

1 Calcular la derivada de las siguientes funciones:

$$a) y = \frac{x+1}{3x-5}$$

$$b) y = \frac{x^2+5}{2x-3}$$

$$c) y = \frac{10}{1+x^2}$$

$$d) y = \frac{x^3+1}{x^2-x-2}$$

$$e) y = (x^2+a^2)^5$$

$$f) y = \sqrt{x^2+a^2}$$

$$g) y = (1+\sqrt{x+a})^2$$

$$h) y = -\frac{1}{\sqrt{x-a}}$$

$$i) y = (x-\sqrt{1-x^2})^2$$

$$j) y = \sqrt[6]{2x^5-3x^2}$$

$$k) y = x^2\sqrt{x^3}$$

$$l) y = \sqrt{\frac{1+x}{1-x}}$$

$$m) y = \frac{1-x}{\sqrt{1-x^2}}$$

$$n) y = \frac{a+\sqrt{x}}{a-\sqrt{x}}$$

$$o) y = \sqrt{\frac{a+x}{a-x}}$$

SOLUCIÓN:

$$a) y = \frac{x+1}{3x-5}$$

$$y' = \frac{3x-5-(x+1)\cdot 3}{(3x-5)^2} = \frac{3x-5-3x-3}{(3x-5)^2} = \frac{-8}{(3x-5)^2}$$

$$b) y = \frac{x^2+5}{2x-3}$$

$$y' = \frac{2x(2x-3)-(x^2+5)\cdot 2}{(2x-3)^2} = \frac{4x^2-6x-2x^2-10}{(2x-3)^2} = \frac{2x^2-6x-10}{(2x-3)^2}$$

$$c) y = \frac{10}{1+x^2}$$

$$y' = \frac{-10\cdot 2x}{(1+x^2)^2} = \frac{-20x}{(1+x^2)^2}$$

$$d) y = \frac{x^3+1}{x^2-x-2}$$

$$y' = \frac{3x^2(x^2-x-2)-(x^3+1)(2x-1)}{(x^2-x-2)^2} = \frac{3x^4-3x^3-6x^2-2x^4+x^3-2x+1}{(x^2-x-2)^2} = \frac{x^4-2x^3-6x^2-2x+1}{(x^2-x-2)^2} =$$

$$= \frac{(x^2-4x+1)(x+1)^2}{(x-2)^2(x+1)^2} = \frac{x^2-4x+1}{(x-2)^2}$$

$$e) y = (x^2+a^2)^5$$

$$y' = 5 \cdot (x^2+a^2)^4 \cdot 2x = 10x \cdot (x^2+a^2)^4$$

$$f) y = \sqrt{x^2 + a^2}$$

$$y' = \frac{2x}{2\sqrt{x^2 + a^2}} = \frac{x}{\sqrt{x^2 + a^2}}$$

$$g) y = (1 + \sqrt{x+a})^2$$

$$y' = 2(1 + \sqrt{x+a}) \cdot \frac{1}{2\sqrt{x+a}} = \frac{1 + \sqrt{x+a}}{\sqrt{x+a}}$$

$$h) y = -\frac{1}{\sqrt{x-a}}$$

$$y' = -\frac{1}{\sqrt{x-a}} = -\frac{1}{x-a} \cdot \frac{1}{2\sqrt{x-a}} = \frac{1}{2(x-a)\sqrt{x-a}} = \frac{\sqrt{x-a}}{2(x-a)^2}$$

$$i) y = (x - \sqrt{1-x^2})^2$$

$$\begin{aligned} y' &= 2(x - \sqrt{1-x^2}) \left(1 - \frac{-2x}{2\sqrt{1-x^2}} \right) = 2(x - \sqrt{1-x^2}) \left(1 + \frac{x}{\sqrt{1-x^2}} \right) = 2(x - \sqrt{1-x^2}) \left(\frac{\sqrt{1-x^2} + x}{\sqrt{1-x^2}} \right) = \\ &= 2 \left(\frac{x^2 - 1 + x^2}{\sqrt{1-x^2}} \right) = \frac{2(2x^2 - 1)}{\sqrt{1-x^2}} \end{aligned}$$

$$j) y = \sqrt[6]{2x^5 - 3x^2}$$

$$y' = \frac{1}{6\sqrt[6]{(2x^5 - 3x^2)^5}} \cdot (10x^4 - 6x) = \frac{5x^4 - 3x}{3\sqrt[6]{(2x^5 - 3x^2)^5}}$$

$$k) y = x^2 \sqrt{x^3}$$

$$y = x^2 \sqrt{x^3} = x^3 \sqrt{x} = x^{\frac{7}{2}} \quad \rightarrow y' = \frac{7}{2} x^{\frac{5}{2}} = \frac{7}{2} x^2 \sqrt{x}$$

$$l) y = \sqrt{\frac{1+x}{1-x}}$$

$$y' = \frac{1}{2\sqrt{\frac{1+x}{1-x}}} \cdot \frac{1-x - (1+x)(-1)}{(1-x)^2} = \frac{\sqrt{1-x}}{2\sqrt{1+x}} \cdot \frac{2}{(1-x)^2} = \frac{1}{\sqrt{1+x}\sqrt{1-x}(1-x)} = \frac{1}{\sqrt{1-x^2}(1-x)}$$

$$m) y = \frac{1-x}{\sqrt{1-x^2}}$$

$$y' = \frac{-\sqrt{1-x^2} - (1-x) \cdot \frac{-2x}{2\sqrt{1-x^2}}}{1-x^2} = \frac{-\sqrt{1-x^2} + (1-x) \cdot \frac{x}{\sqrt{1-x^2}}}{1-x^2} = \frac{-(1-x^2) + x - x^2}{\sqrt{1-x^2} \cdot 1-x^2} =$$

$$= \frac{x-1}{(1-x^2)\sqrt{1-x^2}} = \frac{(x-1)\sqrt{1-x^2}}{(1-x^2)^2} = \frac{\sqrt{1-x^2}}{(x+1)^2(x-1)}$$

$$n) y = \frac{a+\sqrt{x}}{a-\sqrt{x}}$$

$$y' = \frac{a+\sqrt{x}}{a-\sqrt{x}} = \frac{\frac{1}{2\sqrt{x}}(a-\sqrt{x}) - (a+\sqrt{x})\left(-\frac{1}{2\sqrt{x}}\right)}{(a-\sqrt{x})^2} = \frac{a-\sqrt{x}+a+\sqrt{x}}{2\sqrt{x}(a-\sqrt{x})^2} = \frac{a}{\sqrt{x}(a-\sqrt{x})^2}$$

$$o) y = \sqrt{\frac{a+x}{a-x}}$$

$$y' = \sqrt{\frac{a+x}{a-x}} \cdot \frac{1}{2\sqrt{\frac{a+x}{a-x}}} \cdot \frac{(a-x) - (a+x)(-1)}{(a-x)^2} = \frac{\sqrt{a-x}}{2\sqrt{a+x}} \cdot \frac{2a}{(a-x)^2} = \frac{a}{(a-x)\sqrt{a^2-x^2}}$$

