

$$1^o) f(x) = x^4 - 2x^3 - 3$$

① Dom $f(x) = \mathbb{R}$

② Puntos de corte

Eje X $y=0$

$$\begin{array}{c|ccc|c} 1 & -2 & 0 & 0 & -3 \\ -1 & 1 & 3 & -3 & -3 \\ \hline 1 & -3 & 3 & -3 & 0 \end{array}$$

No tiene más

$(1, 0)$

Eje Y $x=0$

$$y = -3$$

$(0, -3)$

③ Simetría

$$f(-x) = (-x)^4 - 2(-x)^3 - 3 = x^4 + 2x^3 - 3$$

④ Signo

No se puede estudiar

⑤ Monotonía

$$f'(x) = 4x^3 - 6x^2$$

$$2x^2(2x-3) = 0$$

$x=0$ Posibles

$x=\frac{3}{2}$ Máximos

Mínimos

Crece $(-\infty, 0) \cup (1, \infty)$

decrece $(0, 1) \cup (1, \frac{3}{2})$

⑥ Curvatura

$$f''(x) = 12x^2 - 12x \quad 12x^2 - 12x = 0 \quad \begin{cases} x=0 \text{ Posibles puntos} \\ x=1 \text{ de inflexión} \end{cases}$$

Comprobación de máximos y mínimos

$$f''(0) = 0 \text{ no es ni máximo ni mínimo}$$

$$f''\left(\frac{3}{2}\right) = 12 \cdot \frac{9}{4} - 12 \cdot \frac{3}{2} = \frac{54}{2} - \frac{36}{2} = \frac{18}{2} = 9 > 0 \text{ Es mínimo}$$

$$\text{mínimo } \left(\frac{3}{2}, -\frac{43}{16}\right) \quad f\left(\frac{3}{2}\right) = \frac{81}{16} - \frac{54}{8} - 3 = \frac{81 - 108 - 48}{16} = \frac{-75}{16}$$

Curvatura

$$\begin{array}{c|c|c} (+) & (-) & (+) \\ \hline 0 & 1 & \end{array}$$

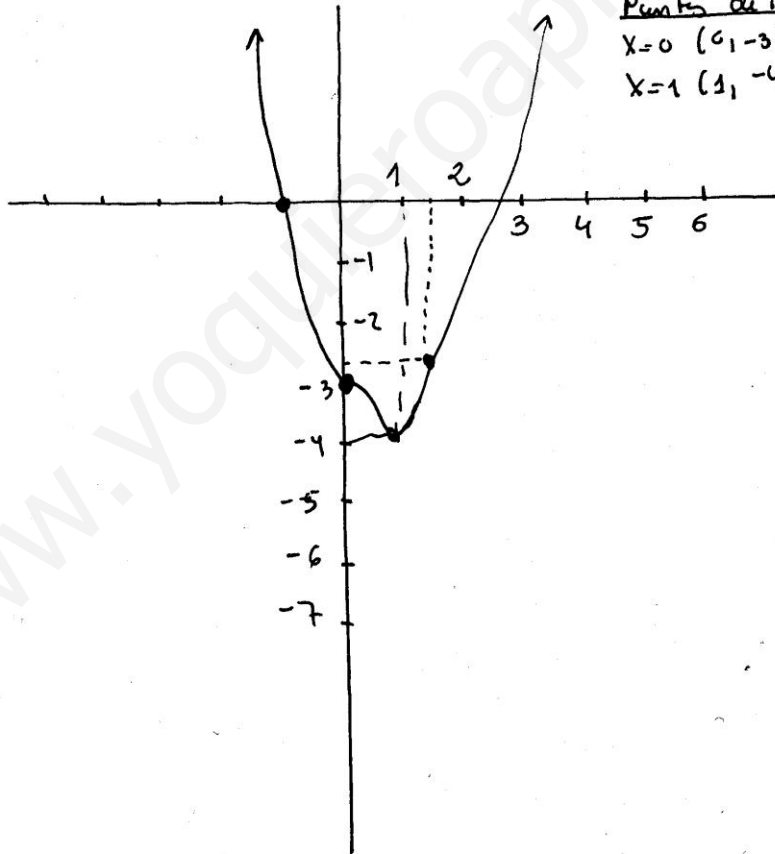
Concava $(-\infty, 0) \cup (1, \infty)$

Convexa $(0, 1)$

Puntos de inflexión

$x=0 (0, -3)$

$x=1 (1, -4)$

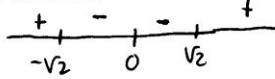


2º) $f(x) = x^4 - 2x^2$

① Dominio $\mathbb{R}$

② Puntos de corte
 $x^2(x^2 - 2) = 0 \Rightarrow x = \pm\sqrt{2}$
 $(0,0) (\sqrt{2},0) (-\sqrt{2},0)$

③ Signo



④ Simetría

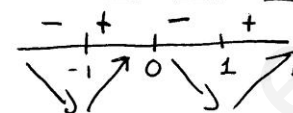
$f(-x) = (-x)^4 - 2(-x)^2 = x^4 - 2x^2 = f(x)$
PAR

