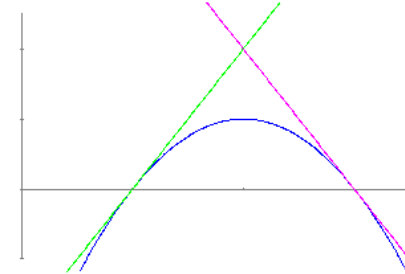


AREAS DE FIGURAS PLANAS

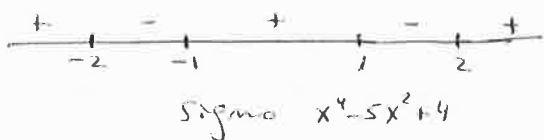
Calcular la superficie comprendida entre las funciones:

1. $f(x) = x^4 - 5x^2 + 4$; $x = -3$; $x = 3$ y el eje X
2. $f(x) = x^3 - 4x$ y el eje OX .
3. $y^2 = 4x$ y $x = 3$
4. $x = y^2$ e $y - x + 2 = 0$.
5. $f(x) = \text{sen } x$ y $g(x) = \cos x$ en $[0, 2\pi]$
6. $f(x) = x^2 - x$ y $g(x) = 3x - x^2$
7. $f(x) = 2x - x^2$ y $g(x) = -x$.
8. $f(x) = |x|$ y $g(x) = |x^2 - 2|$
9. $y = x^2$ e $y^2 = x$
10. $x^2 + y^2 = 1$ y $f(x) = x^2 - 2x + 1$. Dibujar el recinto, 1 unidad = 2 cm.
11. Hallar el área del triángulo mixtilíneo formado por $y^2 = 6x$ y sus rectas tangentes en los puntos de abscisa $x = 6$.
12. La figura muestra una parte de la curva de ecuación $y = (x - 1)(3 - x)$ junto con las tangentes a la curva en sus puntos de intersección con el eje X . Halla el área del triángulo curvilíneo encerrado entre la curva y sus tangentes.
13. $f(x) = |x^2 - 1| + 1$ y $g(x) = 2$
14. Hallar m para que el área comprendida entre $y = x^3$ e $y = mx$ sea $8u^2$.
15. Sea $f : x \rightarrow \frac{\text{sen } x}{x}$, $\pi \leq x \leq 3\pi$. Halle el área encerrada por la gráfica de f y el eje OX (Usar calculadora gráfica)



① $y = x^4 - 5x^2 + 4$

$y > 0 \Rightarrow x^4 - 5x^2 + 4 = 0 \Rightarrow x^2 = \frac{5 \pm \sqrt{25-16}}{2} = \frac{5 \pm 3}{2} \rightarrow 4 \Rightarrow x = \pm 2$
 $\rightarrow 1 \Rightarrow x = \pm 1$



Area = $\int_{-3}^3 |x^4 - 5x^2 + 4| dx = \int_{-3}^{-2} f(x) dx - \int_{-2}^{-1} f(x) dx + \int_{-1}^1 f(x) dx - \int_1^2 f(x) dx + \int_2^3 f(x) dx =$

$= 2 \int_0^1 f(x) dx - 2 \int_1^2 f(x) dx + 2 \int_2^3 f(x) dx =$

$= 2 \left(\frac{x^5}{5} - \frac{5x^3}{3} + 4x \right) \Big|_0^1 - 2 \left(\frac{x^5}{5} - \frac{5x^3}{3} + 4x \right) \Big|_1^2 + 2 \left(\frac{x^5}{5} - \frac{5x^3}{3} + 4x \right) \Big|_2^3 =$

$= 2 \left(\frac{1}{5} - \frac{5}{3} + 4 \right) - 2 \left(\frac{32}{5} - \frac{40}{3} + 8 \right) + 2 \left(\frac{1}{5} - \frac{5}{3} + 4 \right) + 2 \left(\frac{243}{5} - \frac{135}{3} + 12 \right) - 2 \left(\frac{32}{5} - \frac{40}{3} + 8 \right) =$

