

2. Operaciones con derivadas

2.1. Calcula la derivada de las siguientes funciones:

1. $f(x) = 2x^2$	2. $f(x) = -5x^2$	3. $f(x) = 3x^3$	4. $f(x) = 3x^4$
5. $f(x) = -2x^4$	6. $f(x) = -4x^5$	7. $f(x) = \frac{1}{2}x^6$	8. $f(x) = \frac{2}{3}x^3$
9. $f(x) = -\frac{5}{6}x^4$	10. $f(x) = \frac{5}{4}x^8$	11. $f(x) = -\frac{3}{2}x^6$	12. $f(x) = -\frac{3x^5}{5}$
13. $f(x) = \frac{2}{x}$	14. $f(x) = -\frac{5}{x^3}$	15. $f(x) = \frac{3}{4x^4}$	16. $f(x) = -\frac{2}{5x^2}$

2.2. Hallar la derivada simplificada de las siguientes funciones, pasándolas previamente a forma de potencia:

1. $f(x) = 2\sqrt{x}$	2. $f(x) = 3\sqrt{5x}$	3. $f(x) = \frac{\sqrt{x}}{2}$	4. $f(x) = \frac{\sqrt{3x}}{5}$
5. $f(x) = \sqrt[3]{x}$	6. $f(x) = \sqrt[4]{x^3}$	7. $f(x) = \sqrt[5]{2x^2}$	8. $f(x) = \sqrt[3]{3x^2}$
9. $f(x) = x^2 \cdot \sqrt{x}$	10. $f(x) = x \cdot \sqrt[4]{x^3}$	11. $f(x) = \frac{4}{\sqrt{x}}$	12. $f(x) = \frac{3}{2\sqrt[5]{x^2}}$
13. $f(x) = \frac{x^2}{\sqrt{x}}$	14. $f(x) = -\frac{3}{\sqrt[3]{x^2}}$	15. $f(x) = -\frac{3x}{\sqrt[3]{x}}$	16. $f(x) = \frac{1}{\sqrt[3]{x^2}}$

2.3. Calcula la derivada de las siguientes funciones polinómicas:

1. $f(x) = 3x^3 - 2x$	2. $f(x) = x^4 + 2x^2 + 12$	3. $f(x) = 3x^4 - 2x^2 + 5$
4. $f(x) = x^4 + 3x^2 + 6$	5. $f(x) = 6x^3 - x^2 + 3$	6. $f(x) = 3x^3 - 2x^2 + 6$
7. $f(x) = \frac{2}{3}x^3 - \frac{1}{4}x^2 + \frac{5}{2}x + 3$	8. $f(x) = \frac{1}{4} - \frac{1}{3}x + x^2 - 3x^3$	9. $f(x) = \frac{2}{3}x^2 + \frac{4}{5}x - \frac{1}{6}$
10. $f(x) = 3x^{-3} + 2x^{-2} - 3x^{-1} + 5$	11. $f(x) = 1 + \frac{2}{x} - \frac{3}{x^2}$	12. $f(x) = \frac{4}{x} - \frac{7}{x^4} + 5$
13. $f(x) = \frac{x^4}{2} - \frac{x^2}{4} - x$	14. $f(x) = \frac{5x^3}{3} - \frac{x}{2} - 1$	15. $f(x) = \frac{1}{4}x^6 - \frac{3}{2}x^3 + 3x^2 - 2x$

2.4. Calcula la derivada de las siguientes funciones:

1. $f(x) = x^2 + \sqrt{x^3}$	2. $f(x) = 3x^{\frac{1}{2}} + 2x^{\frac{2}{3}} - 3$	3. $f(x) = 3x^{-\frac{1}{4}} - 5x^{-\frac{3}{5}}$
4. $f(x) = \frac{x^2}{3} + 3x - \frac{2}{x^4}$	5. $f(x) = x^{-2} - \frac{4}{x^2} + \frac{2}{x^4}$	6. $f(x) = \frac{x^4}{4} + \frac{3x^2}{2} - 2 - \frac{3}{x} + \frac{6}{x^3}$
7. $f(x) = 2\sqrt{x} + \sqrt[3]{x}$	8. $f(x) = \frac{1}{\sqrt{x}} - \frac{3}{\sqrt[3]{x^2}}$	9. $f(x) = \frac{1}{\sqrt{x}} - \frac{3}{\sqrt[3]{x^2}}$
10. $f(x) = \sqrt{x} - \frac{1}{\sqrt{x}}$	11. $f(x) = x^3 - 2\sqrt{x} + \frac{4}{\sqrt{x}}$	12. $f(x) = \frac{-1}{\sqrt[5]{x^3}} - \frac{2}{x^2} + \frac{5}{\sqrt{x}}$
13. $f(x) = \frac{5\sqrt{x} - 3x^2}{2}$	14. $f(x) = 3x^3 + \frac{2}{3}x^2 - x + 3\sqrt{x}$	15. $f(x) = x\sqrt{x} + \frac{1}{x^2\sqrt{x}} - \frac{3}{x^3\sqrt{x^2}}$