Eq: $(0,0)$

⑤ Monotonía

$f'(x) = 4x^3 - 4x = 4x(x^2 - 1)$
 $x = 0$ Punto crítico
 $x = \pm 1$ Posibles máximos o mínimos

crece $(-1,0) \cup (1,\infty)$
 decrece $(-\infty,-1) \cup (0,1)$



⑥ Curvatura y puntos de inflexión

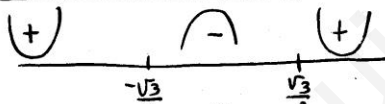
$f''(x) = 12x^2 - 4$ $x = \pm\sqrt{\frac{4}{12}} = \pm\sqrt{\frac{1}{3}} = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$ posibles puntos de inflexión
 comprobación de Máximos y Mínimos

$f''(-1) = 12(-1)^2 - 4 > 0$ es mínimo $(-1, -1)$ $f(-1) = (-1)^4 - 2(-1)^2 = 1 - 2 = -1$

$f''(1) = 12 - 4 > 0$ es mínimo $(1, -1)$ $f(1) = 1 - 2 = -1$

$f''(0) = -4 < 0$ es máximo $(0, 0)$

Estudio de la curvatura

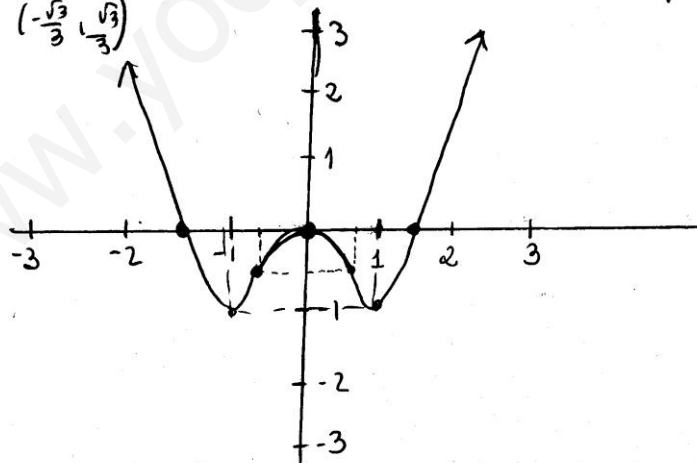


cóncava $(-\infty, -\frac{\sqrt{3}}{3}) \cup (\frac{\sqrt{3}}{3}, \infty)$

convexa $(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3})$

Puntos de inflexión

$(-\frac{\sqrt{3}}{3}, -5/4)$ $f(-\frac{\sqrt{3}}{3}) = (-\frac{\sqrt{3}}{3})^4 - 2(-\frac{\sqrt{3}}{3})^2 = \frac{1}{9} - \frac{2}{3} = -\frac{5}{9}$
 $(\frac{\sqrt{3}}{3}, -5/4)$



$$3^o) f(x) = -3x^4 + 4x^3$$

① Dom $f(x) = \mathbb{R}$

② Punto de corte

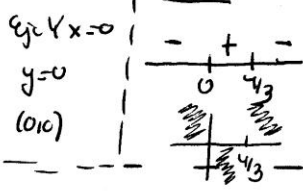
$$y \text{ en } x=0$$

$$x^3(-3x+4)=0 \rightarrow x=0$$

$$x^3(-3x+4)=0 \rightarrow x=\frac{4}{3}$$

$$(0,0) \quad (\frac{4}{3}, 10)$$

③ Signo



④ Simetría

$$f(-x) = -3(-x)^4 + 4(-x)^3 = -3x^4 - 4x^3$$

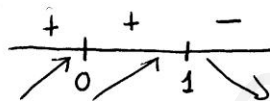
No simétrica.

⑤ Monotonía

$$f'(x) = -12x^3 + 12x^2$$

$$x^2(-12x+12)=0 \rightarrow x=0$$

Puntos críticos
posibles máximos
o mínimos



crece $(-\infty, 0) \cup (0, 1)$

decrece $(1, \infty)$

⑥ Curvatura y punto de inflexión

$$f''(x) = -36x^2 + 24x$$

Comprobación de máximos y mínimos

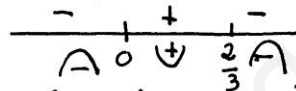
$$f''(0) = 0 \text{ no es máximo ni mínimo}$$

$$f''(1) = -36 + 24 < 0 \text{ es Máximo } (1, 1) \quad f(1) = -3 + 4 = 1$$

Curvatura

$$-36x^2 + 24x = 0 \quad x(-36x + 24) = 0 \rightarrow x = \frac{24}{36} = \frac{2}{3}$$

