# AREAS DE FIGURAS PLANAS

Calcular la superficie comprendida entre las funciones:

1. $f(x) = x^4 - 5x^2 + 4$; $x = -3$; $x = 3$ y el eje X

2. $f(x) = x^3 - 4x$ y el eje OX .

3      $y^2 = 4x$ y $x = 3$

4. $x = y^2$ e $y - x + 2 = 0$.

5. $f(x) = $ sen x y $g(x) = $ cos x en $[0, 2\pi]$

6. $f(x) = x^2 - x$ y $g(x) = 3x - x^2$

7. $f(x) = 2x - x^2$ y $g(x) = -x$.

8. $f(x) = |x|$ y $g(x) = |x^2 - 2|$

9. $y = x^2$ e $y^2 = x$

10. $x^2 + y^2 = 1$ y $f(x) = x^2 - 2x + 1$. Dibujar el recinto, 1 unidad = 2 cm.

11.   Hallar el área del triángulo mixtilíneo formado por $y^2 = 6x$ y sus rectas tangentes en los puntos de abscisa $x = 6$.

12.   La figura muestra una parte de la curva de ecuación $y = (x - 1)(3 - x)$ junto con las tangentes a la curva en sus puntos de intersección con el eje X
Halla el área del triángulo curvilíneo encerrado entre la curva y sus tangentes.

13. $f(x) = |x^2 - 1| + 1$ y $g(x) = 2$

14.   Hallar m para que el área comprendida entre $y = x^3$ e $y = mx$ sea $8u^2$.

15.   Sea $f : x \to \dfrac{senx}{x}$, $\pi \le x \le 3\pi$. Halle el área encerrada por la gráfica de f y el eje OX (Usar calculadora gráfica)
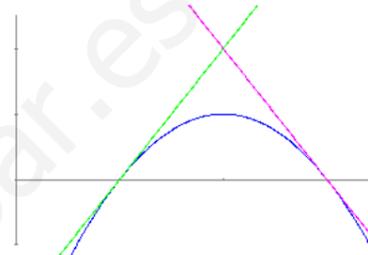
① $y = x^4 - 5x^2 + 4$

$y > 0 \implies x^4 - 5x^2 + 4 = 0 \implies x^2 = \dfrac{5 \pm \sqrt{25-16}}{2} = \dfrac{5 \pm 3}{2} \nearrow 4 \implies x = \pm 2$
$\searrow 1 \implies x = \pm 1$

signo $x^4 - 5x^2 + 4$
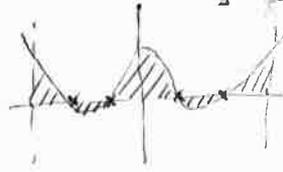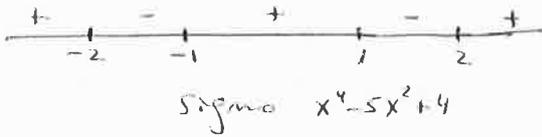
$Area = \displaystyle\int_{-3}^{3} |x^4 - 5x^2 + 4|\, dx = \int_{-3}^{-2} f(x)dx - \int_{-2}^{-1} f(x)dx + \int_{-1}^{1} f(x)dx - \int_{1}^{2} f(x)dx + \int_{2}^{3} f(x)dx =$

$= 2\displaystyle\int_{0}^{1} f(x)dx - 2\int_{1}^{2} f(x)dx + 2\int_{2}^{3} f(x)dx =$

$= 2\left(\dfrac{x^5}{5} - \dfrac{5x^3}{3} + 4x\right)\Big|_{0}^{1} - 2\left(\dfrac{x^5}{5} - \dfrac{5x^3}{3} + 4x\right)\Big|_{1}^{2} + 2\left(\dfrac{x^5}{5} - \dfrac{5x^3}{3} + 4x\right)\Big|_{2}^{3} =$