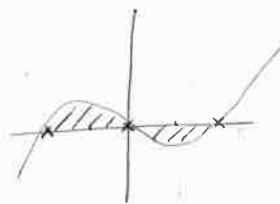
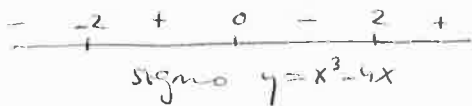
$= 4 \left(\frac{1}{5} - \frac{5}{3} + 4 \right) - 4 \left(\frac{32}{5} - \frac{40}{3} + 8 \right) + 2 \left(\frac{243}{5} - \frac{135}{3} + 12 \right) =$

$= \frac{4 - 128 + 486}{5} + \frac{160 - 20 - 270}{3} + 16 - 32 + 24 = \frac{362}{5} - \frac{130}{3} + 8 =$

$= \frac{1086 - 650 + 120}{15} = \boxed{\frac{556}{15} \text{ uS}}$

② $y = x^3 - 4x$

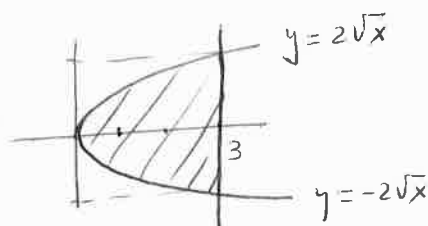
$y = 0 \Rightarrow x(x^2 - 4) = 0 \Rightarrow$
 $x = -2$
 $x = 0$
 $x = 2$



Area = $\int_{-2}^2 |x^3 - 4x| dx = \int_{-2}^0 (x^3 - 4x) dx - \int_0^2 (x^3 - 4x) dx = -2 \int_0^2 (x^3 - 4x) dx =$

$= -2 \left(\frac{x^4}{4} - 2x^2 \right) \Big|_0^2 = -2 \left(\frac{16}{4} - 8 \right) = \boxed{8 \text{ uS}}$

③ $y^2 = 4x \left\{ \Rightarrow y^2 = 12 \Rightarrow y = \pm \sqrt{12} \right.$
 $x = 3$



Area = $2 \int_0^3 2\sqrt{x} dx = 4 \int_0^3 x^{1/2} dx = 4 \left(\frac{x^{3/2}}{3/2} \right) \Big|_0^3 = \frac{8}{3} \sqrt{x^3} \Big|_0^3 =$
 $= \frac{8}{3} \sqrt{27} = \boxed{8\sqrt{3} \text{ uS}}$

④
$$\begin{cases} x = y^2 \\ y - x + 2 = 0 \end{cases} \Rightarrow y - y^2 + 2 = 0 \Rightarrow y^2 - y - 2 = 0$$

$$y = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2} \begin{cases} 2 \Rightarrow x = 4 \\ -1 \Rightarrow x = 1 \end{cases}$$

$$\begin{aligned} \text{Area} &= 2 \int_0^1 \sqrt{x} \, dx + \int_1^4 [\sqrt{x} - (x-2)] \, dx = \\ &= 2 \frac{x^{3/2}}{3/2} \Big|_0^1 + \left(\frac{x^{3/2}}{3/2} - \frac{x^2}{2} + 2x \right) \Big|_1^4 = \\ &= \frac{4}{3} + \frac{16}{3} - 8 + 8 - \frac{2}{3} + \frac{1}{2} - 2 = \boxed{\frac{9}{2} \text{ us}} \end{aligned}$$

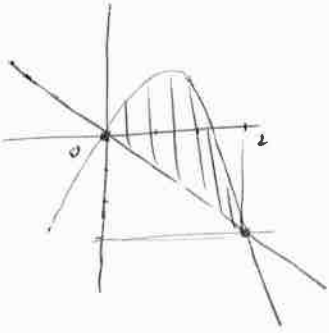
⑤
$$\begin{cases} f(x) = \sin x \\ g(x) = \cos x \end{cases} \Rightarrow \sin x = \cos x \Rightarrow \tan x = 1 \Rightarrow \begin{cases} x = 45^\circ \\ x = 225^\circ \end{cases} \Rightarrow \begin{cases} x = \pi/4 \\ x = 3\pi/4 \end{cases}$$

$$\begin{aligned} \text{Area} &= \int_0^{2\pi} |\sin x - \cos x| \, dx = \int_0^{\pi/4} (\cos x - \sin x) \, dx + \int_{\pi/4}^{3\pi/4} (\sin x - \cos x) \, dx + \int_{3\pi/4}^{2\pi} (\cos x - \sin x) \, dx = \\ &= (\sin x + \cos x) \Big|_0^{\pi/4} + (-\cos x - \sin x) \Big|_{\pi/4}^{3\pi/4} + (\sin x + \cos x) \Big|_{3\pi/4}^{2\pi} = \\ &= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 1 + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + 1 - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = \boxed{2\sqrt{2} \text{ us}} \end{aligned}$$