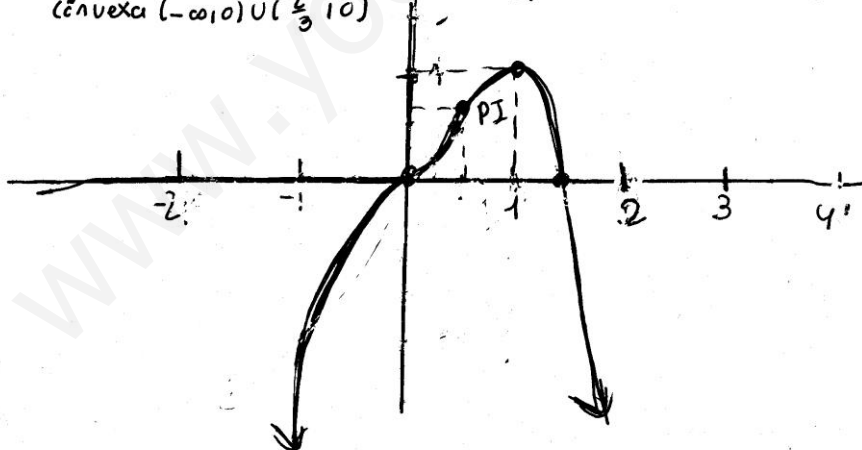
Punto de inflexión



cóncava $(0, \frac{2}{3})$

convexa $(-\infty, 0) \cup (\frac{2}{3}, \infty)$

$$f(\frac{2}{3}) = -3 \cdot \frac{16}{81} + 4 \cdot \frac{8}{27} = -\frac{16}{27} + \frac{32}{27} = \frac{16}{27}$$



$$4^o) f(x) = x^4 - 9x^2$$

① Dom $f(x) = \mathbb{R}$

② Puntos de corte

$$\text{Eje } x \quad y=0$$

$$x^2(x^2-9)=0 \rightarrow x=0$$

$$(0,0); (3,0); (-3,0)$$

$$\text{Eje } y \quad x=0$$

$$y=0$$

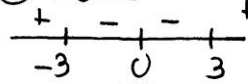
$$(0,0)$$

③ Simetría

$$f(-x) = (-x)^4 - 9(-x)^2 = x^4 - 9x^2 = f(x)$$

P.A.R.

④ Signo



⑤ Monotonía

$$f'(x) = 4x^3 - 18x^2$$

$$x(4x^2 - 18) = 0$$

$$x=0$$

$$x = \pm \sqrt{\frac{9}{2}} = \pm \frac{3}{\sqrt{2}}$$

$$x = \pm \frac{3}{\sqrt{2}}$$

$$x = \pm \frac{3}{\sqrt{2}}$$

$$x = \pm \frac{3}{\sqrt{2}}$$

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$$x = \pm \frac{3}{\sqrt{2}}$$

⑥ Curvatura y puntos de inflexión

$$f''(x) = 12x^2 - 18$$

$$x = \pm \sqrt{\frac{3}{2}}$$

$$x = \pm \sqrt{\frac{3}{2}}$$

$$x = \pm \sqrt{\frac{3}{2}}$$

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$$x = \pm \sqrt{\frac{3}{2}}$$

$$x = \pm \sqrt{\frac{3}{2}}$$

Comprobación de Máximos y mínimos

$$f''(-\sqrt{\frac{3}{2}}) = 54 - 18 > 0 \text{ es mínimo } (-\sqrt{\frac{3}{2}}, -\frac{81}{4})$$

$$f''(0) = -18 < 0 \text{ es máximo } (0, 0)$$

$$f''(\sqrt{\frac{3}{2}}) = 54 - 18 > 0 \text{ es mínimo } (\sqrt{\frac{3}{2}}, -\frac{81}{4})$$

Curvatura



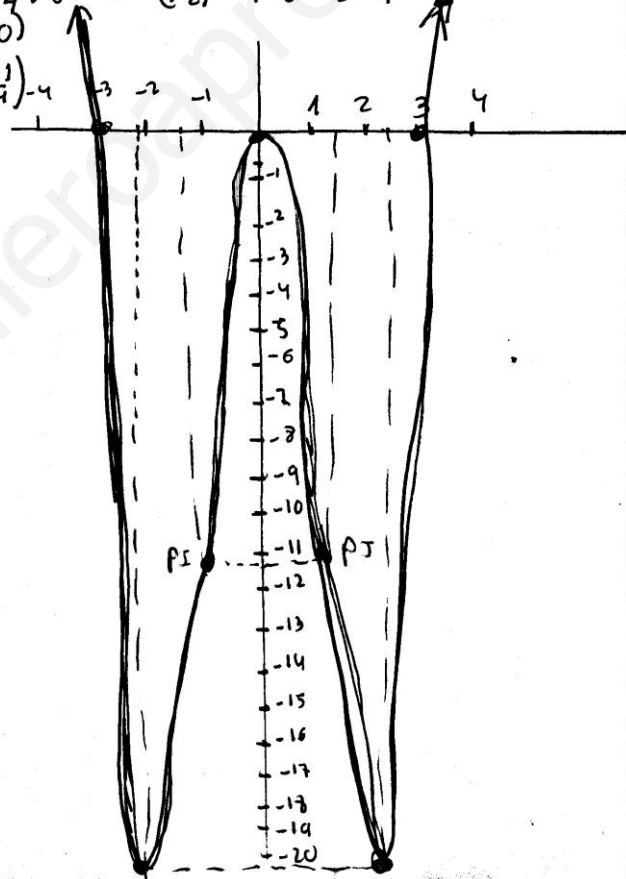
Concava $(-\infty, -\sqrt{\frac{3}{2}}) \cup (\sqrt{\frac{3}{2}}, \infty)$