$= 2\left(\dfrac{1}{5} - \dfrac{5}{3} + 4\right) - 2\left(\dfrac{32}{5} - \dfrac{40}{3} + 8\right) + 2\left(\dfrac{1}{5} - \dfrac{5}{3} + 4\right) + 2\left(\dfrac{243}{5} - \dfrac{135}{3} + 12\right) - 2\left(\dfrac{32}{5} - \dfrac{40}{3} + 8\right)$
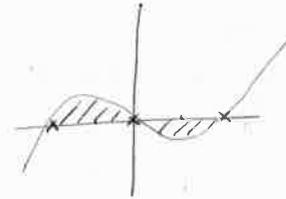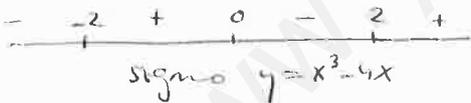
$= 4\left(\dfrac{1}{5} - \dfrac{5}{3} + 4\right) - 4\left(\dfrac{32}{5} - \dfrac{40}{3} + 8\right) + 2\left(\dfrac{243}{5} - \dfrac{135}{3} + 12\right) =$

$= \dfrac{4 - 128 + 486}{5} + \dfrac{160 - 20 - 270}{3} + 16 - 32 + 24 = \dfrac{362}{5} - \dfrac{130}{3} + 8 =$

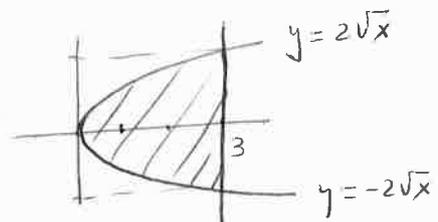$= \dfrac{1086 - 650 + 120}{15} = \boxed{\dfrac{556}{15}\ us}$

② $y = x^3 - 4x$

$y = 0 \implies x(x^2 - 4) = 0 \quad \begin{array}{l} x = -2 \\ x = 0 \\ x = 2 \end{array}$

signo $y = x^3 - 4x$

$Area = \displaystyle\int_{-2}^{2} |x^3 - 4x|\, dx = \int_{-2}^{0} (x^3 - 4x)dx - \int_{0}^{2} (x^3 - 4x)dx = -2\int_{0}^{2} (x^3 - 4x)dx =$

$= -2\left(\dfrac{x^4}{4} - 2x^2\right)\Big|_{0}^{2} = -2\left(\dfrac{16}{4} - 8\right) = \boxed{8\ us}$

③ $\left. \begin{array}{l} y^2 = 4x \\ x = 3 \end{array} \right\} \implies y^2 = 12 \implies y = \pm\sqrt{12}$

$Area = 2\displaystyle\int_{0}^{3} 2\sqrt{x}\, dx = 4\int_{0}^{3} x^{1/2} dx = 4\,\dfrac{x^{3/2}}{3/2}\Big|_{0}^{3} = \dfrac{8}{3}\sqrt{x^3}\Big|_{0}^{3} =$

$= \dfrac{8}{3}\sqrt{27} = \boxed{8\sqrt{3}\ us}$

(4)

$$x = y^2 \Big\} \quad y - y^2 + 2 = 0 \quad \longrightarrow \quad y^2 - y - 2 = 0$$
$$y - x + 2 = 0$$

$$y = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2} \begin{array}{l} \nearrow 2 \Rightarrow x = 4 \\ \searrow -1 \Rightarrow x = 1 \end{array}$$

$y = x - 2$
$y = \sqrt{x}$
$y = -\sqrt{x}$

$$\text{Area} = 2\int_0^1 \sqrt{x}\, dx + \int_1^4 \left[\sqrt{x} - (x-2)\right] dx =$$

$$= 2\, \frac{x^{3/2}}{3/2} \Big|_0^1 + \left(\frac{x^{3/2}}{3/2} - \frac{x^2}{2} + 2x\right)\Big|_1^4 =$$

$$= \frac{4}{3} + \frac{16}{3} - 8 + 8 - \frac{2}{3} + \frac{1}{2} - 2 = \boxed{\frac{9}{2}\ us}$$

(5)

$$f(x) = \operatorname{sen} x \Big\} \Rightarrow \operatorname{sen} x = \cos x \Rightarrow \operatorname{tg} x = 1 \Rightarrow \begin{array}{ll} x = 45° & x = \pi/4 \\ x = 225° & x = 3\pi/4 \end{array}$$
$$g(x) = \cos x$$