2 Calcular la derivada de las siguientes funciones trigonométricas:

a) $y = \text{sen}^3 x$

b) $y = \text{sen } x^3$

c) $y = \text{sen} \frac{x}{2} \cos \frac{x}{2}$

d) $y = \frac{\cos x}{1 - \text{sen } x}$

e) $y = \frac{\text{sen } x}{1 + \text{tg}^2 x}$

f) $y = \sqrt{1 + \text{sen } x}$

g) $y = \frac{\cos x + \text{sen } x}{\cos x - \text{sen } x}$

h) $y = \frac{1 - \text{tg}^2 x}{1 + \text{tg}^2 x}$

i) $y = \text{arcsen} \frac{x}{2}$

j) $y = \text{arccos} \sqrt{1-x^2}$

k) $y = \text{arctg} \frac{2x}{1-x^2}$

l) $y = \text{arctg} \frac{a+x}{1-ax}$

SOLUCIÓN:

a) $y = \text{sen}^3 x$

$$y' = 3 \text{sen}^2 x \cos x$$

b) $y = \text{sen } x^3$

$$y' = \cos x^3 \cdot 3x^2 = 3x^2 \cdot \cos x^3$$

$$c) y = \operatorname{sen} \frac{x}{2} \cos \frac{x}{2}$$

$$y = \operatorname{sen} \frac{x}{2} \cos \frac{x}{2} = \frac{1}{2} \operatorname{sen} x \rightarrow y' = \frac{1}{2} \cos x$$

$$d) y = \frac{\cos x}{1 - \operatorname{sen} x}$$

$$y' = \frac{-\operatorname{sen} x(1 - \operatorname{sen} x) - \cos x(-\cos x)}{(1 - \operatorname{sen} x)^2} = \frac{-\operatorname{sen} x + \operatorname{sen}^2 x + \cos^2 x}{(1 - \operatorname{sen} x)^2} = \frac{1 - \operatorname{sen} x}{(1 - \operatorname{sen} x)^2} = \frac{1}{1 - \operatorname{sen} x}$$

$$e) y = \frac{\operatorname{sen} x}{1 + \operatorname{tg}^2 x}$$

$$y' = \frac{\cos x(1 + \operatorname{tg}^2 x) - \operatorname{sen} x \cdot 2 \operatorname{tg} x(1 + \operatorname{tg}^2 x)}{(1 + \operatorname{tg}^2 x)^2} = \frac{\cos x - 2 \operatorname{sen} x \operatorname{tg} x}{1 + \operatorname{tg}^2 x} = \frac{\cos x - 2 \frac{\operatorname{sen}^2 x}{\cos x}}{\frac{1}{\cos^2 x}} =$$

$$= \frac{\cos^2 x - 2 \operatorname{sen}^2 x}{\frac{1}{\cos^2 x}} = \cos x(1 - \operatorname{sen}^2 x - 2 \operatorname{sen}^2 x) = \cos x(1 - 3 \operatorname{sen}^2 x)$$

$$f) y = \sqrt{1 + \operatorname{sen} x}$$

$$y' = \frac{1}{2\sqrt{1 + \operatorname{sen} x}} \cdot \cos x = \frac{\cos x}{2\sqrt{1 + \operatorname{sen} x}}$$

$$g) y = \frac{\cos x + \operatorname{sen} x}{\cos x - \operatorname{sen} x}$$

$$y' = \frac{(-\operatorname{sen} x + \cos x)(\cos x - \operatorname{sen} x) - (\cos x + \operatorname{sen} x)(-\operatorname{sen} x - \cos x)}{(\cos x - \operatorname{sen} x)^2} =$$

$$= \frac{(\cos x - \operatorname{sen} x)^2 + (\cos x + \operatorname{sen} x)^2}{(\cos x - \operatorname{sen} x)^2} = \frac{(1 - 2 \operatorname{sen} x \cos x) + (1 + 2 \operatorname{sen} x \cos x)}{1 - 2 \operatorname{sen} x \cos x} = \frac{2}{1 - \operatorname{sen} 2x}$$

$$h) y = \frac{1 - \operatorname{tg}^2 x}{1 + \operatorname{tg}^2 x}$$

$$y' = \frac{-2 \operatorname{tg} x(1 + \operatorname{tg}^2 x)^2 - (1 - \operatorname{tg}^2 x) 2 \operatorname{tg} x(1 + \operatorname{tg}^2 x)}{(1 + \operatorname{tg}^2 x)^2} = \frac{-2 \operatorname{tg} x(1 + \operatorname{tg}^2 x + 1 - \operatorname{tg}^2 x)}{1 + \operatorname{tg}^2 x} = \frac{-4 \operatorname{tg} x}{1 + \operatorname{tg}^2 x} = -4 \operatorname{sen} x \cos x$$

$$i) y = \operatorname{arcsen} \frac{x}{2}$$

$$y' = \frac{1}{2} \cdot \frac{1}{\sqrt{1 - \frac{x^2}{4}}} = \frac{1}{2} \cdot \frac{1}{\frac{\sqrt{4 - x^2}}{2}} = \frac{1}{\sqrt{4 - x^2}}$$

$$j) y = \arccos \sqrt{1-x^2}$$

$$y' = \frac{-1}{\sqrt{1-(\sqrt{1-x^2})^2}} \cdot \frac{-2x}{2\sqrt{1-x^2}} = \frac{x}{x\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}}$$

$$k) y = \operatorname{arctg} \frac{2x}{1-x^2}$$

$$y' = \frac{1}{1+\left(\frac{2x}{1-x^2}\right)^2} \cdot \frac{2(1-x^2)-2x(-2x)}{(1-x^2)^2} = \frac{1}{\frac{1+x^4-2x^2+4x^2}{(1-x^2)^2}} \cdot \frac{2-2x^2+4x^2}{(1-x^2)^2} = \frac{2x^2+2}{x^4+2x^2+1} =$$

$$= \frac{2(x^2+1)}{(x^2+1)^2} = \frac{2}{x^2+1}$$

$$l) y = \operatorname{arctg} \frac{a+x}{1-ax}$$

$$y' = \frac{1}{1+\left(\frac{a+x}{1-ax}\right)^2} \cdot \frac{(1-ax)-(a+x)(-a)}{(1-ax)^2} = \frac{1}{\frac{1+a^2x^2-2ax+a^2+x^2+2ax}{(1-ax)^2}} \cdot \frac{1-ax+a^2+ax}{(1-ax)^2} =$$

$$= \frac{1+a^2}{1+a^2+a^2x^2+x^2} = \frac{1+a^2}{(1+a^2)+(a^2+1)x^2} = \frac{1+a^2}{(1+a^2)(x^2+1)} = \frac{1}{x^2+1}$$