Convexa $(-\sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}})$

Puntos de Inflexión

$$(-\sqrt{\frac{3}{2}}, -\frac{45}{4}) \quad f(\sqrt{\frac{3}{2}}) = \frac{9}{4} - \frac{27}{2} = \frac{9-54}{4} = -\frac{45}{4}$$

$$(\sqrt{\frac{3}{2}}, -\frac{45}{4})$$



5º) $f(x) = x^4 - x^2$

① Dominio $\text{Dom } f(x) = \mathbb{R}$

② Puntos de corte

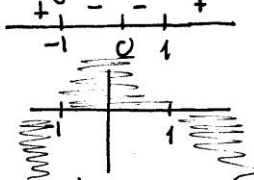
Eje Y $y=0$
 $x^2(x^2-1)=0 \Rightarrow x=0$
 $x= \pm 1$
 $(0,0); (1,0); (-1,0)$

Eje X $x=0$
 $y=0$
 $(0,0)$

③ Simetría

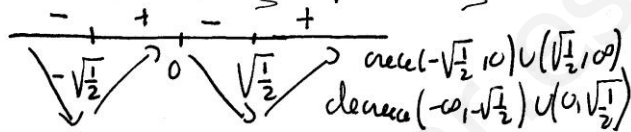
$f(-x) = (-x)^4 - (-x)^2 = x^4 - x^2 = f(x)$
 PAR

④ Signo



⑤ Monotonía

$f'(x) = 4x^3 - 2x = 0$
 $x(4x^2 - 2) = 0 \Rightarrow x=0$
 $x = \pm \sqrt{\frac{1}{2}}$
 Puntos críticos
 Posibles
 Máximos
 Mínimos



⑥ Curvatura

$f''(x) = 12x^2 - 2$ $12x^2 - 2 = 0 \Rightarrow x = \pm \sqrt{\frac{1}{6}}$ Posibles puntos de inflexión

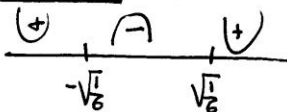
Comprobación de Máximos

$f''(\sqrt{\frac{1}{2}}) = 6 - 2 > 0$ es mínimo $(\sqrt{\frac{1}{2}}, -\frac{1}{2})$ $f(\sqrt{\frac{1}{2}}) = (\sqrt{\frac{1}{2}})^4 - (\sqrt{\frac{1}{2}})^2 = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$

$f''(-\sqrt{\frac{1}{2}}) = 6 - 2 > 0$ es mínimo $(-\sqrt{\frac{1}{2}}, -\frac{1}{2})$

$f''(0) = -2 < 0$ es máximo $(0, 0)$

Curvatura



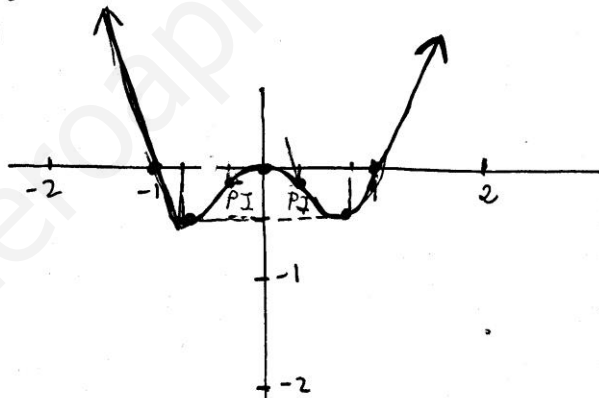
cóncava $(-\infty, -\sqrt{\frac{1}{6}}) \cup (\sqrt{\frac{1}{6}}, \infty)$

convexa $(-\sqrt{\frac{1}{6}}, \sqrt{\frac{1}{6}})$

Puntos de inflexión

$(\sqrt{\frac{1}{6}}, -\frac{5}{36})$ $f(\sqrt{\frac{1}{6}}) = (\frac{1}{6})^2 - (\frac{1}{6}) = \frac{1}{36} - \frac{1}{6} = -\frac{5}{36}$

$(-\sqrt{\frac{1}{6}}, -\frac{5}{36})$



$$6^o) f(x) = -2x^4 + 2x^2$$

① Dom $f(x) = \mathbb{R}$

② Puntos de corte

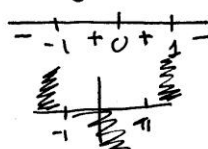
Eje X $y=0$
 $-2x^2(-x^2+1)=0$
 $x=0$
 $x=\pm 1$
 $(0,0); (1,0); (-1,0)$