2.5. Calcula la derivada de los siguientes productos:

$$1. f(x) = (2x^2 + 1)(x^3 + 6)$$

$$4. f(x) = (3x - 4) \cdot (2x + 1)$$

$$7. f(x) = (x - 4) \cdot (x^2 + 3)$$

$$10. f(x) = (x + 2)^2$$

$$2. f(x) = (2x - 1)(x^2 + 1)$$

$$5. f(x) = (4 - x) \cdot (2 - 3x)$$

$$8. f(x) = (x^2 - 4) \cdot (x^4 + 3)$$

$$11. f(x) = (3x - 5)^2$$

$$3. f(x) = (3x + 2)(4x + 5)$$

$$6. f(x) = (5x + 4) \cdot (2 - 6x)$$

$$9. f(x) = 3(3x^2 + 1) \cdot (4 - x)$$

$$12. f(x) = (4 - 3x)^3$$

2.6. Calcula la derivada de las siguientes funciones racionales:

$$1. f(x) = \frac{3x}{3x+1}$$

$$4. f(x) = \frac{2x^4}{1-x^2}$$

$$7. f(x) = \frac{2x+1}{5x-1}$$

$$10. f(x) = \frac{3x^3}{2+4x^2}$$

$$13. f(x) = \frac{x^2 - 2x + 4}{x^2 + 2x + 4}$$

$$2. f(x) = \frac{1-2x}{2-x}$$

$$5. f(x) = \frac{-3}{x^4 + 2x}$$

$$8. f(x) = \frac{x^2}{5x+2}$$

$$11. f(x) = \frac{x^2 + 1}{x^2 - x + 2}$$

$$14. f(x) = \frac{2 - 3x^2}{2x+1}$$

$$3. f(x) = \frac{3x+4}{5x-3}$$

$$6. f(x) = \frac{4}{3x^2 + 1}$$

$$9. f(x) = \frac{x^4 + 4}{x^4 - 4}$$

$$12. f(x) = \frac{x^2 + 5x}{x-5}$$

$$15. f(x) = \frac{(x+2)(x-2)}{x(x-1)}$$

2.7. Calcula la derivada de las siguientes funciones irracionales:

$$1. f(x) = x - \sqrt{x}$$

$$4. f(x) = x^2 \cdot (1 + \sqrt{x})$$

$$7. f(x) = (x-1) \cdot \sqrt[3]{x+1}$$

$$10. f(x) = \frac{2\sqrt{x}}{x^2 + 2}$$

$$2. f(x) = \sqrt{2x} + \sqrt[3]{x}$$

$$5. f(x) = x^2 \cdot \sqrt{7-2x}$$

$$8. f(x) = (x-1)^2 \cdot \sqrt[3]{x^2-1}$$

$$11. f(x) = \frac{x+1}{\sqrt{x^3}}$$

$$3. f(x) = \frac{1}{\sqrt{x+1}}$$

$$6. f(x) = (x+2) \cdot \sqrt{x^3+4x}$$

$$9. f(x) = \frac{x+2}{\sqrt{x^2+4}}$$

$$12. f(x) = \frac{x^2+1}{2\sqrt{x}}$$

2.8. Calcula la derivada de las siguientes funciones:

$$1. f(x) = \sqrt{x} \cdot (x+2) \cdot (x-1)$$

$$4. f(x) = \frac{\sqrt{2+6x}}{x}$$

$$7. f(x) = \frac{1-\sqrt[3]{x}}{\sqrt{x}}$$

$$10. f(x) = \frac{3x}{x-1} + \frac{2x}{x^2-1}$$

$$13. f(x) = \frac{1}{(1-x)^2}$$

$$16. f(x) = 3\sqrt{x^2-3}$$

$$19. f(x) = \sqrt{\frac{1+x}{1-x}}$$

$$2. f(x) = \sqrt{1+3x} \cdot \sqrt[3]{1+2x}$$

$$5. f(x) = \frac{\sqrt{x}}{1-x}$$

$$8. f(x) = \frac{1-\sqrt{x}}{1+\sqrt{x}}$$

$$11. f(x) = \sqrt{\frac{1-x}{1+x}}$$

$$14. f(x) = \frac{x^2}{(2x-1)^2}$$

$$17. f(x) = \sqrt[3]{x^2+x+1}$$

$$20. f(x) = \frac{1-x}{\sqrt{1-x^2}}$$

$$3. f(x) = (2\sqrt{x}-1) \cdot (3-4\sqrt{x})$$

$$6. f(x) = \frac{\sqrt{x}}{1+\sqrt{x}}$$

$$9. f(x) = \frac{x^2}{2-\sqrt{x}}$$

$$12. f(x) = \sqrt{\frac{3x+2}{3x-2}}$$

$$15. f(x) = \frac{2x^2-4x}{(x-1)^2}$$

$$18. f(x) = (x - \sqrt{1-x^2})^2$$

$$21. f(x) = x \cdot \sqrt{x^2+2}$$

