

CÁLCULO DE DERIVADAS

1.- Determinar la derivada de las siguientes funciones irracionales:

a) $f(x) = \sqrt{4x + x^2}$

b) $f(x) = \sqrt{3x} + \frac{1}{x}$

c) $f(x) = \sqrt[3]{x^2 + 1}$

d) $f(x) = (1+x)\sqrt{1-x}$

e) $f(x) = \sqrt{\frac{1+x}{1-x}}$

f) $f(x) = \frac{x}{\sqrt{1+x}}$

2.- Obtener la derivada de las siguientes funciones exponenciales y logarítmicas:

a) $f(x) = (x-1) \cdot e^x$

b) $f(x) = a^{2x+3}$

c) $f(x) = \frac{1 + \ln x}{x}$

d) $f(x) = \log_a(5x^2 - 3)$

e) $f(x) = \ln(ax + b)$

f) $f(x) = \ln(\ln x)$

g) $f(x) = \frac{e^x - 1}{e^x + 1}$

h) $f(x) = \frac{2e^x}{e^x + 1}$

i) $f(x) = \log_2[x^2 + 1]$

3.- Obtener la derivada de las siguientes funciones trigonométricas:

a) $f(x) = \sin^2 x$

b) $f(x) = \sin x^2$

c) $f(x) = \sin^2 x - \cos^2 x$

d) $f(x) = \frac{\sin x \cdot \cos x}{2}$

e) $f(x) = \frac{\cos x}{1 - \sin x}$

f) $f(x) = \frac{\sin x}{1 + \operatorname{tg}^2 x}$

g) $f(x) = \ln \cos x$

h) $f(x) = \ln \sin x$

i) $f(x) = \frac{\sin x}{1 + \cos x}$

4.- Determinar la derivada de las funciones:

a) $f(x) = \sec x$

b) $g(x) = \operatorname{cosec} x$

c) $h(x) = \operatorname{cotg} x$

d) $f(x) = \frac{\operatorname{cosec} x}{3 \sec x}$

e) $f(x) = (1 + \operatorname{tg}^2 x) \cdot \sec^2 x$

f) $f(x) = \operatorname{cosec} x \cdot \sec x$

g) $f(x) = \arccos(x^2 - 1)$

h) $f(x) = \operatorname{arctg}(2x + 1)$

i) $f(x) = \frac{1}{3} \operatorname{arcsen} \frac{3x}{2}$

5.- Obtener la derivada de las siguientes funciones:

1) $f(x) = \frac{x^3}{1+x^2}$

2) $f(x) = \frac{6x}{2+x^4}$

3) $f(x) = \frac{1}{(1-x)^2}$

4) $f(x) = x \cdot \sqrt{x^2 + 2}$

5) $f(x) = \frac{1-x}{\sqrt{1-x^2}}$

6) $f(x) = \frac{2x^2 - 1}{\sqrt{x}}$

7) $f(x) = x(\ln x - 1)$

8) $f(x) = \ln\left(\frac{1+x}{1-x}\right)$

9) $f(x) = \frac{e^x + e^{-x}}{2}$

10) $f(x) = \ln\left(\frac{e^x}{e^x + 1}\right)$

11) $f(x) = \ln(1 + \sqrt{x})$

12) $f(x) = \ln(x + \sqrt{1+x^2})$

13) $f(x) = \sqrt{1 + \sin x}$

14) $f(x) = \frac{1 + \cos x}{1 - \cos x}$

15) $f(x) = \ln \sqrt{\frac{1 + \sin x}{1 - \sin x}}$

16) $f(x) = \operatorname{arcsen}(\cos x)$

17) $f(x) = \operatorname{arctg}(\operatorname{cotg} x)$

18) $f(x) = \arccos \sqrt{1-x^2}$

6.- Calcular la derivada de las siguientes funciones, aplicando la derivación logarítmica:

a) $y = x^x$

b) $y = x^{\ln x}$

c) $y = x^{\cos x}$

d) $f(x) = (\operatorname{tg} x)^{\operatorname{tg} x}$

e) $f(x) = (\cos x)^x$

f) $f(x) = (\operatorname{tg} x)^x$

SOLUCIONES

1.- Determinar la derivada de las siguientes funciones irracionales:

$$\text{a) } f(x) = \sqrt{4x+x^2} \Rightarrow f'(x) = \frac{1}{2\sqrt{4x+x^2}} \cdot (4+2x) = \frac{2+x}{\sqrt{4x+x^2}}$$

$$\text{b) } f(x) = \sqrt{3x} + \frac{1}{x} \Rightarrow f'(x) = \sqrt{3x} + \frac{1}{x} = \frac{3}{2\sqrt{3x}} - \frac{1}{x^2}$$

$$\text{c) } f(x) = \sqrt[3]{x^2+1} \Rightarrow f'(x) = \frac{2x}{3\sqrt[3]{(x^2+1)^2}}$$

$$\text{d) } f(x) = (1+x)\sqrt{1-x} \Rightarrow f'(x) = \sqrt{1-x} + (1+x) \cdot \frac{-1}{2\sqrt{1-x}} = \frac{2(1-x) - (1+x)}{2\sqrt{1-x}} = \frac{1-3x}{2\sqrt{1-x}}$$

$$\text{e) } f(x) = \sqrt{\frac{1+x}{1-x}} \Rightarrow f'(x) = \frac{1}{2\sqrt{\frac{1+x}{1-x}}} \cdot \frac{1-x - (1+x)(-1)}{(1-x)^2} = \frac{\sqrt{1-x}}{2\sqrt{1+x}} \cdot \frac{2}{(1-x)^2} = \frac{\sqrt{1-x^2}}{(1+x)(1-x)^2}$$

$$\text{f) } f(x) = \frac{x}{\sqrt{1+x}} \Rightarrow f'(x) = \frac{x}{\sqrt{1+x}} = \frac{\sqrt{1+x} - x \cdot \frac{1}{2\sqrt{1+x}}}{1+x} = \frac{2+2x-x}{2\sqrt{1+x}} = \frac{2+x}{2(1+x)\sqrt{1+x}} = \frac{(2+x)\sqrt{1+x}}{2(1+x)^2}$$

2.- Obtener la derivada de las siguientes funciones exponenciales y logarítmicas:

$$\text{a) } f(x) = (x-1) \cdot e^x \Rightarrow f'(x) = (x-1) \cdot e^x = e^x + (x-1) \cdot e^x = xe^x$$

$$\text{b) } f(x) = a^{2x+3} \Rightarrow f'(x) = a^{2x+3} \cdot \ln a \cdot 2 = 2a^{2x+3} \cdot \ln a$$

$$\text{c) } f(x) = \frac{1+\ln x}{x} \Rightarrow f'(x) = \frac{\frac{1}{x} \cdot x - (1+\ln x)}{x^2} = \frac{\ln x}{x^2}$$

$$\text{d) } f(x) = \log_a(5x^2-3) \Rightarrow f'(x) = \frac{1}{5x^2-3} \cdot \frac{1}{\ln a} \cdot 10x = \frac{10x}{5x^2-3} \cdot \frac{1}{\ln a}$$

$$\text{e) } f(x) = \ln(ax+b) \Rightarrow f'(x) = \frac{1}{ax+b} \cdot a = \frac{a}{ax+b}$$

$$\text{f) } f(x) = \ln(\ln x) \Rightarrow f'(x) = \frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x \ln x}$$

$$\text{g) } f(x) = \frac{e^x-1}{e^x+1} \Rightarrow f'(x) = \frac{e^x(e^x+1) - (e^x-1)e^x}{(e^x+1)^2} = \frac{2e^x}{(e^x+1)^2}$$

$$\text{h) } f(x) = \frac{2e^x}{e^x+1} \Rightarrow f'(x) = \frac{2e^x(e^x+1) - 2e^x \cdot e^x}{(e^x+1)^2} = \frac{2e^x}{(e^x+1)^2}$$

$$\text{i) } f(x) = \log_2[x^2+1] \Rightarrow f'(x) = \frac{1}{x^2+1} \cdot \frac{1}{\ln 2} \cdot (2x) = \frac{2x}{x^2+1} \cdot \frac{1}{\ln 2}$$