Eje Y $x=0$
 $y=0$
 $(0,0)$

③ Simetría

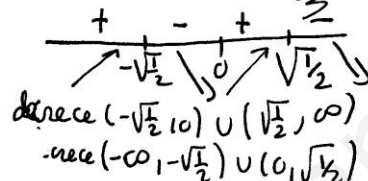
$f(-x) = -2(-x)^4 + 2(-x)^2$
 $= -2x^4 + 2x^2$
 PAR

④ Signo



④ Monotonía

$f'(x) = -8x^3 + 4x$
 $-4x(x^2 - 1) = 0$
 $x=0$
 $x=\pm \sqrt{1/2}$
 Punto crítico
 Posibles Máximos
 Mínimos



⑤ Convexidad

$f''(x) = -24x^2 + 4$
 $-24x^2 + 4 = 0$
 $x = \pm \sqrt{1/6}$
 Posibles puntos de inflexión

Comprobación de Máximos y mínimos

$x=0$ $f''(0) = 4 > 0$ es mínimo $(0,0)$

$x = \sqrt{1/2}$ $f''(\sqrt{1/2}) = -24(\sqrt{1/2})^2 + 4 < 0$ es máximo $(\sqrt{1/2}, 1/2)$ $f(\sqrt{1/2}) = -2(\sqrt{1/2})^4 + 2(\sqrt{1/2})^2 = -1/2 + 1 = 1/2$

$x = -\sqrt{1/2}$ $f''(-\sqrt{1/2}) = -24(-\sqrt{1/2})^2 + 4 < 0$ es máximo $(-\sqrt{1/2}, 1/2)$

Convexidad



Concava $(-\sqrt{1/6}, \sqrt{1/6})$

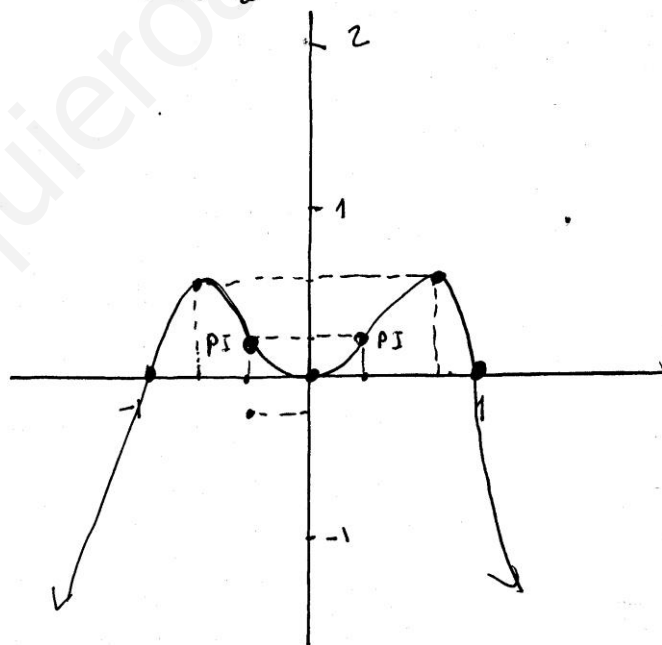
Convexa $(-\infty, -\sqrt{1/6}) \cup (\sqrt{1/6}, \infty)$

Puntos de inflexión

$x = \pm \sqrt{1/6}$ $(-\sqrt{1/6}, 10/39)$

$f(-\sqrt{1/6}) = -2(-\sqrt{1/6})^4 + 2(-\sqrt{1/6})^2 =$
 $= -2 \frac{1}{36} + 2 \frac{1}{6} = -\frac{2}{36} + \frac{12}{36} = \frac{10}{36} = \frac{5}{18}$

$x = \sqrt{1/6}$ $(\sqrt{1/6}, 10/39)$



$$7^o) f(x) = 3x^4 - 4x^3$$

① Dom $f(x) = \mathbb{R}$

② Puntos de corte

Eje X $y=0$

$$x^3(3x-4)=0 \rightarrow \begin{cases} x=0 \\ x=\frac{4}{3} \end{cases}$$

$(0,0); (\frac{4}{3},0)$

Eje Y $x=0$

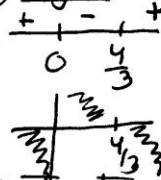
$y=0$
 $(0,0)$

③ Simetría

$$f(-x) = 3(-x)^4 - 4(-x)^3 = 3x^4 + 4x^3 \neq f(x) \neq f(-x)$$

ASIMÉTRICA

④ Signo

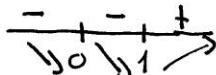


⑤ Monotonía

$$f'(x) = 12x^3 - 12x^2$$

$$12x^2(x-1)=0 \rightarrow \begin{cases} x=0 \\ x=1 \end{cases}$$

Puntos
Máximos
o Mínimos



decrece $(-\infty, 0) \cup (0, 1)$
crece $(1, \infty)$

⑥ Curvatura

$$f''(x) = 36x^2 - 24x$$

$$x(36x-24)=0 \rightarrow \begin{cases} x=0 \\ x=\frac{24}{36} = \frac{2}{3} \end{cases}$$