2.9. Calcula la derivada de las siguientes funciones exponenciales:

1. $f(x) = e^{3x}$

4. $f(x) = e^{x^2}$

7. $f(x) = x^2 \cdot e^x$

10. $f(x) = e^x + x^2$

13. $f(x) = \frac{e^{4x}}{\sqrt{x}}$

16. $f(x) = (5x^2 + x) \cdot e^{x+1}$

19. $f(x) = \frac{x^2 - 1}{e^x}$

22. $f(x) = \frac{e^{\sqrt{x}}}{\sqrt{x}}$

2. $f(x) = e^{1-x}$

5. $f(x) = 2^x$

8. $f(x) = (x^2 - x + 1)e^x$

11. $f(x) = (3x - 2)e^{2x}$

14. $f(x) = \frac{e^x}{x}$

17. $f(x) = e^{3x^2 - 2x + 1}$

20. $f(x) = \frac{e^x + 1}{x}$

23. $f(x) = \frac{e^x + 2}{e^x - 2}$

3. $f(x) = e^{3+2x}$

6. $f(x) = x \cdot 2^x$

9. $f(x) = \sqrt{x} \cdot e^{\frac{x}{2}}$

12. $f(x) = \frac{2^x}{x}$

15. $f(x) = \frac{1}{e^x}$

18. $f(x) = -2e^{\sqrt{x}}$

21. $f(x) = \frac{e^x + e^{-x}}{2}$

24. $f(x) = \frac{x + e^x}{x - e^x}$

2.10. Calcula la derivada de las siguientes funciones logarítmicas:

1. $f(x) = \ln(2x - 1)$

4. $f(x) = \ln(x^4 - 3x - 1)$

7. $f(x) = \log_3 x$

10. $f(x) = (x + 1) \cdot \ln x$

13. $f(x) = \frac{x}{\ln x}$

16. $f(x) = \frac{\ln(2x + 1)}{x^2}$

2. $f(x) = \ln(2 - x)$

5. $f(x) = \ln \sqrt{1 + x^2}$

8. $f(x) = \ln x^3$

11. $f(x) = x \cdot \ln(1 - x^2)$

14. $f(x) = \frac{\ln x}{x}$

17. $f(x) = \frac{1 + \ln x}{x}$

3. $f(x) = \ln(x^2 - 1)$

6. $f(x) = \ln \sqrt{x(1-x)}$

9. $f(x) = \ln\left(\frac{1}{x}\right)$

12. $f(x) = x^2 \cdot \ln(2 - x)$

15. $f(x) = \frac{\ln x}{\sqrt{x}}$

18. $f(x) = \frac{\ln x}{1 + x^2}$

2.11. Calcula la derivada de las siguientes funciones trigonométricas:

1. $f(x) = \sin(5x^2 + 2x)$

4. $f(x) = \operatorname{tg}(2x^2 + 1)$

7. $f(x) = \sin^2 x$

10. $f(x) = \sin x \cdot \cos x$

13. $f(x) = x \cdot \sin x$

16. $f(x) = \operatorname{ctg} x$

19. $f(x) = \cos x \cdot (1 - \cos x)$

22. $f(x) = \ln \cos x$

2. $f(x) = \cos(x^2 + 2)$

5. $f(x) = \sin(3x^3 - 2x + 1)$

8. $f(x) = \cos^2 x$

11. $f(x) = \sin x^2 + \cos 2x$

14. $f(x) = x^2 \cdot \cos x$

17. $f(x) = \sec x$

20. $f(x) = \sin x + \operatorname{tg} x$

23. $f(x) = \ln \sin x$

3. $f(x) = \cos(x^4)$

6. $f(x) = \operatorname{tg} \sqrt{x}$

9. $f(x) = \operatorname{tg}^2 x$

12. $f(x) = \sin \sqrt{x+1}$

15. $f(x) = x \cdot \operatorname{tg} x$

18. $f(x) = \operatorname{cosec} x$

21. $f(x) = \sin^2 x + \sin x^2$

24. $f(x) = \ln \operatorname{tg} x$

2.12. Calcula la derivada de las siguientes funciones:

$$1. f(x) = \frac{1}{2} e^{1+x^2}$$

$$4. f(x) = \ln(x^2 + x)$$

$$7. f(x) = \ln \sqrt{\frac{1+x}{1-x}}$$

$$10. f(x) = \frac{1}{x} + 2 \ln x - \frac{\ln x}{x}$$

$$13. f(x) = \frac{\operatorname{sen} x}{1 - \cos x}$$

$$16. f(x) = \frac{1 + \operatorname{tg} x}{1 - \operatorname{tg} x}$$