3 Calcular la derivada de las siguientes funciones logarítmicas y exponenciales:

$$a) y = \frac{1+\ln x}{x}$$

$$b) y = (x-1) \cdot e^x$$

$$c) y = \ln(\ln x)$$

$$d) y = \log_a(3x^2+5)$$

$$e) y = \ln\left(\frac{1+x^2}{1-x^2}\right)$$

$$f) y = \ln\left(\frac{e^x}{1+e^x}\right)$$

$$g) y = \ln(x+\sqrt{1+x^2})$$

$$h) y = (x^2-2x) \cdot e^x$$

$$i) y = \frac{e^x-1}{e^x+1}$$

$$j) y = 2a \ln(a+x) - x$$

$$k) y = \frac{x^3}{e^x}$$

$$l) y = e^x \cdot \cos x$$

$$m) y = \frac{\ln x}{x^3}$$

$$n) y = \ln\left(\frac{x}{\sqrt{1+x^2}}\right)$$

$$o) y = 2^x \cdot x^2$$

SOLUCIÓN:

$$a) y = \frac{1+\ln x}{x}$$

$$y' = \frac{1+\ln x}{x} = \frac{1}{x^2} \cdot \left(\frac{1}{x} \cdot x - (1+\ln x)\right) = \frac{-\ln x}{x^2}$$

$$b) y = (x - 1) \cdot e^x$$

$$y' = e^x + (x - 1) e^x = x e^x$$

$$c) y = \ln(\ln x)$$

$$y' = \frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x \cdot \ln x}$$

$$d) y = \log_a(3x^2 + 5)$$

$$y' = \frac{6x}{3x^2 + 5} \cdot \frac{1}{\ln a}$$

$$e) y = \ln\left(\frac{1+x^2}{1-x^2}\right)$$

$$y' = \frac{1-x^2}{1+x^2} \cdot \frac{2x(1-x^2) - (1+x^2)(-2x)}{(1-x^2)^2} = \frac{1}{1+x^2} \cdot \frac{2x(1-x^2+1+x^2)}{1-x^2} = \frac{4x}{1-x^4}$$

$$f) y = \ln\left(\frac{e^x}{1+e^x}\right)$$

$$y' = \frac{1+e^x}{e^x} \cdot \frac{e^x(1+e^x) - e^x \cdot e^x}{(1+e^x)^2} = \frac{1}{e^x} \cdot \frac{e^x(1+e^x - e^x)}{1+e^x} = \frac{1}{1+e^x}$$

$$g) y = \ln(x + \sqrt{1+x^2})$$

$$y' = \frac{1}{x + \sqrt{1+x^2}} \cdot \left(1 + \frac{2x}{2\sqrt{1+x^2}}\right) = \frac{1}{x + \sqrt{1+x^2}} \cdot \left(\frac{\sqrt{1+x^2} + x}{\sqrt{1+x^2}}\right) = \frac{1}{\sqrt{1+x^2}}$$

$$h) y = (x^2 - 2x) \cdot e^x$$

$$y' = (2x - 2) e^x + (x^2 - 2x) \cdot e^x = (2x - 2 + x^2 - 2x) \cdot e^x = (x^2 - 2) \cdot e^x$$

$$i) y = \frac{e^x - 1}{e^x + 1}$$

$$y' = \frac{e^x - 1}{e^x + 1} = \frac{e^x(e^x + 1) - (e^x - 1)e^x}{(e^x + 1)^2} = \frac{e^x(e^x + 1 - e^x + 1)}{(e^x + 1)^2} = \frac{2e^x}{(e^x + 1)^2}$$

$$j) y = 2a \ln(a + x) - x$$

$$y' = \frac{2a}{a+x} - 1 = \frac{2a - a - x}{a+x} = \frac{a-x}{a+x}$$

$$k) y = \frac{x^3}{e^x}$$

$$y' = \frac{3x^2 \cdot e^x - x^3 \cdot e^x}{e^{2x}} = \frac{(3x^2 - x^3) \cdot e^x}{e^{2x}} = \frac{3x^2 - x^3}{e^x}$$

$$l) y = e^x \cdot \cos x$$

$$y' = e^x \cdot \cos x + e^x \cdot (-\operatorname{sen} x) = e^x (\cos x - \operatorname{sen} x)$$

$$m) y = \frac{\ln x}{x^3}$$

$$y' = \frac{\frac{1}{x} \cdot x^3 - \ln x (3x^2)}{x^6} = \frac{x^2(1-3\ln x)}{x^6} = \frac{1-3\ln x}{x^4}$$

$$n) y = \ln\left(\frac{x}{\sqrt{1+x^2}}\right)$$

$$y' = \frac{\frac{\sqrt{1+x^2}}{x} \cdot \frac{\sqrt{1+x^2} - \frac{x}{\sqrt{1+x^2}} \cdot 2x}{1+x^2}}{x} = \frac{\sqrt{1+x^2}}{x} \cdot \frac{1+x^2-x^2}{(1+x^2)\sqrt{1+x^2}} = \frac{1}{x(1+x^2)}$$

$$o) y = 2^x \cdot x^2$$

$$y' = 2^x \cdot \ln 2 \cdot x^2 + 2^x \cdot 2x = 2^x x (x \ln 2 + 2)$$

4

Calcular la derivada de las siguientes funciones, aplicando la derivación logarítmica:

a) $y = x^x$

b) $y = x^{\ln x}$

c) $y = x^{\cos x}$

d) $y = (\operatorname{tg} x)^{\operatorname{tg} x}$

SOLUCIÓN:

a) $y = x^x$

$$\ln y = \ln x^x \rightarrow \ln y = x \ln x \rightarrow \frac{y'}{y} = \ln x + x \cdot \frac{1}{x} \rightarrow y' = y(1 + \ln x) \rightarrow y' = (1 + \ln x) \cdot x^x$$

b) $y = x^{\ln x}$

$$\ln y = \ln x^{\ln x} \rightarrow \ln y = (\ln x)^2 \rightarrow \frac{y'}{y} = 2 \ln x \cdot \frac{1}{x} \rightarrow y' = y \left(\frac{2}{x} \cdot \ln x \right) \rightarrow y' = \left(\frac{2}{x} \cdot \ln x \right) \cdot x^{\ln x}$$

c) $y = x^{\cos x}$

$$\ln y = \cos x \cdot \ln x \rightarrow \frac{y'}{y} = -\operatorname{sen} x \cdot \ln x + \cos x \cdot \frac{1}{x}$$

$$y' = y \left(-\operatorname{sen} x \cdot \ln x + \cos x \cdot \frac{1}{x} \right) \rightarrow y' = \left(-\operatorname{sen} x \cdot \ln x + \cos x \cdot \frac{1}{x} \right) \cdot x^{\cos x}$$

d) $y = (\operatorname{tg} x)^{\operatorname{tg} x}$

$$\ln y = \operatorname{tg} x \cdot \ln(\operatorname{tg} x) \rightarrow \frac{y'}{y} = (1 + \operatorname{tg}^2 x) \cdot \ln(\operatorname{tg} x) + \operatorname{tg} x \cdot \frac{1}{\operatorname{tg} x} (1 + \operatorname{tg}^2 x) \rightarrow \frac{y'}{y} = (1 + \operatorname{tg}^2 x) \cdot (1 + \ln(\operatorname{tg} x))$$

$$y' = (1 + \operatorname{tg}^2 x) \cdot (1 + \ln(\operatorname{tg} x)) \cdot \operatorname{tg} x^{\operatorname{tg} x}$$