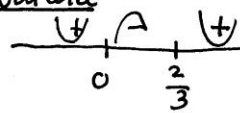
Puntos de inflexión

Comprobación de máximos y mínimos

$x=0$ $f''(0)=0$ no es máximo ni mínimo

$x=1$ $f''(1)=36-24>0$ es mínimo $mínimo=(1,-1)$ $f(1)=3-4=-1$

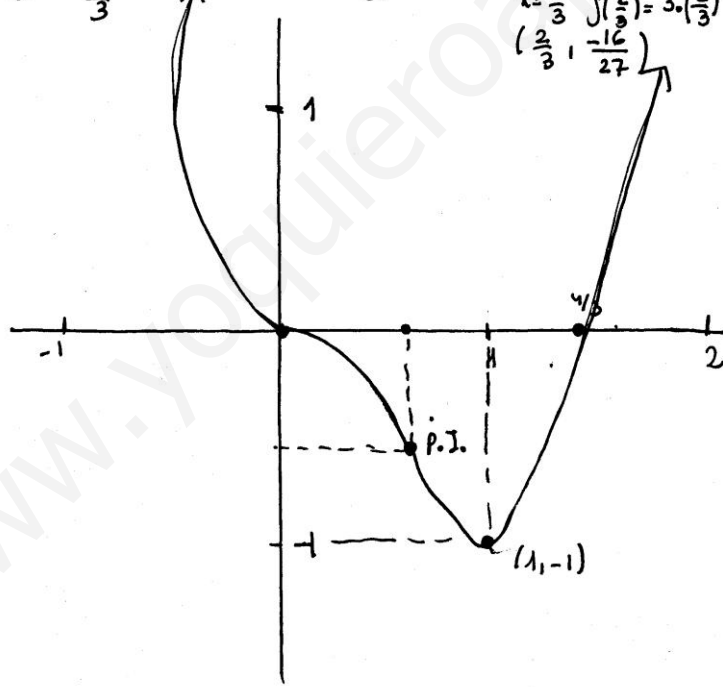
Curvatura



cóncava $(-\infty, 0) \cup (\frac{2}{3}, \infty)$
cóncava $(0, \frac{2}{3})$

Puntos de inflexión

$x=0$ $f(0)=0$ $(0,0)$
 $x=\frac{2}{3}$ $f(\frac{2}{3}) = 3 \cdot (\frac{2}{3})^4 - 4(\frac{2}{3})^3 = \frac{16}{27} - \frac{32}{27} = -\frac{16}{27}$
 $(\frac{2}{3}, -\frac{16}{27})$



$$8^o) f(x) = x^4 - 8x^2 + 7$$

① $\text{Dom } f(x) = \mathbb{R}$

② Eje X $y=0$ Puntos de corte

$$x^4 - 8x^2 + 7 = 0 \quad x = \pm \sqrt{7}$$

$$x^2 = y \quad y^2 - 8y + 7 = 0 \quad x = \pm \sqrt{1} = \pm 1$$

$$y = \frac{8 \pm \sqrt{64 - 28}}{2} = \frac{8 \pm 6}{2} \rightarrow 7, 1$$

$$(1, 0); (-1, 0); (-\sqrt{7}, 0); (\sqrt{7}, 0)$$

Eje Y $x=0$

$$y = 7$$

③ Simetría

$$f(-x) = (-x)^4 - 8(-x)^2 + 7 = x^4 - 8x^2 + 7 = f(x)$$

$$x^4 - 8x^2 + 7 \text{ PAR}$$

④ Monotonía

$$f'(x) = 4x^3 - 16x$$

$$x(4x^2 - 16) = 0$$

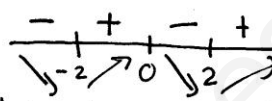
$$x = 0$$

$$x = \pm \sqrt{4} = \pm 2$$

dobles

Máximos

Mínimos



crece $(-\infty, -2) \cup (0, 2)$

decrece $(-2, 0) \cup (2, \infty)$

⑤ Curvatura y punto de inflexión

$$f''(x) = 12x^2 - 16$$

$$12x^2 - 16 = 0$$

$$x = \pm \sqrt{\frac{16}{12}} = \pm \sqrt{\frac{4}{3}} = \pm \frac{2}{\sqrt{3}}$$

