

$$-14x = -8$$

$$x = \frac{-8}{-14}$$

$$x = \frac{4}{7}$$

8. Ecuaciones con fracciones con numerador compuesto y con paréntesis.

a)

$$\frac{3x-1}{4} - \frac{2x+1}{5} = \frac{7x-13}{20}$$

$$\frac{15x-5}{20} - \frac{8x+4}{20} = \frac{7x-13}{20}$$

$$\frac{15x-5-8x-4}{20} = \frac{7x-13}{20}$$

$$15x-5-8x-4 = 7x-13$$

$$7x-7x = -13+5+4$$

$$0 = -4 \ !$$

No tiene solución

b)

$$2 + \frac{2}{5}(x+1) = x - \frac{2x+3}{5}$$

$$2 + \frac{2x}{5} + \frac{2}{5} = x - \frac{2x+3}{5}$$

$$\frac{10}{5} + \frac{2x}{5} + \frac{2}{5} = \frac{5x}{5} - \frac{2x+3}{5}$$

$$\frac{10+2x+2}{5} = \frac{5x-2x-3}{5}$$

$$10+2x+2 = 5x-2x-3$$

$$2x-5x+2x = -3-10-2$$

$$-x = -15$$

$$x = 15$$

d)

$$\frac{2}{3}(1-3x) + \frac{3(x-1)}{4} = \frac{5}{12}(1-x)$$

$$\frac{2}{3} - \frac{6x}{3} + \frac{3x-3}{4} = \frac{5}{12} - \frac{5x}{12}$$

$$\frac{8}{12} - \frac{24x}{12} + \frac{9x-9}{12} = \frac{5}{12} - \frac{5x}{12}$$

$$\frac{8-24x+9x-9}{12} = \frac{5-5x}{12}$$

$$8-24x+9x-9 = 5-5x$$

$$-15x+5x = 5-8+9$$

$$-10x = 6$$

$$x = \frac{6}{-10}$$

$$x = -\frac{3}{5}$$

e)

$$\frac{3}{5}\left(\frac{x-1}{3}+1\right) + x = \frac{3}{4}\left(x - \frac{2}{3}\right)$$

$$\frac{3x-3}{15} + \frac{3}{5} + x = \frac{3x}{4} - \frac{6}{12}$$

$$\frac{12x-12}{60} + \frac{36}{60} + \frac{60x}{60} = \frac{45x}{60} - \frac{30}{60}$$

$$\frac{12x-12+36+60x}{60} = \frac{45x-30}{60}$$

$$12x-12+36+60x = 45x-30$$

$$72x-45x = -30+12-36$$

$$27x = -54$$

$$x = \frac{-54}{27}$$

$$x = -2$$