4

Calcular la derivada de las siguientes funciones, simplificando el resultado:

a) $y = (x - \sqrt{1-x^2})^2$

b) $y = \sqrt{\frac{1-\cos x}{1+\cos x}}$

c) $y = \ln \sqrt{\frac{1+\operatorname{sen} x}{1-\operatorname{sen} x}}$

d) $y = \ln \left(\frac{x^2}{\sqrt{1-x^2}} \right)$

e) $y = \ln \left(\frac{1+\operatorname{sen} x}{\operatorname{tg} x} \right)$

f) $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

SOLUCIÓN:

a) $y = (x - \sqrt{1-x^2})^2$

$$y' = 2(x - \sqrt{1-x^2}) \left(1 - \frac{-2x}{2\sqrt{1-x^2}} \right) = 2(x - \sqrt{1-x^2}) \left(\frac{x + \sqrt{1-x^2}}{\sqrt{1-x^2}} \right) = \frac{2(x^2 - 1 + x^2)}{\sqrt{1-x^2}} = \frac{2(2x^2 - 1)}{\sqrt{1-x^2}}$$

b) $y = \sqrt{\frac{1-\cos x}{1+\cos x}}$

$$y' = \frac{1}{2\sqrt{\frac{1-\cos x}{1+\cos x}}} \cdot \frac{\operatorname{sen} x(1+\cos x) - (1-\cos x)(-\operatorname{sen} x)}{(1+\cos x)^2} = \frac{\sqrt{1+\cos x}}{2\sqrt{1-\cos x}} \cdot \frac{2\operatorname{sen} x}{(1+\cos x)^2} =$$

Multiplicamos y dividimos por $\sqrt{1+\cos x}$

$$= \frac{1+\cos x}{\sqrt{1-\cos^2 x}} \cdot \frac{\operatorname{sen} x}{(1+\cos x)^2} = \frac{1}{1+\cos x}$$

c) $y = \ln \sqrt{\frac{1+\operatorname{sen} x}{1-\operatorname{sen} x}}$

$$y' = \frac{1}{\sqrt{\frac{1+\operatorname{sen} x}{1-\operatorname{sen} x}}} \cdot \frac{1}{2\sqrt{\frac{1+\operatorname{sen} x}{1-\operatorname{sen} x}}} \cdot \frac{\cos x(1-\operatorname{sen} x) - (1+\operatorname{sen} x)(-\cos x)}{(1-\operatorname{sen} x)^2} = \frac{1}{2\left(\frac{1+\operatorname{sen} x}{1-\operatorname{sen} x}\right)} \cdot \frac{2\cos x}{(1-\operatorname{sen} x)^2} =$$

$$= \frac{1}{(1+\operatorname{sen} x)} \cdot \frac{\cos x}{(1-\operatorname{sen} x)} = \frac{\cos x}{1-\operatorname{sen}^2 x} = \frac{1}{\cos x}$$

d) $y = \ln \left(\frac{x^2}{\sqrt{1-x^2}} \right)$

$$y' = \frac{\sqrt{1-x^2}}{x^2} \cdot \frac{2x\sqrt{1-x^2} - x^2 \cdot \frac{-2x}{2\sqrt{1-x^2}}}{(\sqrt{1-x^2})^2} = \frac{\sqrt{1-x^2}}{x^2} \cdot \frac{2x(1-x^2) + x^3}{(1-x^2)\sqrt{1-x^2}} = \frac{1}{x^2} \cdot \frac{2x - x^3}{(1-x^2)} = \frac{2-x^2}{(1-x^2)x}$$

$$e) y = \ln\left(\frac{1+\operatorname{sen} x}{\operatorname{tg} x}\right)$$

$$y' = \frac{\operatorname{tg} x}{1+\operatorname{sen} x} \cdot \frac{\cos x \cdot \operatorname{tg} x - (1+\operatorname{sen} x)(1+\operatorname{tg}^2 x)}{\operatorname{tg}^2 x} = \frac{1}{1+\operatorname{sen} x} \cdot \frac{\operatorname{sen} x - \frac{1+\operatorname{sen} x}{\cos^2 x}}{\operatorname{tg} x} = \frac{1}{1+\operatorname{sen} x} \cdot \frac{\operatorname{sen} x - \frac{1+\operatorname{sen} x}{1-\operatorname{sen}^2 x}}{\operatorname{tg} x} =$$

$$= \frac{1}{1+\operatorname{sen} x} \cdot \frac{\operatorname{sen} x - \frac{1}{1-\operatorname{sen} x}}{\operatorname{tg} x} = \frac{1}{1+\operatorname{sen} x} \cdot \frac{\operatorname{sen} x - \operatorname{sen}^2 x - 1}{\operatorname{tg} x(1-\operatorname{sen} x)} = \frac{\operatorname{sen} x - \operatorname{sen}^2 x - 1}{\operatorname{tg} x(1-\operatorname{sen}^2 x)} = \frac{\operatorname{sen} x - \operatorname{sen}^2 x - 1}{\cos x \cdot \operatorname{sen} x}$$

$$f) y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$y' = \frac{e^x - e^{-x}}{e^x + e^{-x}} \cdot \frac{(e^x + e^{-x})(e^x + e^{-x}) - (e^x - e^{-x})(e^x - e^{-x})}{(e^x + e^{-x})^2} = \frac{(e^{2x} + e^{-2x} + 2) - (e^{2x} + e^{-2x} - 2)}{(e^x + e^{-x})^2} = \frac{4}{(e^x + e^{-x})^2}$$

5 Calcular la derivada de las siguientes funciones, simplificando el resultado:

$$a) y = \operatorname{arcsen}\left(\frac{x^2-1}{x^2}\right)$$

$$b) y = \operatorname{arctg}\left(\frac{1+\operatorname{sen} x}{\cos x}\right)$$

$$c) y = \operatorname{arccos}\left(\frac{1-x^2}{1+x^2}\right)$$

$$d) y = \operatorname{arcsen}\left(\frac{x}{\sqrt{1+x^2}}\right)$$

$$e) y = \operatorname{arctg}(x + \sqrt{1+x^2})$$

$$f) y = \operatorname{arctg}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

$$g) y = \operatorname{arcsen}\left(\frac{2\operatorname{tg} x}{1+\operatorname{tg}^2 x}\right)$$

$$h) y = \operatorname{arctg}\left(\frac{\cos x}{\sqrt{1+\operatorname{sen}^2 x}}\right)$$

$$i) y = \operatorname{arcsen}\left(\frac{1+x^2}{x}\right)$$

$$j) y = \operatorname{arcsen}\sqrt{1-x^2}$$

$$k) y = \operatorname{arctg}\sqrt{\frac{1-x}{1+x}}$$

$$l) y = \operatorname{arccos}\sqrt{1-\frac{1}{x^2}}$$

$$m) y = \operatorname{arctg}(3^x)$$

$$n) y = \operatorname{arctg}(2x+1)$$

$$o) y = \operatorname{arctg}\left(\frac{x+a}{1-ax}\right)$$

SOLUCIÓN:

$$a) y = \operatorname{arcsen}\left(\frac{x^2-1}{x^2}\right)$$

$$y' = \frac{1}{\sqrt{1-\left(\frac{x^2-1}{x^2}\right)^2}} \cdot \frac{2x \cdot x^2 - (x^2-1) \cdot 2x}{x^4} = \frac{x^4}{\sqrt{x^4 - (x^2-1)^2}} \cdot \frac{2x}{x^4} = \frac{2x}{\sqrt{2x^2-1}}$$

$$b) y = \operatorname{arctg}\left(\frac{1+\operatorname{sen} x}{\cos x}\right)$$

$$y' = \frac{1}{1+\left(\frac{1+\operatorname{sen} x}{\cos x}\right)^2} \cdot \frac{\cos^2 x - (1+\operatorname{sen} x)(-\operatorname{sen} x)}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x + (1+\operatorname{sen} x)^2} \cdot \frac{\cos^2 x + \operatorname{sen} x + \operatorname{sen}^2 x}{\cos^2 x} =$$