$$x = \pm \frac{2}{\sqrt{3}}$$

posibles punto de inflexión

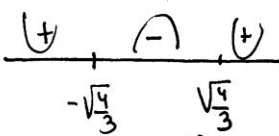
Comprobación de máximos y mínimos

$$x=0 \quad f''(0) = -16 < 0 \text{ es Máximo } (0, 7)$$

$$x=2 \quad f''(2) = 48 - 16 > 0 \text{ es Mínimo}$$

$$x=-2 \quad f''(-2) = 48 - 16 > 0 \text{ es Mínimo}$$

Curvatura



Convexa $(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}})$

Concava $(-\infty, -\frac{2}{\sqrt{3}}) \cup (\frac{2}{\sqrt{3}}, \infty)$

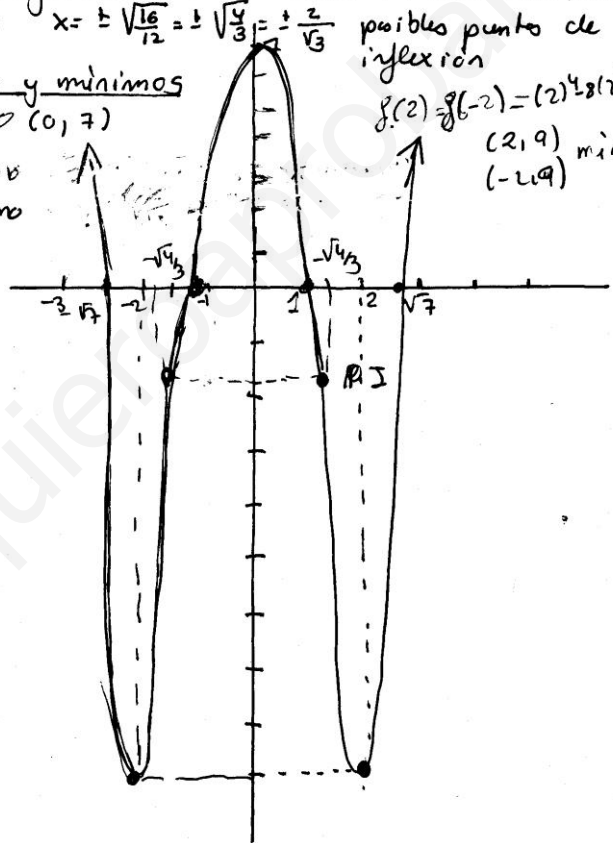
Punto de inflexión

$$x = \frac{2}{\sqrt{3}} \quad f\left(\frac{2}{\sqrt{3}}\right) = \left(\frac{2}{\sqrt{3}}\right)^4 - 8\left(\frac{2}{\sqrt{3}}\right)^2 + 7$$

$$= \frac{16}{9} - \frac{32}{3} + 7 = \frac{16 - 96 + 63}{9} = -\frac{17}{9}$$

$$\left(\frac{2}{\sqrt{3}}, -\frac{17}{9}\right)$$

$$\left(-\frac{2}{\sqrt{3}}, -\frac{17}{9}\right)$$



$$9^o) f(x) = 3x^4 + 4x^3 - 36x^2$$

① Dom $f(x) = \mathbb{R}$

② Punto de Corte

Eje X $y=0$

$$3x^4 + 4x^3 - 36x^2 = 0$$

$$x^2(3x^2 + 4x - 36) = 0 \rightarrow \begin{cases} x=0 \\ x=-4/3 \\ x=2/9 \end{cases}$$

$(0,0); (-4/3,0); (2/9,0)$

Eje Y $x=0$

$y=0$
 $(0,0)$

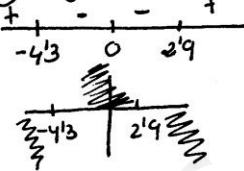
③ Simetría

$$f(-x) = 3(-x)^4 + 4(-x)^3 - 36(-x)^2$$

$$= 3x^4 - 4x^3 - 36x^2$$

ASIMÉTRICA

④ Signo



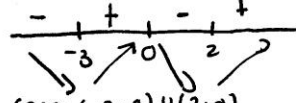
⑤ Monotonía

$$f'(x) = 12x^3 + 12x^2 - 72x$$

$$12x(x^2 + x - 6) = 0$$

Possible
Máximo
Mínimo

$x=0$
 $x=2$
 $x=-3$



cruce $(-3,0)$ $(2,0)$
decrece $(-\infty, -3)$ $(0, 2)$

⑥ Curvatura

$$f''(x) = 36x^2 - 24x - 72$$

$$x = \frac{24 \pm \sqrt{10944}}{72} \rightarrow \pm 1/4$$

Possible points
de inflexión

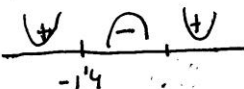
Comprobación máximos y mínimos

$$f''(0) = -72 < 0 \rightarrow \text{Máximo } (0,0)$$

$$f''(-3) = 32670 > 0 \rightarrow \text{Mínimo } (-3, -135) \quad f(-3) = 3(-3)^4 + 4(-3)^3 - 36(-3)^2 = 81 - 108 - 108 = -135$$

$$f''(2) = 2470 > 0 \rightarrow \text{Mínimo } (2, -64) \quad f(2) = 48 + 32 - 144 = -64$$