3.- Obtener la derivada de las siguientes funciones trigonométricas:

$$a) f(x) = \operatorname{sen}^2 x \Rightarrow f'(x) = 2 \operatorname{sen} x \cos x = \operatorname{sen} 2x$$

$$b) f(x) = \operatorname{sen} x^2 \Rightarrow f'(x) = \cos x^2 \cdot 2x = 2x \cos x^2$$

$$c) f(x) = \operatorname{sen}^2 x - \cos^2 x \Rightarrow f'(x) = 2 \operatorname{sen} x \cos x - 2 \cos x (-\operatorname{sen} x) = 4 \operatorname{sen} x \cos x$$

$$d) f(x) = \frac{\operatorname{sen} x \cdot \cos x}{2} \Rightarrow f'(x) = \frac{1}{2} (\cos x \cdot \cos x - \operatorname{sen} x \cdot \operatorname{sen} x) = \frac{\cos^2 x - \operatorname{sen}^2 x}{2} = \frac{\cos 2x}{2}$$

$$e) f(x) = \frac{\cos x}{1 - \operatorname{sen} x} \Rightarrow f'(x) = \frac{\cos x}{1 - \operatorname{sen} x} = \frac{-\operatorname{sen} x (1 - \operatorname{sen} x) - \cos x (-\cos x)}{(1 - \operatorname{sen} x)^2} = \frac{-\operatorname{sen} x + \operatorname{sen}^2 x + \cos^2 x}{(1 - \operatorname{sen} x)^2} =$$
$$= \frac{1 - \operatorname{sen} x}{(1 - \operatorname{sen} x)^2} = \frac{1}{1 - \operatorname{sen} x}$$

$$f) f(x) = \frac{\operatorname{sen} x}{1 + \operatorname{tg}^2 x} \Rightarrow f'(x) = \frac{\operatorname{sen} x}{1 + \operatorname{tg}^2 x} = \frac{\cos x \cancel{(1 + \operatorname{tg}^2 x)} - \operatorname{sen} x \cdot 2 \operatorname{tg} x \cdot \cancel{(1 + \operatorname{tg}^2 x)}}{(1 + \operatorname{tg}^2 x)^2} = \frac{\cos x - 2 \operatorname{sen} x \operatorname{tg} x}{1 + \operatorname{tg}^2 x} =$$
$$= \frac{\cos^2 x - 2 \operatorname{sen}^2 x}{\sec^2 x \cdot \cos x} = \frac{1 - \operatorname{sen}^2 x - 2 \operatorname{sen}^2 x}{\sec x} = (1 - 3 \operatorname{sen}^2 x) \cos x$$

$$g) f(x) = \ln \cos x \Rightarrow f'(x) = \frac{-\operatorname{sen} x}{\cos x} = -\operatorname{tg} x$$

$$h) f(x) = \ln \operatorname{sen} x \Rightarrow f'(x) = \frac{\cos x}{\operatorname{sen} x} = \operatorname{ctg} x$$

$$i) f(x) = \frac{\operatorname{sen} x}{1 + \cos x} \Rightarrow f'(x) = \frac{\cos x (1 + \cos x) - \operatorname{sen} x \cdot (-\operatorname{sen} x)}{(1 + \cos x)^2} = \frac{\cos x}{(1 + \cos x)^2}$$