⑥
$$\begin{cases} y = x^2 - x \\ y = 3x - x^2 \end{cases} \Rightarrow x^2 - x = 3x - x^2 \Rightarrow 2x^2 = 4x \Rightarrow 2x(x-2) = 0 \Rightarrow \begin{cases} x = 0 \\ x = 2 \end{cases}$$

$$\begin{aligned} \text{Area} &= \int_0^2 (3x - x^2) - (x^2 - x) \, dx = \int_0^2 (4x - 2x^2) \, dx = \\ &= \left(2x^2 - \frac{2}{3}x^3 \right) \Big|_0^2 = 8 - \frac{16}{3} = \boxed{\frac{8}{3} \text{ us}} \end{aligned}$$

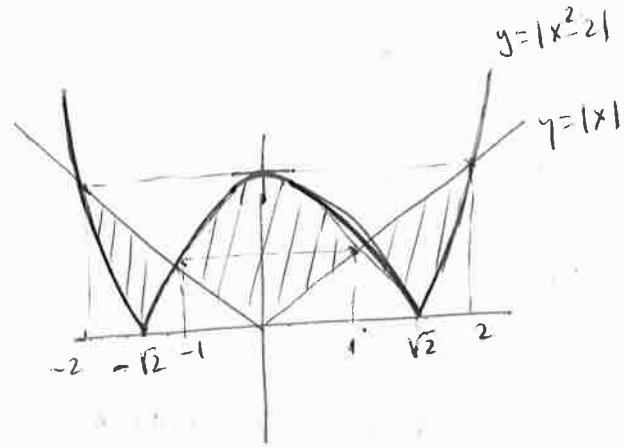
$$\textcircled{7} \quad \begin{cases} y = 2x - x^2 \\ y = -x \end{cases} \Rightarrow 2x - x^2 = -x \Rightarrow 0 = x^2 - 3x \Rightarrow 0 = x(x-3) \begin{cases} x=0 \\ x=3 \end{cases}$$



$$\begin{aligned} \text{Area} &= \int_0^3 [(2x - x^2) - (-x)] dx = \\ &= \int_0^3 (3x - x^2) dx = \left(\frac{3x^2}{2} - \frac{x^3}{3} \right) \Big|_0^3 = 6 - \frac{8}{3} = \boxed{\frac{10}{3} \text{ u.s.}} \end{aligned}$$

$$\textcircled{8} \quad y = |x| = \begin{cases} -x & \text{si } x \leq 0 \\ x & \text{si } x > 0 \end{cases}$$

$$y = |x^2 - 2| = \begin{cases} x^2 - 2 & \text{si } x \leq -\sqrt{2} \\ 2 - x^2 & \text{si } -\sqrt{2} \leq x \leq \sqrt{2} \\ x^2 - 2 & \text{si } x \geq \sqrt{2} \end{cases}$$



$$\begin{aligned} |x| &= |x^2 - 2| \\ \downarrow & \quad \downarrow \\ x = x^2 - 2 & \quad -x = x^2 - 2 \rightarrow 0 = x^2 + x - 2 \\ x^2 - x - 2 = 0 & \quad x = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2} \begin{cases} 1 \\ -2 \end{cases} \\ x = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2} & \quad \begin{cases} 2 \\ -1 \end{cases} \end{aligned}$$

$$\text{Area} = \int_{-2}^2 | |x| - |x^2 - 2| | dx = 2 \int_0^2 | |x| - |x^2 - 2| | dx = 2 \int_0^1 [(2-x^2) - x] dx +$$