Signo $\operatorname{sen} x - \cos x$

$y = \cos x$
$y = \operatorname{sen} x$

$$\text{Area} = \int_0^{2\pi} |\operatorname{sen} x - \cos x|\, dx = \int_0^{\pi/4} (\cos x - \operatorname{sen} x)\, dx + \int_{\pi/4}^{3\pi/4} (\operatorname{sen} x - \cos x)\, dx + \int_{3\pi/4}^{2\pi} (\cos x - \operatorname{sen} x)\, dx =$$

$$= (\operatorname{sen} x + \cos x)\Big|_0^{\pi/4} + (-\cos x - \operatorname{sen} x)\Big|_{\pi/4}^{3\pi/4} + (\operatorname{sen} x + \cos x)\Big|_{3\pi/4}^{2\pi} =$$
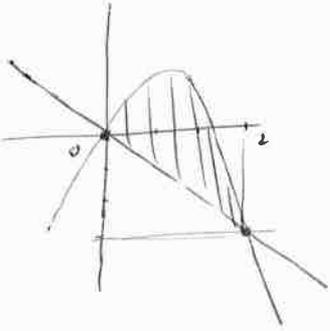
$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 1 + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + 1 - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = \boxed{2\sqrt{2}\ us}$$

(6)

$$y = x^2 - x \Big\} \Rightarrow x^2 - x = 3x - x^2 \Rightarrow 2x^2 = 4x \Rightarrow 2x(x-2) = 0 \begin{array}{l}\nearrow x = 0 \\ \searrow x = 2\end{array}$$
$$y = 3x - x^2$$

$y = x^2 - x$
$y = 3x - x^2$

$$\text{Area} = \int_0^2 (3x - x^2) - (x^2 - x)\, dx = \int_0^2 (4x - 2x^2)\, dx =$$

$$= \left(2x^2 - \frac{2}{3}x^3\right)\Big|_0^2 = 8 - \frac{16}{3} = \boxed{\frac{8}{3}\ us}$$

⑦

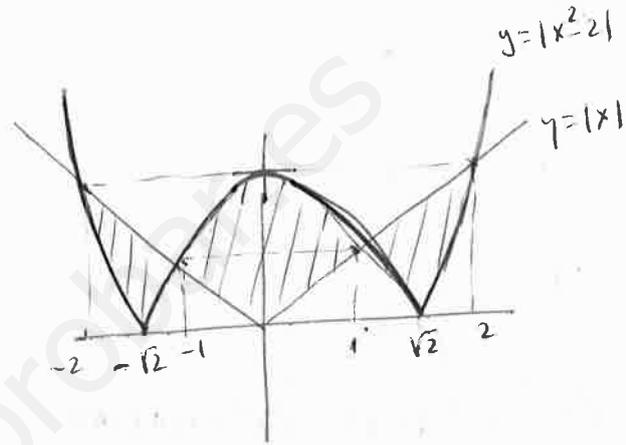$$y=2x-x^2 \left. \right| \implies 2x-x^2=-x \implies 0=x^2-3x \implies 0=x(x-3) \begin{cases} x=0 \\ x=3 \end{cases}$$
$$y=-x$$

$$\text{Area} = \int_0^2 \left[(2x-x^2)-(-x)\right] dx =$$

$$= \int_0^2 (3x-x^2)\,dx = \left(\frac{3x^2}{2}-\frac{x^3}{3}\right)\Big|_0^2 = 6-\frac{8}{3} = \boxed{\frac{10}{3}\ us}$$

⑧

$$y=|x| = \begin{cases} -x & si & x\leq 0 \\ x & si & x>0 \end{cases}$$

$$y=|x^2-2| = \begin{cases} x^2-2 & si & x\leq -\sqrt{2} \\ 2-x^2 & si & -\sqrt{2}\leq x \leq \sqrt{2} \\ x^2-2 & si & x\geq \sqrt{2} \end{cases}$$

$$|x| = |x^2-2|$$

$$x = x^2-2 \qquad\qquad -x = x^2-2 \longrightarrow 0 = x^2+x-2$$
$$x^2-x-2=0 \qquad\qquad\qquad\qquad x = \frac{-1\pm\sqrt{1+8}}{2} = \frac{-1\pm 3}{2} \begin{cases} 1 \\ -2 \end{cases}$$
$$x = \frac{1\pm\sqrt{1+8}}{2} = \frac{1\pm 3}{2} = \begin{cases} 2 \\ -1 \end{cases}$$

$$\text{Area} = \int_{-2}^{2} \Big||x|-|x^2-2|\Big|\,dx = 2\int_0^2 \Big||x|-|x^2-2|\Big|\,dx = 2\int_0^1 \left[(2-x^2)-x\right]dx \; +$$