$$= \frac{1}{\cos^2 x + 1 + \sin^2 x + 2\sin x} \cdot (1 + \sin x) = \frac{1}{2 + 2\sin x} \cdot (1 + \sin x) = \frac{1}{2}$$

$$c) y = \arccos\left(\frac{1-x^2}{1+x^2}\right)$$

$$y' = -\frac{1}{\sqrt{1-\left(\frac{1-x^2}{1+x^2}\right)^2}} \cdot \frac{-2x \cdot (1+x^2) - (1-x^2) \cdot 2x}{(1+x^2)^2} = -\frac{1+x^2}{\sqrt{(1+x^2)^2 - (1-x^2)^2}} \cdot \frac{-4x}{(1+x^2)^2} =$$

$$= \frac{1+x^2}{\sqrt{4x^2}} \cdot \frac{4x}{(1+x^2)^2} = \frac{1}{2x} \cdot \frac{4x}{1+x^2} = \frac{2}{1+x^2}$$

$$d) y = \arcsen\left(\frac{x}{\sqrt{1+x^2}}\right)$$

$$y' = \frac{1}{\sqrt{1-\left(\frac{x}{\sqrt{1+x^2}}\right)^2}} \cdot \frac{\sqrt{1+x^2} - x \cdot \frac{x}{\sqrt{1+x^2}}}{(\sqrt{1+x^2})^2} = \frac{\sqrt{1+x^2}}{\sqrt{1+x^2-x^2}} \cdot \frac{1+x^2-x^2}{(\sqrt{1+x^2})^2 \sqrt{1+x^2}} = \frac{1}{1+x^2}$$

$$e) y = \arctg(x + \sqrt{1-x^2})$$

$$y' = \frac{1}{1+(x+\sqrt{1+x^2})^2} \cdot \left(1 + \frac{x}{\sqrt{1+x^2}}\right) = \frac{1}{1+(x+\sqrt{1+x^2})^2} \cdot \frac{\sqrt{1+x^2} + x}{\sqrt{1+x^2}} =$$

$$= \frac{1}{1+x^2+1+x^2+2x\sqrt{1+x^2}} \cdot \frac{x+\sqrt{1+x^2}}{\sqrt{1+x^2}} = \frac{1}{2(1+x^2+x\sqrt{1+x^2})} \cdot \frac{x+\sqrt{1+x^2}}{\sqrt{1+x^2}} =$$

$$= \frac{1}{2\sqrt{1+x^2}(\sqrt{1+x^2}+x)} \cdot \frac{x+\sqrt{1+x^2}}{\sqrt{1+x^2}} = \frac{1}{2(1+x^2)}$$

$$f) y = \arctg\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

$$y' = \frac{1}{1+\frac{x^2}{a^2-x^2}} \cdot \frac{\sqrt{a^2-x^2} - x \cdot \frac{-2x}{2\sqrt{a^2-x^2}}}{a^2-x^2} = \frac{a^2-x^2}{a^2-x^2+x^2} \cdot \frac{\sqrt{a^2-x^2} - x \cdot \frac{-2x}{2\sqrt{a^2-x^2}}}{a^2-x^2} =$$

$$= \frac{1}{a^2} \cdot \frac{a^2-x^2+x^2}{\sqrt{a^2-x^2}} = \frac{1}{\sqrt{a^2-x^2}}$$

$$g) y = \arcsen\left(\frac{2\operatorname{tg}x}{1+\operatorname{tg}^2x}\right)$$

$$y' = \frac{1}{\sqrt{1 - \left(\frac{2 \operatorname{tg} x}{1 + \operatorname{tg}^2 x}\right)^2}} \cdot \frac{2(1 + \operatorname{tg}^2 x)^{\cancel{2}} - 2 \operatorname{tg} x \cdot 2 \operatorname{tg} x \cdot (1 + \operatorname{tg}^2 x)}{(1 + \operatorname{tg}^2 x)^{\cancel{2}}} = \frac{1 + \operatorname{tg}^2 x}{\sqrt{(1 + \operatorname{tg}^2 x)^2 - 4 \operatorname{tg}^2 x}} \cdot \frac{2 + 2 \operatorname{tg}^2 x - 4 \operatorname{tg}^2 x}{1 + \operatorname{tg}^2 x} =$$

$$= \frac{2 - 2 \operatorname{tg}^2 x}{\sqrt{1 + \operatorname{tg}^4 x - 2 \operatorname{tg}^2 x}} = \frac{2(1 - \operatorname{tg}^2 x)}{\sqrt{(1 - \operatorname{tg}^2 x)^2}} = \frac{2(1 - \operatorname{tg}^2 x)}{(1 - \operatorname{tg}^2 x)} = 2$$

$$h) y = \operatorname{arctg}\left(\frac{\cos x}{\sqrt{1 + \operatorname{sen}^2 x}}\right)$$

$$y' = \frac{1}{1 + \frac{\cos^2 x}{1 + \operatorname{sen}^2 x}} \cdot \frac{-\operatorname{sen} x \sqrt{1 + \operatorname{sen}^2 x} - \cos x \cdot \frac{2 \operatorname{sen} x \cos x}{2 \sqrt{1 + \operatorname{sen}^2 x}}}{1 + \operatorname{sen}^2 x} =$$

$$= \frac{1 + \operatorname{sen}^2 x}{1 + \operatorname{sen}^2 x + \cos^2 x} \cdot \frac{-\operatorname{sen} x(1 + \operatorname{sen}^2 x) - \operatorname{sen} x \cos^2 x}{(1 + \operatorname{sen}^2 x) \sqrt{1 + \operatorname{sen}^2 x}} = \frac{1}{2} \cdot \frac{-\operatorname{sen} x(1 + \operatorname{sen}^2 x + \cos^2 x)}{\sqrt{1 + \operatorname{sen}^2 x}} =$$

$$= \frac{1}{2} \cdot \frac{-2 \operatorname{sen} x}{\sqrt{1 + \operatorname{sen}^2 x}} = \frac{-\operatorname{sen} x}{\sqrt{1 + \operatorname{sen}^2 x}}$$

$$i) y = \operatorname{arcsen}\left(\frac{1 + x^2}{x}\right)$$

$$y' = \frac{1}{\sqrt{1 - \left(\frac{1 + x^2}{x}\right)^2}} \cdot \frac{2x^2 - (1 + x^2)}{x^2} = \frac{x}{\sqrt{x^2 - 1 - 2x^2 - x^4}} \cdot \frac{x^2 - 1}{x^2} = \frac{x^2 - 1}{x \sqrt{-1 - x^2 - x^4}}$$

$$j) y = \operatorname{arcsen} \sqrt{1 - x^2}$$

$$y' = \frac{1}{\sqrt{1 - (\sqrt{1 - x^2})^2}} \cdot \frac{-2x}{2 \sqrt{1 - x^2}} = \frac{-x}{\sqrt{1 - 1 + x^2} \sqrt{1 - x^2}} = \frac{-1}{\sqrt{1 - x^2}}$$