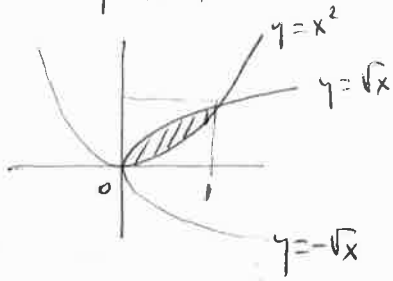
$$+ 2 \int_1^{\sqrt{2}} [x - (2-x^2)] dx + 2 \int_{\sqrt{2}}^2 [x - (x^2-2)] dx =$$

$$= 2 \left(2x - \frac{x^3}{3} - \frac{x^2}{2} \right) \Big|_0^1 + 2 \left(\frac{x^2}{2} - 2x + \frac{x^3}{3} \right) \Big|_1^{\sqrt{2}} + 2 \left(\frac{x^2}{2} - \frac{x^3}{3} + 2x \right) \Big|_{\sqrt{2}}^2 =$$

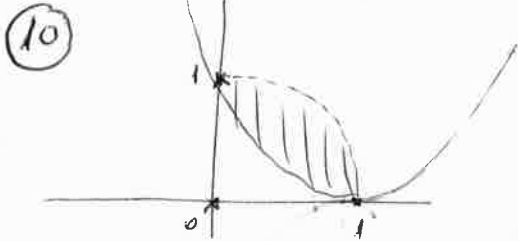
$$= 2 \left(2 - \frac{1}{3} - \frac{1}{2} \right) + 2 \left(1 - 2\sqrt{2} + \frac{2\sqrt{2}}{3} \right) - 2 \left(\frac{1}{2} - 2 + \frac{1}{3} \right) + 2 \left(2 - \frac{8}{3} + 4 \right) - 2 \left(1 - \frac{2\sqrt{2}}{3} + 2\sqrt{2} \right)$$

$$= 2 \left(10 - \frac{10}{3} - \frac{8\sqrt{2}}{3} - 1 \right) = 2 \frac{27 - 10 - 8\sqrt{2}}{3} = \boxed{\frac{34 - 16\sqrt{2}}{3} \text{ u.s.}}$$

9 $y = x^2$
 $y^2 = x$ $\Rightarrow y = (y^2)^2 \Rightarrow y = y^4 \Rightarrow y(1-y^3) = 0 \Rightarrow \begin{cases} y=0 \Rightarrow x=0 \\ y=1 \Rightarrow x=1 \end{cases}$



$$\text{Area} = \int_0^1 (\sqrt{x} - x^2) dx = \left(\frac{x^{3/2}}{3/2} - \frac{x^3}{3} \right) \Big|_0^1 = \frac{2}{3} - \frac{1}{3} = \boxed{\frac{1}{3} \text{ us}}$$



$$x^2 + y^2 = 1 \Rightarrow y = \pm \sqrt{1-x^2}$$

$$y = x^2 - 2x + 1$$

$$\pm \sqrt{1-x^2} = x^2 - 2x + 1$$

$$1-x^2 = x^4 + 4x^2 + 1 - 4x^3 + 2x^2 - 4x$$

$$0 = x^4 - 4x^3 + 7x^2 - 4x$$

$$0 = x(x^3 - 4x^2 + 7x - 4) \Rightarrow \boxed{x=0}$$

1	-4	7	-4
	1	-3	4
			0

$$x^2 - 3x + 4 = 0$$

$$x = \frac{3 \pm \sqrt{9-16}}{2} \quad \times$$

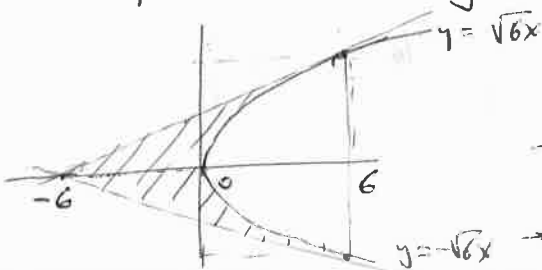
$$\text{Area} = \int_0^1 \sqrt{1-x^2} - (x^2 - 2x + 1) dx$$