$$+ 2\int_1^{\sqrt{2}} \left[x-(2-x^2)\right]dx \; + 2\int_{\sqrt{2}}^{2} \left[x-(x^2-2)\right]dx =$$

$$= 2\left(2x-\frac{x^3}{3}-\frac{x^2}{2}\right)\Big|_0^1 + 2\left(\frac{x^2}{2}-2x+\frac{x^3}{3}\right)\Big|_1^{\sqrt{2}} + 2\left(\frac{x^2}{2}-\frac{x^3}{3}+2x\right)\Big|_{\sqrt{2}}^{2} =$$
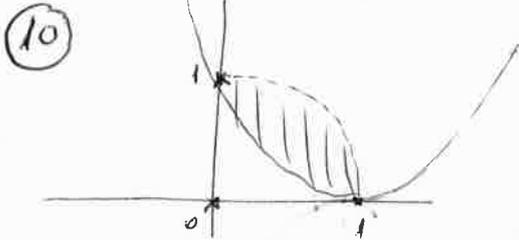
$$= 2\left(2-\frac{1}{3}-\frac{1}{2}\right) + 2\left(1-2\sqrt{2}+\frac{2\sqrt{2}}{3}\right) - 2\left(\frac{1}{2}-2+\frac{1}{3}\right) +2\left(2-\frac{8}{3}+4\right) - 2\left(1-\frac{2\sqrt{2}}{3}+2\sqrt{2}\right)$$

$$= 2\left(10-\frac{10}{3}-\frac{8\sqrt{2}}{3}-1\right) = 2\frac{27-10-8\sqrt{2}}{3} = \boxed{\frac{34-16\sqrt{2}}{3}\ us}$$

⑨

$$y = x^2$$
$$y^2 = x$$ $\Biggr\}$ $\Rightarrow$ $y = (y^2)^2$ $\Rightarrow$ $y = y^4$ $\Rightarrow$ $y(1-y^3)=0$ $\Rightarrow$ $\begin{cases} y=0 \Rightarrow x=0 \\ y=1 \Rightarrow x=1 \end{cases}$
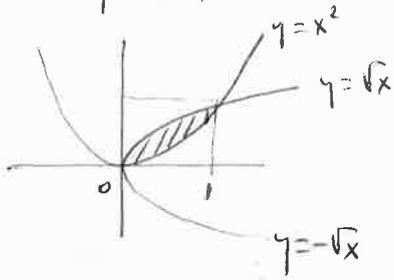
$$Area = \int_0^1 (\sqrt{x} - x^2)\, dx = \left(\frac{x^{3/2}}{3/2} - \frac{x^3}{3} \right)\Biggr|_0^1 =$$

$$= \frac{2}{3} - \frac{1}{3} = \boxed{\frac{1}{3} \ u^2}$$

⑩

$$\left. \begin{array}{l} x^2 + y^2 = 1 \\ y = x^2 - 2x + 1 \end{array} \right\} \rightarrow y = \pm\sqrt{1-x^2}$$

$$\pm\sqrt{1-x^2} = x^2 - 2x + 1$$
$$1 - x^2 = x^4 + 4x^2 + 1 - 4x^3 + 2x^2 - 4x$$
$$0 = x^4 - 4x^3 + 7x^2 - 4x$$
$$0 = x(x^3 - 4x^2 + 7x - 4) \rightarrow \boxed{x=0}$$

$$\boxed{1}\begin{array}{c|cccc} & 1 & -4 & 7 & -4 \\ & & 1 & -3 & 4 \\ \hline & 1 & -3 & 4 & \boxed{0} \end{array}$$

$$x^2 - 3x + 4 = 0$$
$$x = \frac{3 \pm \sqrt{9-16}}{2} \quad \times$$

$$Area = \int_0^1 \sqrt{1-x^2} - (x^2 - 2x + 1)\, dx$$

$$\int_0^1 \sqrt{1-x^2}\, dx = \int_0^{\pi/2} \cos^2 t\, dt = \int_0^{\pi/2} \frac{1 + \cos(2t)}{2}\, dt = \frac{t}{2} + \frac{\operatorname{sen}(2t)}{4}\Biggr|_0^{\pi/2} = \frac{\pi}{4}$$
$$x = \operatorname{sen} t$$
$$dx = \cos t\, dt$$

$$\int_0^1 (x^2 - 2x + 1)\, dx = \left(\frac{x^3}{3} - x^2 + x \right)\Biggr|_0^1 = \frac{1}{3} - 1 + 1 = \frac{1}{3}$$

