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CÁLCULO DE LÍMITES

1. Calcular los siguientes límites:

a) $\lim\limits_{x\to 0} \dfrac{x^3-x^2-2x}{x^3-x}$ b) $\lim\limits_{x\to -5} \dfrac{x^2+3x-10}{x^3+8x^2+5x-50}$ c) $\lim\limits_{x\to +\infty} \dfrac{(x^2+2)(x^2-2)}{4x^2\cdot(x+2)^2}$

d) $\lim\limits_{x\to 1} \dfrac{x^5-1}{x^4+2x^3-x-2}$ e) $\lim\limits_{x\to \infty} \dfrac{x(2x+1)^2-8x^3}{5x(x+2)^2}$ f) $\lim\limits_{x\to 2} \dfrac{x^4-6x^3+12x^2-8x}{x^3-x^2-8x+12}$

g) $\lim\limits_{x\to a} \dfrac{x^2-ax}{x^2+ax-2a^2}$ h) $\lim\limits_{x\to 0} \dfrac{(a+x)^2-a^2}{x}$ i) $\lim\limits_{x\to a} \dfrac{x^2-(a+1)x+a}{x^2-a^2}$

Solución:

a) $$\lim_{x\to 0} \frac{x^3-x^2-2x}{x^3-x} = \frac{0}{0} = \lim_{x\to 0} \frac{x(x^2-x-2)}{x(x^2-1)} = \lim_{x\to 0} \frac{x^2-x-2}{x^2-1} = 2$$

b) $$\lim_{x\to -5} \frac{x^2+3x-10}{x^3+8x^2+5x-50} = \frac{0}{0} = \lim_{x\to -5} \frac{(\cancel{x+5})(x-3)}{(\cancel{x+5})^2(x-2)} = \lim_{x\to -5} \frac{x-3}{(x+5)(x-2)} = \frac{-8}{0} = +\infty$$

c) $$\lim_{x\to +\infty} \frac{(x^2+2)(x^2-2)}{4x^2\cdot(x+2)^2} = \frac{\infty}{\infty}$$ Dividimos por x^4:

$$\lim_{x\to +\infty} \frac{\left(1+\frac{2}{x^2}\right)\left(1-\frac{2}{x^2}\right)}{4\cdot\left(1+\frac{2}{x}\right)^2} = \frac{1}{4}$$

d) $$\lim_{x\to 1} \frac{x^5-1}{x^4+2x^3-x-2} = \frac{0}{0} = \lim_{x\to 1} \frac{(\cancel{x-1})(x^4+x^3+x^2+x+1)}{(\cancel{x-1})(x+2)(x^2+x+1)} = \lim_{x\to 1} \frac{x^4+x^3+x^2+x+1}{(x+2)(x^2+x+1)} = \frac{5}{9}$$

e) $$\lim_{x\to \infty} \frac{x(2x+1)^2-8x^3}{5x(x+2)^2} = \frac{\infty}{\infty} = \lim_{x\to \infty} \frac{(2x+1)^2-8x^2}{5(x+2)^2}$$ Dividimos por x^2:

$$\lim_{x\to \infty} \frac{\left(2+\frac{1}{x}\right)^2-8}{5\left(1+\frac{2}{x}\right)^2} = \frac{4-8}{5} = -\frac{4}{5}$$

f) $$\lim_{x\to 2} \frac{x^4-6x^3+12x^2-8x}{x^3-x^2-8x+12} = \frac{0}{0} = \lim_{x\to 2} \frac{x(x-2)^{\cancel{3}}}{(\cancel{x-2})^2(x+3)} = \lim_{x\to 2} \frac{x(x-2)}{x+3} = 0$$

g) $$\lim_{x\to a} \frac{x^2-ax}{x^2+ax-2a^2} = \frac{0}{0} = \lim_{x\to a} \frac{x(x-a)}{(x-a)(x+2a)} = \lim_{x\to a} \frac{x}{x+2a} = \frac{a}{3a} = \frac{1}{3}$$

h) $$\lim_{x\to 0} \frac{(a+x)^2-a^2}{x} = \frac{0}{0} = \lim_{x\to 0} \frac{a^2+2ax+x^2-a^2}{x} = \lim_{x\to 0} \frac{x(2a+x)}{x} = \lim_{x\to 0} (2a+x) = 3a$$

i) $$\lim_{x\to a} \frac{x^2-(a+1)x+a}{x^2-a^2} = \frac{0}{0} = \lim_{x\to a} \frac{(x-a)(x-1)}{(x-a)(x+a)} = \lim_{x\to a} \frac{x-1}{x+a} = \frac{a-1}{2a}$$