$$\int_0^1 \sqrt{1-x^2} dx = \int_0^{\pi/2} \cos^2 t dt \quad \begin{matrix} x = \sin t \\ dx = \cos t dt \end{matrix} = \int_0^{\pi/2} \frac{1 + \cos(2t)}{2} dt = \frac{t}{2} + \frac{\sin(2t)}{4} \Big|_0^{\pi/2} = \frac{\pi}{4}$$

$$\int_0^1 (x^2 - 2x + 1) dx = \left(\frac{x^3}{3} - x^2 + x \right) \Big|_0^1 = \frac{1}{3} - 1 + 1 = \frac{1}{3}$$

$$\text{Area} = \boxed{\frac{\pi}{4} - \frac{1}{3}}$$

11 $y^2 = 6x$
 $2y y' = 6 \Rightarrow y' = \frac{3}{y}$



$$\begin{cases} x=6 \Rightarrow y^2=36 \Rightarrow y = \pm 6 \\ P(6,6) \Rightarrow \text{slope} = \frac{3}{6} = \frac{1}{2} \\ P(6,-6) \Rightarrow \text{slope} = \frac{3}{-6} = -\frac{1}{2} \end{cases}$$

$$\boxed{y-6 = \frac{1}{2}(x-6)}$$

$$\boxed{y+6 = -\frac{1}{2}(x-6)}$$

$$\rightarrow \boxed{y = \frac{x+6}{2}}$$

$$\rightarrow \boxed{y = \frac{-x-6}{2}}$$

$$\text{Área} = 2 \int_{-6}^0 \frac{x+6}{2} dx + 2 \int_0^6 \left(\frac{x+6}{2} - \sqrt{6x} \right) dx = \left(\frac{x^2}{2} + 6x \right) \Big|_{-6}^0 + \left(\frac{x^2}{2} + 6x - 2\sqrt{6} \frac{x^{3/2}}{3/2} \right) \Big|_0^6 =$$

$$= 0 - \left(\frac{36}{2} - 36 \right) + \left(\frac{36}{2} + 36 - \frac{4\sqrt{6}}{3} \sqrt{6^3} \right) - 0 = \boxed{24 \text{ us}}$$

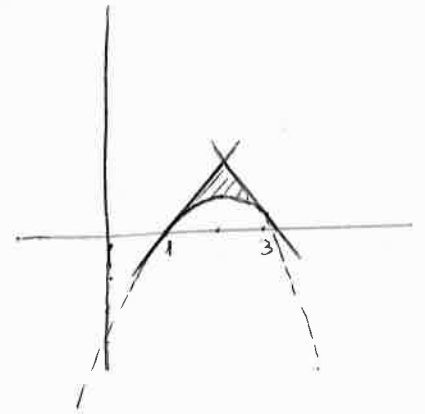
(12) $y = (x-1)(3-x)$

$$y = -x^2 + 4x - 3$$

$$y' = -2x + 4$$

$$x=1 \begin{cases} y=0 \\ y'=2 \end{cases} \quad y=0 = 2(x-1) \rightarrow \boxed{y = 2x-2}$$

$$x=3 \begin{cases} y=0 \\ y'=-2 \end{cases} \quad y=0 = -2(x-3) \rightarrow \boxed{y = 6-2x}$$



~~Área~~ $y = 2x-2$ $\left\{ \begin{array}{l} \rightarrow 2x-2 = 6-2x \\ \rightarrow 4x=8 \end{array} \right. \rightarrow \boxed{x=2 \rightarrow y=2}$

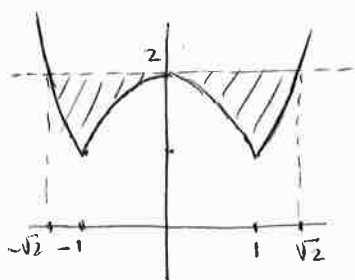
$$\text{Área} = \int_1^2 [(2x-2) - (-x^2+4x-3)] dx + \int_2^3 [(6-2x) - (-x^2+4x-3)] dx =$$

$$= \int_1^2 (x^2 - 2x + 1) dx + \int_2^3 (x^2 - 6x + 9) dx = \left(\frac{x^3}{3} - x^2 + x \right) \Big|_1^2 + \left(\frac{x^3}{3} - 3x^2 + 9x \right) \Big|_2^3$$

$$= \left(\frac{7}{3} - 2 \right) + \left(\frac{19}{3} - 6 \right) = \frac{1}{3} + \frac{1}{3} = \boxed{\frac{2}{3}}$$