4.- Determinar la derivada de las funciones:

$$a) f(x) = \sec x \Rightarrow f(x) = \frac{1}{\cos x} \quad f'(x) = \frac{\operatorname{sen} x}{\cos^2 x} = \operatorname{tg} x \cdot \sec x$$

$$b) g(x) = \operatorname{cosec} x \Rightarrow f(x) = \frac{1}{\operatorname{sen} x} \Rightarrow f'(x) = \frac{-\cos x}{\operatorname{sen}^2 x} = -\operatorname{ctg} x \cdot \operatorname{cosec} x$$

$$c) h(x) = \operatorname{cotg} x \Rightarrow f(x) = \frac{\cos x}{\operatorname{sen} x} \Rightarrow f'(x) = \frac{-\operatorname{sen} x \cdot \operatorname{sen} x - \cos x \cos x}{\operatorname{sen}^2 x} = \frac{-\operatorname{sen}^2 x - \cos^2 x}{\operatorname{sen}^2 x} = \frac{-1}{\operatorname{sen}^2 x}$$

$$d) f(x) = \frac{\operatorname{cosec} x}{3 \sec x} \Rightarrow f(x) = \frac{\cos x}{3 \operatorname{sen} x} = \frac{1}{3} \operatorname{cotg} x \Rightarrow f'(x) = \frac{-1}{3 \operatorname{sen}^2 x}$$

$$e) f(x) = (1 + \operatorname{tg}^2 x) \cdot \sec^2 x \Rightarrow f(x) = \frac{1}{\cos^2 x} \cdot \frac{1}{\cos^2 x} = \frac{1}{\cos^4 x} \Rightarrow f'(x) = \frac{4 \cos^3 x \cdot (-\operatorname{sen} x)}{\cos^8 x} = \frac{-4 \operatorname{sen} x}{\cos^5 x}$$

$$f) f(x) = \frac{1}{\cos x \operatorname{sen} x} \Rightarrow f'(x) = \frac{-(-\operatorname{sen}^2 x + \cos^2 x)}{\cos^2 x \operatorname{sen}^2 x} = \frac{\operatorname{sen}^2 x - \cos^2 x}{\cos^2 x \cdot \operatorname{sen}^2 x} = \frac{1}{\cos^2 x} - \frac{1}{\operatorname{sen}^2 x} = \sec^2 x - \operatorname{cosec}^2 x$$

$$g) f(x) = \arccos(x^2 - 1) \Rightarrow f'(x) = -\frac{2x}{\sqrt{1-(x^2-1)^2}} = -\frac{2x}{\sqrt{2x^2-x^4}} = -\frac{2}{\sqrt{2-x^2}}$$

$$h) f(x) = \operatorname{arctg}(2x+1) \Rightarrow f'(x) = \frac{2}{1+(2x+1)^2} = \frac{2}{4x^2+4x+2} = \frac{1}{2x^2+2x+1}$$

$$i) f(x) = \frac{1}{3} \operatorname{arcsen} \frac{3x}{2} \Rightarrow f'(x) = \frac{1}{3} \cdot \frac{\frac{3}{2}}{\sqrt{1-\frac{9x^2}{4}}} = \frac{1}{2} \cdot \frac{1}{\frac{1}{2}\sqrt{4-9x^2}} = \frac{1}{\sqrt{4-9x^2}}$$

5.- Calcular la derivada de los siguientes productos y divisiones:

$$1) f(x) = \frac{x^3}{1+x^2} \Rightarrow f'(x) = \frac{3x^2(1+x^2) - x^3 \cdot 2x}{(1+x^2)^2} = \frac{3x^2 + x^4}{(1+x^2)^2}$$

$$2) f(x) = \frac{6x}{2+x^4} \Rightarrow f'(x) = \frac{6(2+x^4) - 6x \cdot 4x^3}{(2+x^4)^2} = \frac{12-18x^4}{(2+x^4)^2}$$