2. Calcular los siguientes límites:

a) $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+10x}-x\right)$ b) $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+5x+6}-x\right)$ c) $\lim\limits_{x\to+\infty}\left(\sqrt{4x^2+5x}-2x\right)$

d) $\lim\limits_{x\to+\infty}\left(\sqrt{x^2-x-6}-x\right)$ e) $\lim\limits_{x\to+\infty}\left(\sqrt{4x^2+x}-\sqrt{4x^2-2x}\right)$ f) $\lim\limits_{x\to+\infty}\dfrac{\sqrt{9x^2+x}+5x}{x+3\sqrt{x^2+x}}$

g) $\lim\limits_{x\to3}\dfrac{2x-6}{2-\sqrt{x^2-5}}$ h) $\lim\limits_{x\to3}\dfrac{x-\sqrt{3x}}{x-3}$ i) $\lim\limits_{x\to4}\dfrac{\sqrt{x+5}-3}{x^2-4x}$

j) $\lim\limits_{x\to1}\dfrac{x^2+x-2}{\sqrt{x}-1}$ k) $\lim\limits_{x\to+\infty}\dfrac{(x-1)\sqrt{4x^3+3}}{\sqrt{x^5}+1}$ l) $\lim\limits_{x\to2}\dfrac{\sqrt{x^2-3x+6}-\sqrt{x+2}}{3x-6}$

Solución:

a) $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+10x}-x\right)=\infty-\infty$ Multiplicamos por el conjugado:

$$\lim_{x\to+\infty}\frac{x^2+10x-x^2}{\sqrt{x^2+10x}+x}=\lim_{x\to+\infty}\frac{10x}{\sqrt{x^2+10x}+x}=\lim_{x\to+\infty}\frac{10}{\sqrt{1+\frac{10}{x}}+1}=\frac{10}{2}=5$$

b) $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+5x+6}-x\right)=\infty-\infty$ Multiplicamos por el conjugado:

$$\lim_{x\to+\infty}\frac{x^2+5x+6-x^2}{\sqrt{x^2+5x+6}+x}=\lim_{x\to+\infty}\frac{5x+6}{\sqrt{x^2+5x+6}+x}=\lim_{x\to+\infty}\frac{5+\frac{6}{x}}{\sqrt{1+\frac{5}{x}+\frac{6}{x^2}}+1}=\frac{5}{2}$$

c) $\lim\limits_{x\to+\infty}\left(\sqrt{4x^2+5x}-2x\right)=\infty-\infty$ Multiplicamos por el conjugado:

$$\lim_{x\to+\infty}\frac{4x^2+5x-4x^2}{\sqrt{4x^2+5x}+2x}=\lim_{x\to+\infty}\frac{5x}{\sqrt{4x^2+5x}+2x}=\lim_{x\to+\infty}\frac{5}{\sqrt{4+\frac{5}{x}}+2}=\frac{5}{4}$$

d) $\lim\limits_{x\to+\infty}\left(\sqrt{x^2-x-6}-x\right)=\infty-\infty$ Multiplicamos por el conjugado:

$$\lim_{x\to+\infty}\frac{x^2-x-6-x^2}{\sqrt{x^2-x-6}+x}=\lim_{x\to+\infty}\frac{-x-6}{\sqrt{x^2-x-6}+x}=\lim_{x\to+\infty}\frac{-1-\frac{6}{x}}{\sqrt{1-\frac{1}{x}-\frac{6}{x^2}}+1}=-\frac{1}{2}$$

e) $\lim\limits_{x\to+\infty}\left(\sqrt{4x^2+x}-\sqrt{4x^2-2x}\right)=\infty-\infty$ Multiplicamos por el conjugado:

$$\lim_{x\to+\infty}\frac{4x^2+x-4x^2+2x}{\sqrt{4x^2+x}+\sqrt{4x^2-2x}}=\lim_{x\to+\infty}\frac{3x}{\sqrt{4x^2+x}+\sqrt{4x^2-2x}}=\lim_{x\to+\infty}\frac{3}{\sqrt{4+\frac{1}{x}}+\sqrt{4-\frac{2}{x}}}=\frac{3}{4}$$