Salen idénticos los valores de área en ambos recintos. Era previsible por la simetría del dibujo. Se podía haber hecho con una sola integral: $\text{Área} = 2 \int_1^2 (x^2 - 2x + 1) dx$

(13) $f(x) = |x^2 - 1| + 1 = \begin{cases} x^2 - 1 + 1 & \text{si } x < -1 \\ 1 - x^2 + 1 & \text{si } -1 \leq x \leq 1 \\ x^2 - 1 + 1 & \text{si } x > 1 \end{cases} = \begin{cases} x^2 & \text{si } x < -1 \\ 2 - x^2 & \text{si } -1 \leq x \leq 1 \\ x^2 & \text{si } x > 1 \end{cases}$



$$y=2 \Rightarrow \begin{cases} x^2=2 \rightarrow \boxed{x = -\sqrt{2}} \\ 2-x^2=2 \rightarrow \boxed{x=0} \\ x^2=2 \rightarrow \boxed{x=\sqrt{2}} \end{cases}$$

$$\text{Área} = 2 \left(\int_0^1 [2 - (2 - x^2)] dx + \int_1^{\sqrt{2}} [2 - x^2] dx \right) =$$

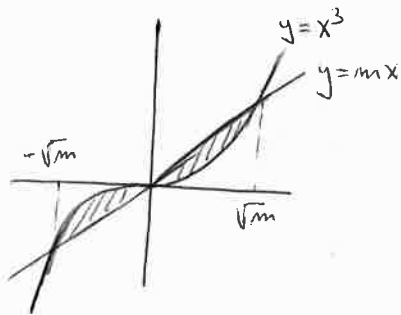
$$= 2 \left(\int_0^1 x^2 dx + \int_1^{\sqrt{2}} (2 - x^2) dx \right) = \frac{2x^3}{3} \Big|_0^1 + 2 \left(2x - \frac{x^3}{3} \right) \Big|_1^{\sqrt{2}} =$$

$$= \frac{2}{3} + \left(4\sqrt{2} - \frac{4\sqrt{2}}{3} \right) - \left(4 - \frac{2}{3} \right) = \frac{2 + 12\sqrt{2} - 4\sqrt{2} - 12 + 2}{3} = \boxed{\frac{8(\sqrt{2}-1)}{3}}$$

14) $y = x^3$
 $y = mx$ $\Rightarrow x^3 = mx \Rightarrow x^3 - mx = 0 \Rightarrow x(x^2 - m) = 0$

$x = 0$
 $x = \sqrt{m}$
 $x = -\sqrt{m}$

m debe ser positivo

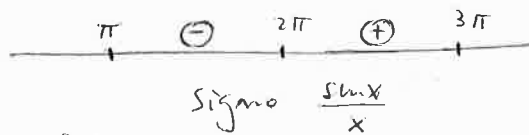


$$\text{Área} = 2 \int_0^{\sqrt{m}} (mx - x^3) dx = \left(2m \frac{x^2}{2} - 2 \frac{x^4}{4} \right) \Big|_0^{\sqrt{m}} =$$

$$= \left(mx^2 - \frac{x^4}{2} \right) \Big|_0^{\sqrt{m}} = m^2 - \frac{m^2}{2} = \frac{m^2}{2}$$

$$\frac{m^2}{2} = 8 \Rightarrow m^2 = 16 \Rightarrow \boxed{m = 4}$$

15) $f(x) = \frac{\sin x}{x}$



$$\text{Área} = - \int_{\pi}^{2\pi} \frac{\sin x}{x} dx + \int_{2\pi}^{3\pi} \frac{\sin x}{x} dx = +0'43379 + 0'25661 = \boxed{0'69040}$$

También se puede utilizar la calculadora gráfica con la función en valor absoluto, sin estudiar previamente su signo ni, en consecuencia, separar en dos integrales.

$$\text{Área} = \int_{\pi}^{3\pi} \left| \frac{\sin x}{x} \right| dx = \boxed{0'69040}$$