$$19. f(x) = \frac{1 - \cos x}{\operatorname{sen}^2 x}$$

$$22. f(x) = \ln \sqrt{\frac{1 + \operatorname{sen} x}{1 - \operatorname{sen} x}}$$

$$2. f(x) = 3^{2x-3}$$

$$5. f(x) = \ln(\sqrt{x^2 + 1} - x)$$

$$8. f(x) = \ln\left(\frac{1+x^2}{1-x^2}\right)$$

$$11. f(x) = \ln \sqrt[3]{1+x^2}$$

$$14. f(x) = \operatorname{sen}^2 x - \cos^2 x$$

$$17. f(x) = \frac{1 - \cos x}{1 + \cos x}$$

$$20. f(x) = \left(\frac{1 + \operatorname{sen} x}{\cos x}\right)^2$$

$$23. f(x) = \ln\left(\frac{1 + \cos x}{\operatorname{sen} x}\right)$$

$$3. f(x) = \frac{a}{2}(e^{x/a} - e^{-x/a})$$

$$6. f(x) = 2\sqrt{x} - 2\ln(2 + \sqrt{x})$$

$$9. f(x) = \ln\left(\frac{e^x}{1+e^x}\right)$$

$$12. f(x) = 3x + \ln e^{2x+1}$$

$$15. f(x) = \cos^2 x + \operatorname{sen} x$$

$$18. f(x) = \frac{\operatorname{sen} x - \cos x}{\operatorname{sen} x + \cos x}$$

$$21. f(x) = \frac{\operatorname{sen} x}{1 + \operatorname{tg}^2 x}$$

$$24. f(x) = \frac{\operatorname{tg}^2 x}{2} + \ln(\cos x)$$

2.13. Calcular la derivada de las siguientes funciones trigonométricas inversas:

$$1. f(x) = \arccos(x^2)$$

$$4. f(x) = \arccos(3x^2 + 4x)$$

$$7. f(x) = \arccos\left(\frac{1-x^2}{1+x^2}\right)$$

$$10. f(x) = \arcsen(\sqrt{1-x^2})$$

$$2. f(x) = \arcsen(5x^2)$$

$$5. f(x) = \arcsen\left(\frac{x^2-1}{x^2}\right)$$

$$8. f(x) = \operatorname{arctg}\left(\frac{x+1}{1-x}\right)$$

$$11. f(x) = \operatorname{arctg}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

$$3. f(x) = \operatorname{arctg}(4x^3)$$

$$6. f(x) = \arcsen(\operatorname{tg} x)$$

$$9. f(x) = \operatorname{arctg}\left(\frac{1 + \operatorname{sen} x}{\cos x}\right)$$

$$12. f(x) = \operatorname{arctg}(x + \sqrt{1+x^2})$$

2.14. Hallar las derivadas **simplificadas** de las siguientes funciones:

$$1. y = \frac{4x - x^2}{(1-2x)^4}$$

$$4. y = \sqrt[5]{(2x^2 + 1)^2}$$

$$7. y = e^x \cdot \cos x$$

$$10. y = \ln(\ln x)$$

$$13. y = \operatorname{tg} x - \operatorname{ctg} x$$

$$16. y = \ln \sqrt{\frac{1-x}{1+x}}$$

$$2. y = \left(\frac{x^2 + 1}{x}\right)^2$$

$$5. y = \sqrt{4x^4 - 2x^2}$$

$$8. y = e^x \cdot \operatorname{sen} x$$

$$11. y = \cos\left(\frac{2x}{x+1}\right)$$

$$14. y = \frac{1 + 2\operatorname{sen} x}{1 - 2\operatorname{sen} x}$$

$$17. y = \ln(x + \sqrt{x^2 - 1})$$

$$3. y = \frac{x}{x-1} + \frac{3x}{x+3}$$

$$6. y = \ln \frac{x^2 + 1}{4x}$$

$$9. y = (x - e^x)^4$$

$$12. y = \frac{1}{\operatorname{tg} x}$$

$$15. y = \ln \sqrt{\frac{1 + \cos x}{1 - \cos x}}$$

$$18. y = \ln \left(\frac{\sqrt{x^2 + 1} - x}{\sqrt{x^2 + 1} + x}\right)$$