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f) $\lim_{x\to+\infty} \frac{\sqrt{9x^2+x}+5x}{x+3\sqrt{x^2+x}} = \frac{\infty}{\infty}$ Dividimos por x:

$$\lim_{x\to+\infty} \frac{\sqrt{9+\frac{1}{x}}+5}{1+3\sqrt{1+\frac{1}{x}}} = \frac{8}{4} = 2$$

g) $\lim_{x\to 3} \frac{2x-6}{2-\sqrt{x^2-5}} = \frac{0}{0} = \lim_{x\to 3} \frac{(2x-6)\left(2+\sqrt{x^2-5}\right)}{4-x^2+5} = \lim_{x\to 3} \frac{2(x-3)\left(2+\sqrt{x^2-5}\right)}{(3-x)(x+3)} = \lim_{x\to 3} \frac{2\left(2+\sqrt{x^2-5}\right)}{-(x+3)} = -\frac{4}{3}$

h) $\lim_{x\to 3} \frac{x-\sqrt{3x}}{x-3} = \frac{0}{0} = \lim_{x\to 3} \frac{x^2-3x}{(x-3)\left(x+\sqrt{3x}\right)} = \lim_{x\to 3} \frac{x(x-3)}{(x-3)\left(x+\sqrt{3x}\right)} = \lim_{x\to 3} \frac{x}{\left(x+\sqrt{3x}\right)} = \frac{3}{6} = \frac{1}{2}$

i) $\lim_{x\to 4} \frac{\sqrt{x+5}-3}{x^2-4x} = \frac{0}{0} = \lim_{x\to 4} \frac{x+5-9}{(x^2-4x)\left(\sqrt{x+5}+3\right)} = \lim_{x\to 4} \frac{x-4}{x(x-4)\left(\sqrt{x+5}+3\right)} = \lim_{x\to 4} \frac{1}{x\left(\sqrt{x+5}+3\right)} = \frac{1}{24}$

j) $\lim_{x\to 1} \frac{x^2+x-2}{\sqrt{x-1}} = \frac{0}{0} = 0$

k) $\lim_{x\to+\infty} \frac{(x-1)\sqrt{4x^3+3}}{\sqrt{x^5+1}} = \frac{\infty}{\infty} = \lim_{x\to+\infty} \frac{(x-1)\sqrt{4x^3}}{\sqrt{x^5}} = \lim_{x\to+\infty} \frac{(x-1)2x\sqrt{x}}{x^2\sqrt{x}} = \lim_{x\to+\infty} \frac{2(x-1)}{x} = \lim_{x\to+\infty} \left(2-\frac{2}{x}\right) = 2$

l) $\lim_{x\to 2} \frac{\sqrt{x^2-3x+6}-\sqrt{x+2}}{3x-6} = \frac{0}{0} =$

$$\lim_{x\to 2} \frac{x^2-3x+6-x-2}{(3x-6)\left(\sqrt{x^2-3x+6}+\sqrt{x+2}\right)} = \lim_{x\to 2} \frac{x^2-2x+4}{(3x-6)\left(\sqrt{x^2-3x+6}+\sqrt{x+2}\right)} =$$

$$= \lim_{x\to 2} \frac{(x-2)^2}{3(x-2)\left(\sqrt{x^2-3x+6}+\sqrt{x+2}\right)} = \lim_{x\to 2} \frac{x-2}{3\left(\sqrt{x^2-3x+6}+\sqrt{x+2}\right)} = 0$$