$$k) y = \operatorname{arctg} \sqrt{\frac{1 - x}{1 + x}}$$

$$y' = \frac{1}{1 + \frac{1 - x}{1 + x}} \cdot \frac{1}{2 \sqrt{\frac{1 - x}{1 + x}}} \cdot \frac{-(1 + x) - (1 - x)}{(1 + x)^2} = \frac{1 + x}{1 + x + 1 - x} \cdot \frac{\sqrt{1 + x}}{2 \sqrt{1 - x}} \cdot \frac{-1 - x - 1 + x}{(1 + x)^2} = \frac{-1}{2 \sqrt{1 - x} \sqrt{1 + x}} = \frac{-1}{2 \sqrt{1 - x^2}}$$

$$l) y = \operatorname{ar} \cos \sqrt{1 - \frac{1}{x^2}}$$

$$y' = \frac{-1}{\sqrt{1 - \left(\sqrt{\frac{x^2 - 1}{x^2}}\right)^2}} \cdot \frac{1}{2 \sqrt{\frac{x^2 - 1}{x^2}}} \cdot \frac{2x^3 - 2x(x^2 - 1)}{x^4} = \frac{-1}{\sqrt{\frac{x^2 - x^2 + 1}{x^2}}} \cdot \frac{x}{2 \sqrt{x^2 - 1}} \cdot \frac{2x}{x^4} = \frac{-1}{x \sqrt{x^2 - 1}}$$

$$m) y = \operatorname{arctg}(3^x)$$

$$y' = \frac{1}{1+3^{2x}} \cdot 3^x \cdot \ln 3 = \frac{3^x \cdot \ln 3}{1+3^{2x}}$$

$$n) y = \operatorname{arctg}(2x+1)$$

$$y' = \frac{2}{1+(2x+1)^2} = \frac{2}{1+4x^2+4x+1} = \frac{1}{2x^2+2x+1}$$

$$o) y = \operatorname{arctg}\left(\frac{x+a}{1-ax}\right)$$

$$y' = \frac{1}{1+\left(\frac{x+a}{1-ax}\right)^2} \cdot \frac{1-ax-(x+a)(-a)}{(1-ax)^2} = \frac{(1-ax)^2}{1+a^2x^2+a^2+x^2} \cdot \frac{1+a^2}{(1-ax)^2} = \frac{1+a^2}{(1+a^2)(1+x^2)} = \frac{1}{1+x^2}$$

6 Calcular la derivada de las siguientes funciones, simplificando el resultado:

$$a) y = \frac{x}{2(1-x^2)} + \frac{1}{4} \ln\left(\frac{1-x}{1+x}\right)$$

$$b) y = -\frac{\cos x}{2\operatorname{sen}^2 x} + \frac{1}{2} \ln\left(\operatorname{tg} \frac{x}{2}\right)$$

$$c) y = \frac{1}{2} \operatorname{tg}^2 x + \ln(\cos x)$$

$$d) y = \ln(x + \sqrt{1+x^2}) - \frac{\sqrt{1+x^2}}{x}$$

$$e) y = \frac{3}{2} \operatorname{arcsen}(x-1) - \frac{x+3}{2} \sqrt{2x-x^2}$$

$$f) y = \operatorname{arctg}\sqrt{\frac{1+x}{1-x}} + \frac{1}{2} \operatorname{arcsen}(\sqrt{1-x^2})$$

$$g) y = \sqrt{2x-x^2} - \operatorname{arccos}(1-x)$$

$$h) y = \sqrt{x^2-1} + x^2 \operatorname{arc} \operatorname{sen}\left(\frac{1}{x}\right)$$

$$i) y = x\sqrt{a^2-x^2} + a^2 \cdot \operatorname{arc} \operatorname{sen}\left(\frac{x}{a}\right)$$

$$j) y = 2\sqrt{ax-a^2} + a \cdot \operatorname{arccos}\left(\frac{x-2a}{x}\right)$$

$$k) y = \sqrt{x^2+x} - \ln(\sqrt{x} + \sqrt{x+1})$$

$$l) y = \ln\left(\frac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}+x}\right)$$

$$m) y = \ln\left(\frac{\sqrt{e^x+1}+1}{\sqrt{e^x+1}-1}\right)$$

$$n) y = \ln\left(\sqrt[4]{\frac{\operatorname{sen} x}{1-\cos x}}\right)$$

$$o) y = \frac{3-2x}{2} + \ln(e^x \cdot \operatorname{tg} x)$$

$$p) y = \sqrt{a^2-x^2} - a \cdot \operatorname{arccos}\left(\frac{x}{a}\right)$$

$$q) y = \operatorname{arctg}\left(\frac{1-e^x}{1+e^x}\right)$$

$$r) y = \ln\left(\sqrt[3]{\frac{1+\cos x}{\operatorname{sen} x}}\right)$$

$$s) y = \ln(x^2 \cdot \sqrt{1-x^2})$$

$$t) y = \ln\left(\frac{x^2}{\sqrt{1-x^2}}\right)$$

$$u) y = \left(\frac{1+\operatorname{sen} x}{\cos x}\right)^2$$

$$v) y = -\operatorname{ctg} x + \ln(\operatorname{sen} x)$$

SOLUCIÓN:

$$a) y = \frac{x}{2(1-x^2)} + \frac{1}{4} \cdot \ln\left(\frac{1-x}{1+x}\right)$$

$$y' = \frac{1}{2} \cdot \frac{1-x^2 - x(-2x)}{(1-x^2)^2} + \frac{1}{4} \cdot \left(\frac{1+x}{1-x}\right) \cdot \frac{-(1+x) - (1-x)}{(1+x)^2} = \frac{1}{2} \cdot \frac{1+x^2}{(1-x^2)^2} + \frac{1}{4} \cdot \left(\frac{1}{1-x}\right) \cdot \frac{-2}{1+x} =$$

$$= \frac{1}{2} \cdot \frac{1+x^2}{(1-x^2)^2} - \frac{1}{2} \cdot \frac{1}{1-x^2} = \frac{1}{2} \cdot \frac{1+x^2 - 1+x^2}{(1-x^2)^2} = \frac{x^2}{(1-x^2)^2}$$

$$b) y = -\frac{\cos x}{2\operatorname{sen}^2 x} + \frac{1}{2} \ln\left(\operatorname{tg} \frac{x}{2}\right)$$

$$y' = -\frac{1}{2} \cdot \frac{-\operatorname{sen} x \cdot \operatorname{sen}^2 x - \cos x \cdot 2\operatorname{sen} x \cos x}{\operatorname{sen}^4 x} + \frac{1}{2} \cdot \frac{1}{\operatorname{tg} \frac{x}{2}} \cdot \left(1 + \operatorname{tg}^2 \frac{x}{2}\right) \cdot \frac{1}{2} = \frac{\operatorname{sen}^2 x + 2\cos^2 x}{2\operatorname{sen}^3 x} + \frac{1}{4} \cdot \frac{1}{\operatorname{tg} \frac{x}{2}} \cdot \frac{1}{\cos^2 \frac{x}{2}} =$$

$$= \frac{1 + \cos^2 x}{2\operatorname{sen}^3 x} + \frac{1}{4 \cdot \operatorname{sen} \frac{x}{2} \cos \frac{x}{2}} = \frac{1 + \cos^2 x}{2\operatorname{sen}^3 x} + \frac{1}{2\operatorname{sen} x} = \frac{1 + \cos^2 x + \operatorname{sen}^2 x}{2\operatorname{sen}^3 x} = \frac{1}{\operatorname{sen}^3 x}$$