$$Area = \boxed{\frac{\pi}{4} - \frac{1}{3}}$$

⑪
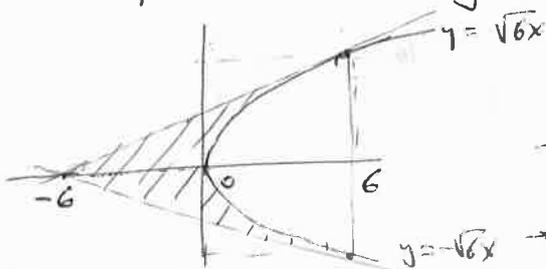
$$y^2 = 6x$$
$$2yy' = 6 \rightarrow y' = \frac{3}{y}$$

$$\begin{cases} x = 6 \Rightarrow y^2 = 36 \Rightarrow y = \pm 6 \\ P(6,6) \Rightarrow pend = \frac{3}{6} = \frac{1}{2} \\ P(6,-6) \Rightarrow pend = \frac{3}{-6} = -\frac{1}{2} \end{cases}$$

$$\boxed{y - 6 = \frac{1}{2}(x-6)} \rightarrow$$
$$\boxed{y + 6 = -\frac{1}{2}(x-6)} \rightarrow$$

$$\rightarrow \boxed{y = \frac{x+6}{2}}$$
$$\rightarrow \boxed{y = \frac{-x-6}{2}}$$

$$Area = 2\int_{-6}^{0} \frac{x+6}{2}\, dx + 2\int_{0}^{6}\left(\frac{x+6}{2} - \sqrt{6x}\right) dx = \left(\frac{x^2}{2} + 6x\right)\Big|_{-6}^{0} + \left(\frac{x^2}{2} + 6x - 2\sqrt{6}\,\frac{x^{3/2}}{3/2}\right)\Big|_{0}^{6} =$$

$$= 0 - \left(\frac{36}{2} - 36\right) + \left(\frac{36}{2} + 36 - \frac{4\sqrt{6}}{3}\sqrt{6^3}\right) - 0 = \boxed{24 \ us}$$
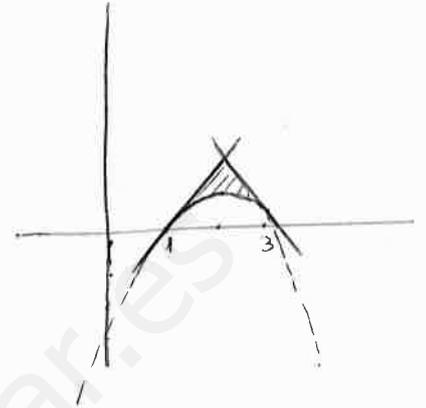
(12)

$$y = (x-1)(3-x)$$

$$y = -x^2 + 4x - 3$$

$$y' = -2x + 4$$

$$x = 1 \begin{cases} y = 0 \\ y' = 2 \end{cases} \quad y - 0 = 2(x-1) \rightarrow \boxed{y = 2x - 2}$$

$$x = 3 \begin{cases} y = 0 \\ y' = -2 \end{cases} \quad y - 0 = -2(x-3) \rightarrow \boxed{y = 6 - 2x}$$

Puntos, $\begin{cases} y = 2x-2 \\ y = 6-2x \end{cases} \rightarrow 2x-2 = 6-2x ; \quad 4x = 8 \rightarrow \boxed{x = 2 \rightarrow y = 2}$

$$Area = \int_{1}^{2} \left[(2x-2) - (-x^2+4x-3)\right] dx + \int_{2}^{3}\left[(6-2x) - (-x^2+4x-3)\right] dx =$$

$$= \int_{1}^{2}(x^2 - 2x + 1)\, dx + \int_{2}^{3}(x^2 - 6x + 9)\, dx = \left(\frac{x^3}{3} - x^2 + x\right)\Big|_{1}^{2} + \left(\frac{x^3}{3} - 3x^2 + 9x\right)\Big|_{2}^{3} =$$

$$= \left(\frac{7}{3} - 2\right) + \left(\frac{19}{3} - 6\right) = \frac{1}{3} + \frac{1}{3} = \boxed{\frac{2}{3}}$$