3. Dada la función:

$$f(x) = \frac{x^3 - x^2 - 2x}{x^4 - x^2}$$

Hallar:

a) $\lim_{x \to -1} f(x)$ b) $\lim_{x \to 2} f(x)$ c) $\lim_{x \to +\infty} f(x)$ d) $\lim_{x \to 0} f(x)$

Solución:

a) $\lim_{x \to -1} f(x) = \frac{0}{0} = \lim_{x \to -1} \frac{x^3 - x^2 - 2x}{x^4 - x^2} = \lim_{x \to -1} \frac{x(x-2)(x+1)}{x^2(x+1)(x-1)} = \lim_{x \to -1} \frac{x-2}{x(x-1)} = -\frac{3}{2}$

b) $\lim_{x \to 2} f(x) = \lim_{x \to 2} \frac{x-2}{x(x-1)} = 0$

c) $\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{x-2}{x^2 - x} = \lim_{x \to +\infty} \frac{\frac{1}{x} - \frac{2}{x^2}}{1 - \frac{1}{x}} = 0$

d) $\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{x-2}{x(x-1)} = \frac{-2}{0} = -\infty$

4. Dada la función:

$$f(x) = \begin{cases} \frac{x^2 - 4x + 3}{x^2 - 1} & \text{si } x < 1 \\ \frac{1-x}{\sqrt{x} - 1} & \text{si } x \geq 1 \end{cases}$$

a) Calcular $\lim_{x \to -\infty} f(x)$, $\lim_{x \to +\infty} f(x)$, $\lim_{x \to 0} f(x)$

b) Demuestra que no existe $\lim_{x \to 1} f(x)$

Solución:

a) $\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{x^2 - 4x + 3}{x^2 - 1} = \lim_{x \to -\infty} \frac{1 - \frac{4}{x} + \frac{3}{x^2}}{1 - \frac{1}{x^2}} = 1$

$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{1-x}{\sqrt{x} - 1} = \lim_{x \to +\infty} -\left(\sqrt{x} + 1\right) = -\infty$

$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{x^2 - 4x + 3}{x^2 - 1} = -3$

b) $\lim_{x \to 1^+} f(x) = \lim_{x \to 1} \frac{1-x}{\sqrt{x} - 1} = \lim_{x \to 1} -\left(\sqrt{x} + 1\right) = -2$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1} \frac{x^2 - 4x + 3}{x^2 - 1} = \lim_{x \to 1} \frac{(x-3)(x-1)}{(x-1)(x+1)} = \lim_{x \to 1} \frac{x-3}{x+1} = -1$

Luego, $\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x) \rightarrow$ No existe $\lim_{x \to 1} f(x)$

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5. Dada la función:

$$f(x) = \frac{3x^3 - 7x^2 + 5x - 1}{mx^3 - 3x + 2}$$

Se sabe que $\lim\limits_{x\to 2} f(x) = \frac{5}{4}$. Se pide:

a) Hallar m

b) $\lim\limits_{x\to +\infty} f(x)$ y $\lim\limits_{x\to 1} f(x)$.

Solución:

a) $\lim\limits_{x\to 2} f(x) = \frac{5}{4} \to \lim\limits_{x\to 2} \frac{3x^3 - 7x^2 + 5x - 1}{mx^3 - 3x + 2} = \frac{5}{8m-4} = \frac{5}{4} \to 8m - 4 = 4 \to m = 1$

b) $\lim\limits_{x\to +\infty} f(x) = \lim\limits_{x\to +\infty} \frac{3x^3 - 7x^2 + 5x - 1}{x^3 - 3x + 2} = \lim\limits_{x\to +\infty} \frac{3 - \frac{7}{x} + \frac{5}{x^2} - \frac{1}{x^3}}{1 - \frac{3}{x^2} + \frac{2}{x^3}} = 3$

$\lim\limits_{x\to 1} f(x) = \frac{0}{0} = \lim\limits_{x\to 0} \frac{3x^3 - 7x^2 + 5x - 1}{x^3 - 3x + 2} = \lim\limits_{x\to 0} \frac{(x-1)^2(3x-1)}{(x-1)^2(x+2)} = \lim\limits_{x\to 0} \frac{3x-1}{x+2} = -\frac{1}{2}$