$$c) y = \frac{1}{2} \operatorname{tg}^2 x + \ln(\cos x)$$

$$y' = \frac{1}{2} \cdot 2 \cdot \operatorname{tg} x (1 + \operatorname{tg}^2 x) + \frac{-\operatorname{sen} x}{\cos x} = \operatorname{tg} x (1 + \operatorname{tg}^2 x - 1) = \operatorname{tg}^3 x$$

$$d) y = \ln(x + \sqrt{1+x^2}) - \frac{\sqrt{1+x^2}}{x}$$

$$y' = \frac{1}{x + \sqrt{1+x^2}} \cdot \left(1 + \frac{2x}{2\sqrt{1+x^2}}\right) - \frac{\frac{2x^2}{2\sqrt{1+x^2}} - \sqrt{1+x^2}}{x^2} = \frac{1}{x + \sqrt{1+x^2}} \cdot \left(\frac{\sqrt{1+x^2} + x}{\sqrt{1+x^2}}\right) - \frac{x^2 - 1 - x^2}{x^2 \sqrt{1+x^2}} =$$

$$= \frac{1}{\sqrt{1+x^2}} + \frac{1}{x^2 \sqrt{1+x^2}} = \frac{x^2 + 1}{x^2 \sqrt{1+x^2}} = \frac{\sqrt{1+x^2}}{x^2}$$

$$e) y = \frac{3}{2} \operatorname{arcsen}(x-1) - \frac{x+3}{2} \sqrt{2x-x^2}$$

$$y' = \frac{3}{2} \cdot \frac{1}{\sqrt{1-(x-1)^2}} - \frac{1}{2} \left(\sqrt{2x-x^2} + (x+3) \cdot \frac{2-2x}{2\sqrt{2x-x^2}} \right) = \frac{3}{2\sqrt{2x-x^2}} - \frac{1}{2} \left(\frac{2x-x^2 + (x+3) \cdot (1-x)}{\sqrt{2x-x^2}} \right) =$$

$$= \frac{3}{2\sqrt{2x-x^2}} - \frac{1}{2} \cdot \frac{3-2x^2}{\sqrt{2x-x^2}} = \frac{x^2}{\sqrt{2x-x^2}}$$

$$f) y = \operatorname{arctg} \sqrt{\frac{1+x}{1-x}} + \frac{1}{2} \operatorname{arcsen}(\sqrt{1-x^2})$$

$$y' = \frac{1}{1 + \frac{1+x}{1-x}} \cdot \frac{1}{2\sqrt{\frac{1+x}{1-x}}} \cdot \frac{1-x - (1+x)(-1)}{(1-x)^2} + \frac{1}{2} \cdot \frac{1}{\sqrt{1-(1-x^2)}} \cdot \frac{-2x}{2\sqrt{1-x^2}} =$$

$$= \frac{1-x}{2} \cdot \frac{\sqrt{1-x}}{2\sqrt{1+x}} \cdot \frac{2}{(1-x)^2} - \frac{1}{2} \cdot \frac{1}{x} \cdot \frac{x}{\sqrt{1-x^2}} = \frac{1}{2\sqrt{1-x^2}} - \frac{1}{2\sqrt{1-x^2}} = 0$$

$$g) y = \sqrt{2x-x^2} - \arccos(1-x)$$

$$y' = \frac{2-2x}{2\sqrt{2x-x^2}} - \frac{1}{\sqrt{1-(1-x)^2}} = \frac{1-x}{\sqrt{2x-x^2}} - \frac{1}{\sqrt{2x-x^2}} = \frac{-x}{\sqrt{2x-x^2}}$$

$$h) y = \sqrt{x^2-1} + x^2 \arcsen\left(\frac{1}{x}\right)$$

$$y' = \frac{2x}{2\sqrt{x^2-1}} + 2x \cdot \arcsen\left(\frac{1}{x}\right) + \frac{x^2}{\sqrt{1-\frac{1}{x^2}}} \cdot \left(-\frac{1}{x^2}\right) = \frac{x}{\sqrt{x^2-1}} + 2x \cdot \arcsen\left(\frac{1}{x}\right) - \frac{x}{\sqrt{x^2-1}} = 2x \cdot \arcsen\left(\frac{1}{x}\right)$$

$$i) y = x\sqrt{a^2-x^2} + a^2 \cdot \arcsen\left(\frac{x}{a}\right)$$

$$y' = \sqrt{a^2-x^2} + x \cdot \frac{-2x}{2\sqrt{a^2-x^2}} + a^2 \cdot \frac{1}{\sqrt{1-\frac{x^2}{a^2}}} \cdot \frac{1}{a} = \sqrt{a^2-x^2} - \frac{x^2}{\sqrt{a^2-x^2}} + \frac{a^2}{\sqrt{a^2-x^2}} =$$

$$= \sqrt{a^2-x^2} + \frac{a^2-x^2}{\sqrt{a^2-x^2}} = 2\sqrt{a^2-x^2}$$

$$j) y = 2\sqrt{ax-a^2} + a \cdot \arccos\left(\frac{x-2a}{x}\right)$$

$$y' = 2 \frac{a}{2\sqrt{ax-a^2}} + a \cdot \frac{-1}{\sqrt{1-\left(\frac{x-2a}{x}\right)^2}} \cdot \frac{x-(x-2a)}{x^2} = \frac{a}{\sqrt{ax-a^2}} - \frac{ax}{\sqrt{x^2-x^2+4ax-4a^2}} \cdot \frac{2a}{x^2} =$$

$$= \frac{a}{\sqrt{ax-a^2}} - \frac{2a^2}{2x\sqrt{ax-a^2}} = \frac{ax-a^2}{\sqrt{ax-a^2}} = \sqrt{ax-a^2}$$

$$k) y = \sqrt{x^2+x} - \ln(\sqrt{x} + \sqrt{x+1})$$

$$y' = \frac{2x+1}{2\sqrt{x^2+x}} - \frac{1}{\sqrt{x} + \sqrt{x+1}} \cdot \left(\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{x+1}}\right) = \frac{2x+1}{2\sqrt{x^2+x}} - \frac{1}{\sqrt{x} + \sqrt{x+1}} \cdot \frac{2(\sqrt{x} + \sqrt{x+1})}{4\sqrt{x^2+x}} =$$

$$= \frac{2x+1}{2\sqrt{x^2+x}} - \frac{1}{2\sqrt{x^2+x}} = \frac{2x}{2\sqrt{x^2+x}} = \frac{x}{\sqrt{x^2+x}}$$

$$l) y = \ln \left(\frac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}+x} \right)$$

$$y' = \frac{\sqrt{x^2+1}+x \cdot \left(\frac{2x}{2\sqrt{x^2+1}} - 1 \right) \cdot (\sqrt{x^2+1}+x) - (\sqrt{x^2+1}-x) \cdot \left(\frac{2x}{2\sqrt{x^2+1}} + 1 \right)}{(\sqrt{x^2+1}+x)^2} =$$