$$3) f(x) = \frac{1}{(1-x)^2} \Rightarrow f'(x) = \frac{-2(1-x)}{(1-x)^4} = \frac{-2}{(1-x)^3}$$

$$4) f(x) = x \cdot \sqrt{x^2+2} \Rightarrow f'(x) = x \cdot \sqrt{x^2+2} = \sqrt{x^2+2} + x \cdot \frac{x}{\sqrt{x^2+2}} = \frac{2x^2+2}{\sqrt{x^2+2}}$$

$$5) f(x) = \frac{1-x}{\sqrt{1-x^2}} \Rightarrow f'(x) = \frac{1-x}{\sqrt{1-x^2}} = \frac{-\sqrt{1-x^2} - (1-x) \cdot \frac{-x}{\sqrt{1-x^2}}}{1-x^2} = \frac{-(1-x^2) + (1-x)x}{(1-x^2)\sqrt{1-x^2}} = \frac{-1+x^2+x-x^2}{(1-x^2)\sqrt{1-x^2}} = \frac{-1}{(1+x)\sqrt{1-x^2}}$$

$$6) f(x) = \frac{2x^2-1}{\sqrt{x}} \Rightarrow f'(x) = \frac{2x^2-1}{\sqrt{x}} = \frac{4x\sqrt{x} - (2x^2-1) \cdot \frac{1}{2\sqrt{x}}}{x} = \frac{8x^2 - (2x^2-1)}{2x\sqrt{x}} = \frac{6x^2+1}{2x\sqrt{x}}$$

$$7) f(x) = x(\ln x - 1) \Rightarrow f'(x) = (\ln x - 1) + x \cdot \frac{1}{x} = \ln x$$

$$8) f(x) = \ln\left(\frac{1+x}{1-x}\right) \Rightarrow f'(x) = \frac{1-x}{1+x} \cdot \frac{1-x-(1+x)}{(1-x)^2} = \frac{-2x}{1-x^2}$$

$$9) f(x) = \frac{e^x + e^{-x}}{2} \Rightarrow f'(x) = \frac{1}{2}(e^x - e^{-x}) = \frac{1}{2}\left(e^x - \frac{1}{e^x}\right) = \frac{e^{2x}-1}{2e^x}$$

$$10) f(x) = \ln\left(\frac{e^x}{e^x+1}\right) \Rightarrow f'(x) = \frac{e^x+1}{e^x} \cdot \frac{e^x(e^x+1) - e^x \cdot e^x}{(e^x+1)^2} = \frac{1}{e^x+1}$$

$$11) f(x) = \ln(1+\sqrt{x}) \Rightarrow f'(x) = \frac{1}{1+\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2(\sqrt{x}+x)}$$

$$12) f(x) = \ln(x + \sqrt{1+x^2}) \Rightarrow f'(x) = \frac{1}{x + \sqrt{1+x^2}} \cdot \left(1 + \frac{2x}{2\sqrt{1+x^2}}\right) = \frac{1}{x + \sqrt{1+x^2}} \cdot \frac{x + \sqrt{1+x^2}}{\sqrt{1+x^2}} = \frac{1}{\sqrt{1+x^2}}$$

$$13) f(x) = \sqrt{1 + \sin x} \Rightarrow f'(x) = \frac{\cos x}{2\sqrt{1 + \sin x}}$$

$$14) f(x) = \frac{1 + \cos x}{1 - \cos x} \Rightarrow f'(x) = \frac{1 + \cos x}{1 - \cos x} = \frac{-\sin x(1 - \cos x) - (1 + \cos x)\sin x}{(1 - \cos x)^2} = \frac{-2\sin x}{(1 - \cos x)^2}$$

$$15) f(x) = \ln \sqrt{\frac{1 + \sin x}{1 - \sin x}} \Rightarrow f'(x) = \frac{1}{2} \cdot \frac{1}{\sqrt{\frac{1 + \sin x}{1 - \sin x}}} \cdot \frac{\cos x(1 - \sin x) - (1 + \sin x)(-\cos x)}{(1 - \sin x)^2} =$$