Curvatura



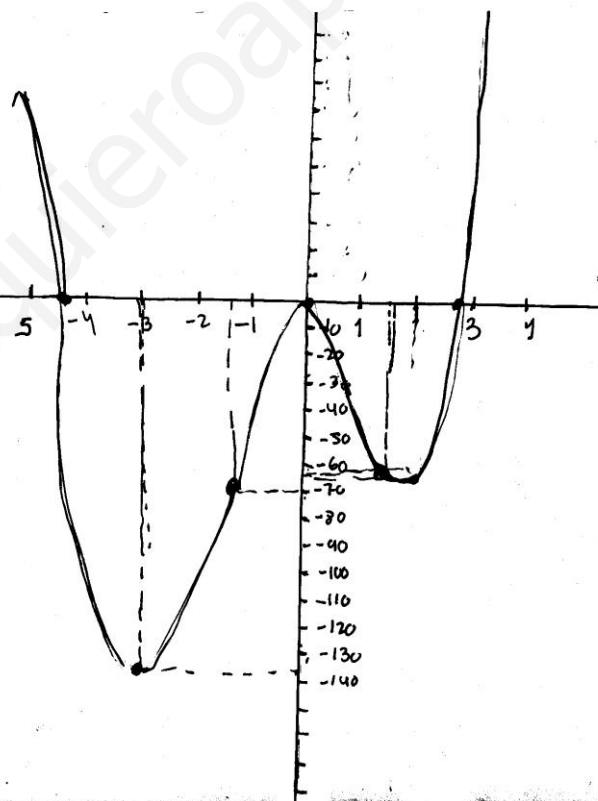
convexa $(-1/4, \infty)$

cóncava $(-\infty, -1/4) \cup (1/4, \infty)$

Puntos de Inflexión

$$x = -1/4 \quad (-1/4, -70) \quad f(-1/4) = -70$$

$$x = 1/3 \quad (1/3, -61/3) \quad f(1/3) = -61/3$$



$$10^o) f(x) = x^4 - 4x^3 - 2x^2 + 12x$$

① Dom $f(x) = \mathbb{R}$

② Puntos de Corte

Eje X $y=0$

$$x^4 - 4x^3 - 2x^2 + 12x = 0$$

$$x(x^3 - 4x^2 - 2x + 12) = 0$$

$$\begin{array}{r|rrrr} & 1 & -4 & -2 & 12 \\ +2 & & +2 & -4 & -12 \\ \hline & 1 & -2 & -6 & 0 \end{array}$$

$$x^2 - 2x - 6 = 0$$

$$x = \frac{2 \pm \sqrt{4+24}}{2}$$

Puntos de corte

$(0,0)$ $(2,0)$ $(3,6)$ $(-1,6)$

⑥ Curvatura

$$f''(x) = 12x^2 - 24x - 4$$

$$3x^2 - 6x - 1 = 0$$

$$x = \frac{6 \pm \sqrt{36+12}}{6} \rightarrow \frac{6 \pm \sqrt{48}}{6} \rightarrow \frac{6 \pm 4\sqrt{3}}{6} \rightarrow 1 \pm \frac{2\sqrt{3}}{3}$$

Comprobación Máximos y mínimos

$$f''(1) = -4 < 0 \text{ Máximo } (1, 7) \quad f(1) = 1 - 4 - 2 + 12 = 7$$

$$f''(-1) = 20 > 0 \text{ Mínimo } (-1, -9) \quad f(-1) = 1 + 4 - 2 - 12 = -9$$

$$f''(3) = 8 > 0 \text{ Mínimo } (3, -9) \quad f(3) = 81 - 108 - 18 + 36 = -9$$

Puntos de inflexión

$$x = 2 \quad (2, -24) \quad f(2) = -24$$

$$x = 0 \quad (0, 24) \quad f(0) = 24$$

Eje Y $x=0$

$$y=0$$

$$(0,0)$$

③ Signo

$$\begin{array}{ccccccc} + & - & + & - & + \\ -1 & 0 & 2 & 3 & 6 \end{array}$$

④ Simetría

$$f(-x) = (-x)^4 - 4(-x)^3 - 2(-x)^2 + 12(-x)$$

$$= x^4 + 4x^3 - 2x^2 - 12x$$

ASIMÉTRICA

⑤ Monotonía

$$f'(x) = 4x^3 - 12x^2 - 4x + 12$$

$$\begin{array}{r|rrrr} & 4 & -12 & -4 & 12 \\ 1 & & 4 & -8 & +12 \\ \hline & 4 & -8 & -12 & 0 \end{array}$$

$$4x^2 - 8x - 12 = 0$$

$$x^2 - 2x - 3 = 0$$

$$x = \frac{2 \pm \sqrt{4+12}}{2} = \frac{2 \pm 4}{2} \rightarrow 3, -1$$

$$\begin{array}{ccccccc} - & + & - & + \\ -1 & 1 & 3 & \end{array}$$

decrece $(-\infty, -1) \cup (1, 3)$ crece $(-1, 1) \cup (3, \infty)$

$$\begin{array}{ccc} + & - & + \\ -0.2 & 2.2 & \end{array}$$

Concava $(-\infty, 0.3) \cup (1.3, \infty)$

Convexa $(0.3, 1.3)$