Salen idénticos los valores de área en ambos recintos. Era previsible por la simetría del dibujo. Se podía haber hecho con una sola integral: $Area = 2\int_{1}^{2}(x^2 - 2x + 1)\, dx$

(13) $f(x) = |x^2 - 1| + 1 = \begin{cases} x^2 - 1 + 1 & Si \ x < -1 \\ 1 - x^2 + 1 & Si \ -1 \le x \le 1 \\ x^2 - 1 + 1 & Si \ x > 1 \end{cases} = \begin{cases} x^2 & Si \ x < -1 \\ 2 - x^2 & Si \ -1 \le x \le 1 \\ x^2 & Si \ x > 1 \end{cases}$
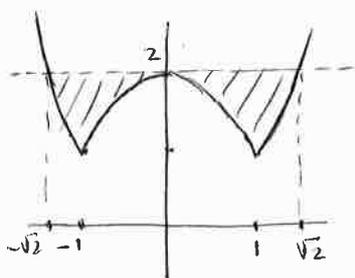
$$y = 2 \Rightarrow \begin{cases} x^2 = 2 \rightarrow \boxed{x = -\sqrt{2}} \\ 2 - x^2 = 2 \rightarrow \boxed{x = 0} \\ x^2 = 2 \rightarrow \boxed{x = 2} \end{cases}$$

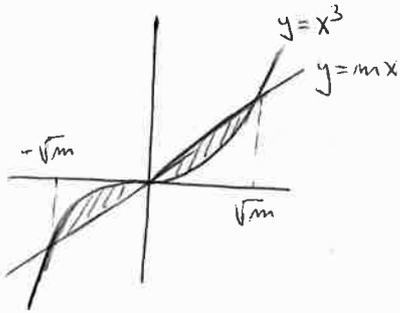$$Area = 2\left(\int_{0}^{1}\left[2 - (2-x^2)\right] dx + \int_{1}^{\sqrt{2}}\left[2 - x^2\right] dx\right) =$$

$$= 2\int_{0}^{1} x^2\, dx + 2\int_{1}^{\sqrt{2}}(2 - x^2)\, dx = \frac{2x^3}{3}\Big|_{0}^{1} + 2\left(2x - \frac{x^3}{3}\right)\Big|_{1}^{\sqrt{2}} =$$

$$= \frac{2}{3} + \left(4\sqrt{2} - \frac{4\sqrt{2}}{3}\right) - \left(4 - \frac{2}{3}\right) = \frac{2 + 12\sqrt{2} - 4\sqrt{2} - 12 + 2}{3} = \boxed{\frac{8(\sqrt{2}-1)}{3}}$$

(14)
$$y=x^3 \quad \Big\} \Rightarrow \quad x^3=mx \Rightarrow x^3-mx=0 \rightarrow x(x^2-m)=0$$
$$y=mx \quad$$

$$\begin{cases} x=0 \\ x=\sqrt{m} \\ x=-\sqrt{m} \end{cases}$$

m debe ser positivo

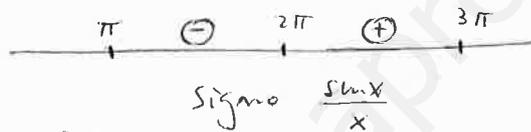$$Area = 2\int_0^{\sqrt{m}}(mx-x^3)dx = \left(2m\frac{x^2}{2} - 2\frac{x^4}{4}\right)\Big|_0^{\sqrt{m}} =$$

$$= \left(mx^2 - \frac{x^4}{2}\right)\Big|_0^{\sqrt{m}} = m^2 - \frac{m^2}{2} = \frac{m^2}{2}$$

$$\frac{m^2}{2} = 8 \Rightarrow m^2 = 16 \Rightarrow \boxed{m=4}$$

(15) $f(x) = \dfrac{\sin x}{x}$

$$\pi \quad \ominus \quad 2\pi \quad \oplus \quad 3\pi$$

Signo $\dfrac{\sin x}{x}$

$$Area = -\int_\pi^{2\pi}\frac{\sin x}{x}dx + \int_{2\pi}^{3\pi}\frac{\sin x}{x}dx = +0'43379 + 0'25661 = \boxed{0'69040}$$

También se puede utilizar la calculadora gráfica con la función en valor absoluto, sin estudiar previamente su signo ni, en consecuencia, separar en dos integrales.

$$Area = \int_\pi^{3\pi}\left|\frac{\sin x}{x}\right|dx = \boxed{0'69040}$$