$$= \frac{1}{2} \cdot \frac{1 - \sin x}{1 + \sin x} \cdot \frac{2 \cos x}{(1 - \sin x)^2} = \frac{\cos x}{1 - \sin^2 x} = \frac{1}{\cos x}$$

$$16) f(x) = \arcsen(\cos x) \Rightarrow f'(x) = \frac{-\sin x}{\sqrt{1 - \cos^2 x}} = -\frac{\sin x}{\sin x} = -1$$

$$17) f(x) = \operatorname{arctg}(\cot g x) \Rightarrow f'(x) = \frac{1}{1 + \cot^2 x} \cdot \left(-\frac{1}{\sin^2 x}\right) = \frac{1}{1 + \frac{\cos^2 x}{\sin^2 x}} \cdot \left(-\frac{1}{\sin^2 x}\right) = \sin^2 x \cdot \left(-\frac{1}{\sin^2 x}\right) = -1$$

$$18) f(x) = \arccos \sqrt{1 - x^2} \Rightarrow f'(x) = -\frac{-2x}{1 - 1 + x^2} = \frac{2x}{x^2} = \frac{2}{x}$$

18.- Calcula la derivada de las siguientes funciones, aplicando la derivación logarítmica:

$$a) y = x^x \Rightarrow \ln y = x \ln x \Rightarrow \frac{y'}{y} = \ln x + x \cdot \frac{1}{x} \Rightarrow y' = (\ln x + 1)x^x$$

$$b) y = x^{\ln x} \Rightarrow \ln y = \ln x \cdot \ln x \Rightarrow \frac{y'}{y} = 2 \ln x \cdot \frac{1}{x} \Rightarrow y' = \frac{2 \ln x}{x} \cdot x^{\ln x}$$

$$c) y = x^{\cos x} \Rightarrow \ln y = \cos x \cdot \ln x \Rightarrow \frac{y'}{y} = -\sin x \cdot \ln x + \frac{1}{x} \cos x \Rightarrow y' = \left(-\sin x \cdot \ln x + \frac{1}{x} \cos x\right) \cdot x^{\cos x}$$

$$d) f(x) = (\operatorname{tg} x)^{\operatorname{tg} x} \Rightarrow \ln y = \operatorname{tg} x \cdot \ln(\operatorname{tg} x) \Rightarrow \frac{y'}{y} = (1 + \operatorname{tg}^2 x) \cdot \ln(\operatorname{tg} x) + \operatorname{tg} x \cdot \frac{1}{\operatorname{tg} x} (1 + \operatorname{tg}^2 x)$$

$$\frac{y'}{y} = (1 + \operatorname{tg}^2 x) \cdot [1 + \ln(\operatorname{tg} x)] \Rightarrow y' = \operatorname{tg} x^{\operatorname{tg} x} (1 + \operatorname{tg}^2 x) \cdot [1 + \ln(\operatorname{tg} x)]$$

$$e) f(x) = (\cos x)^x \Rightarrow \ln y = x \cdot \ln \cos x \Rightarrow \frac{y'}{y} = \ln(\cos x) + x \cdot \frac{-\sin x}{\cos x} = \ln(\cos x) - x \operatorname{tg} x$$

$$y' = (\cos x)^x \cdot [\ln(\cos x) - x \operatorname{tg} x]$$

$$f) f(x) = (\operatorname{tg} x)^x \Rightarrow \ln y = x \ln(\operatorname{tg} x) \Rightarrow \frac{y'}{y} = \ln \operatorname{tg} x + \frac{(1 + \operatorname{tg}^2 x)}{\operatorname{tg} x} \Rightarrow y' = (\operatorname{tg} x)^x \cdot \left[\ln \operatorname{tg} x + \frac{(1 + \operatorname{tg}^2 x)}{\operatorname{tg} x}\right]$$