$$= \frac{1}{\sqrt{x^2+1}-x} \cdot \frac{\left(\frac{x-\sqrt{x^2+1}}{\sqrt{x^2+1}} \right) \cdot (\sqrt{x^2+1}+x) - (\sqrt{x^2+1}-x) \cdot \left(\frac{x+\sqrt{x^2+1}}{\sqrt{x^2+1}} \right)}{\sqrt{x^2+1}+x} =$$

$$= \frac{\left(\frac{x^2-x^2-1}{\sqrt{x^2+1}} \right) - \left(\frac{x^2+1-x^2}{\sqrt{x^2+1}} \right)}{x^2+1-x^2} = \frac{-2}{\sqrt{x^2+1}}$$

$$m) y = \ln \left(\frac{\sqrt{e^x+1}+1}{\sqrt{e^x+1}-1} \right)$$

$$y' = \frac{\sqrt{e^x+1}-1 \cdot \frac{e^x}{2\sqrt{e^x+1}} \cdot (\sqrt{e^x+1}-1) - (\sqrt{e^x+1}+1) \cdot \frac{e^x}{2\sqrt{e^x+1}}}{(\sqrt{e^x+1}-1)^2} =$$

$$= \frac{1}{\sqrt{e^x+1}+1} \cdot \frac{\frac{e^x}{2\sqrt{e^x+1}} \cdot (\sqrt{e^x+1}-1-\sqrt{e^x+1}-1)}{\sqrt{e^x+1}-1} = \frac{-2e^x}{e^x+1-1} = \frac{-e^x}{e^x\sqrt{e^x+1}} = \frac{-1}{\sqrt{e^x+1}}$$

$$n) y = \ln \left(\sqrt[4]{\frac{\sin x}{1-\cos x}} \right)$$

$$y' = \frac{1}{\sqrt[4]{\frac{\sin x}{1-\cos x}}} \cdot \frac{1}{4 \sqrt[4]{\left(\frac{\sin x}{1-\cos x} \right)^3}} \cdot \frac{\cos x(1-\cos x) - \sin x \cdot \sin x}{(1-\cos x)^2} = \frac{1}{\frac{\sin x}{1-\cos x}} \cdot \frac{\cos x - 1}{(1-\cos x)^2} = \frac{-1}{\sin x}$$

$$o) y = \frac{3-2x}{2} + \ln(e^x \cdot \operatorname{tg} x)$$

$$y' = -1 + \frac{e^x \cdot \operatorname{tg} x + e^x \cdot (1+\operatorname{tg}^2 x)}{e^x \cdot \operatorname{tg} x} = -1 + 1 + \frac{1+\operatorname{tg}^2 x}{\operatorname{tg} x} = \frac{1}{\cos^2 x} \cdot \frac{\sin x}{\cos x} = \frac{1}{\sin x \cdot \cos x}$$

$$p) y = \sqrt{a^2-x^2} - a \cdot \arccos\left(\frac{x}{a}\right)$$

$$y' = \frac{-2x}{2\sqrt{a^2-x^2}} - \frac{-a}{\sqrt{1-\left(\frac{x}{a}\right)^2}} \cdot \left(\frac{1}{a}\right) = \frac{-x}{\sqrt{a^2-x^2}} + \frac{a}{\sqrt{a^2-x^2}} = \frac{(a-x)\sqrt{a^2-x^2}}{a^2-x^2} = \frac{\sqrt{a^2-x^2}}{a+x}$$

$$q) y = \operatorname{arctg}\left(\frac{1-e^x}{1+e^x}\right)$$

$$y' = \frac{1}{1+\left(\frac{1-e^x}{1+e^x}\right)^2} \cdot \frac{-e^x \cdot (1+e^x) - (1-e^x) \cdot e^x}{(1+e^x)^2} = \frac{-e^x \cdot (1+e^x+1-e^x)}{(1+e^x)^2 + (1-e^x)^2} = \frac{-2e^x}{2+2e^{2x}} = \frac{-e^x}{1+e^{2x}}$$

$$r) y = \ln\left(\sqrt[3]{\frac{1+\cos x}{\operatorname{sen} x}}\right)$$

$$y' = \frac{1}{\sqrt[3]{\frac{1+\cos x}{\operatorname{sen} x}}} \cdot \frac{1}{3\sqrt[3]{\left(\frac{1+\cos x}{\operatorname{sen} x}\right)^2}} \cdot \frac{-\operatorname{sen}^2 x - (1+\cos x)\cos x}{\operatorname{sen}^2 x} = \frac{\operatorname{sen} x}{3(1+\cos x)} \cdot \frac{-1-\cos x}{\operatorname{sen}^2 x} = \frac{-1}{3\operatorname{sen} x}$$

$$s) y = \ln(x^2 \cdot \sqrt{1-x^2})$$

$$y' = \frac{1}{x^2 \sqrt{1-x^2}} \cdot \left(2x\sqrt{1-x^2} + x^2 \cdot \frac{-2x}{2\sqrt{1-x^2}}\right) = \frac{1}{x^2 \sqrt{1-x^2}} \cdot \left(2x\sqrt{1-x^2} - \frac{x^3}{\sqrt{1-x^2}}\right) = \frac{1}{x^2 \sqrt{1-x^2}} \cdot \left(\frac{2x-3x^3}{\sqrt{1-x^2}}\right) = \frac{2-3x^2}{x(1-x^2)}$$

$$t) y = \ln\left(\frac{x^2}{\sqrt{1-x^2}}\right)$$

$$y' = \frac{\sqrt{1-x^2}}{x^2} \cdot \frac{2x\sqrt{1-x^2} - x^2 \cdot \frac{-2x}{2\sqrt{1-x^2}}}{1-x^2} = \frac{\sqrt{1-x^2}}{x^2} \cdot \frac{2x\sqrt{1-x^2} + \frac{x^3}{\sqrt{1-x^2}}}{1-x^2} = \frac{\sqrt{1-x^2}}{x^2} \cdot \frac{2x-2x^3+x^3}{(1-x^2)\sqrt{1-x^2}} = \frac{1}{x^2} \cdot \frac{2x-x^3}{(1-x^2)} = \frac{2-x^2}{x(1-x^2)}$$

$$u) y = \left(\frac{1+\operatorname{sen} x}{\cos x}\right)^2$$

$$y' = 2 \cdot \left(\frac{1+\operatorname{sen} x}{\cos x}\right) \cdot \frac{\cos^2 x - (1+\operatorname{sen} x)(-\operatorname{sen} x)}{\cos^2 x} = 2 \cdot \left(\frac{1+\operatorname{sen} x}{\cos x}\right) \cdot \frac{1+\operatorname{sen} x}{\cos^2 x} = 2 \left(\frac{1+\operatorname{sen} x}{\cos x}\right)^2$$

$$v) y = -x \operatorname{ctg} x + \ln(\operatorname{sen} x)$$

$$(\operatorname{ctg} x)' = \left(\frac{\cos x}{\operatorname{sen} x}\right)' = \frac{-\operatorname{sen}^2 x - \cos^2 x}{\operatorname{sen}^2 x} = -1 - \operatorname{ctg}^2 x = \frac{-1}{\operatorname{sen}^2 x}$$

$$\ln(\operatorname{sen} x)' = \frac{\cos x}{\operatorname{sen} x} = \operatorname{ctg} x$$

$$y' = -\operatorname{ctg} x + x(1 + \operatorname{ctg}^2 x) + \operatorname{ctg} x = x(1 + \operatorname{ctg}^2 x)$$