

EJERCICIOS: INTEGRALES INMEDIATAS.

Resuelve las siguientes integrales inmediatas:

1. $\int (3x^3 - 5x^2 + 3x + 4) dx$

2. $\int (\operatorname{sen} x + 5 \cos x - 1) dx$

3. $\int \operatorname{tg}^2 x dx$

4. $\int (\sqrt{x} - 3) dx$

5. $\int \frac{2}{\sqrt{x}} dx$

6. $\int \frac{x^3 - 3x^2 + 5x}{x} dx$

7. $\int (5x - 2)^2 dx$

8. $\int \frac{(3x - 1)^2}{3x} dx$

9. $\int (2\sqrt{x} - \sqrt[3]{x} - x^5) dx$

10. $\int \left(\frac{5}{x} - \frac{x}{5} \right) dx$

11. $\int \frac{2}{1+x^2} dx$

12. $\int (x-5)(x+2) dx$

13. $\int \frac{5e^x + e^{2x}}{e^x} dx$

14. $\int e^{ax+b} dx$

15. $\int \cos(ax+b) dx$

16. $\int \left(\frac{a}{x} - b \right) dx$ $a, b \in \mathbb{R}$ en los tres últimos ejercicios.

Resuelve las siguientes integrales, eligiendo un cambio de variable adecuado:

1. $\int \operatorname{sen} x \cos x dx$

2. $\int \frac{\operatorname{arc\,tg} x}{1+x^2} dx$
3. $\int (x^2-x-1)^5 \cdot (2x-1) dx$
4. $\int \frac{x}{x^2-1} dx$
5. $\int \frac{4x-3}{2x^2-3x-14} dx$
6. $\int \frac{\operatorname{tg} x}{\cos^2 x} dx$
7. $\int 2x \cdot \cos(x^2) dx$
8. $\int (1-\cos x)^3 \cdot \operatorname{sen} x dx$
9. $\int \frac{1}{x \ln x} dx$
10. $\int \operatorname{tg} x dx$
11. $\int x^2 \cdot e^{x^3} dx$
12. $\int \frac{2x}{1+x^4} dx$
13. $\int \frac{e^x}{5+e^x} dx$
14. $\int \frac{\operatorname{sen} x}{e^{\cos x}} dx$

Resuelve, integrando por partes:

1. $\int e^x \operatorname{sen} x dx$
2. $\int (5x-2) \operatorname{sen} x dx$
3. $\int \ln 3x dx$
4. $\int x \cos x dx$
5. $\int x \cdot e^x dx$
6. $\int x^2 \cdot e^x dx$

$$7. \int (2x+4)e^{2x+4} dx$$

$$8. \int \operatorname{arc\,tg} x dx$$

$$9. \int \operatorname{arc\,sen} x dx$$

$$10. \int \frac{x}{e^x} dx$$

Resuelve las siguientes integrales racionales:

$$1. \int \frac{dx}{x^2-1}$$

$$2. \int \frac{x-1}{x^2+x-6} dx$$

$$3. \int \frac{x-5}{(x-1)^3} dx$$

$$4. \int \frac{x+2}{x^3-2x^2} dx$$

$$5. \int \frac{1}{(x-1)(x+2)^2} dx$$

$$6. \int \frac{x^3}{x-2} dx$$

Resuelve, eligiendo el método más adecuado:

$$1. \int \frac{x^2-1}{x^2+1} dx$$

$$2. \int \frac{\ln x}{x} dx$$

$$3. \int \frac{x^2}{1+x^6} dx$$

$$4. \int \frac{\ln x}{x^2} dx$$

$$5. \int (\cos 2x - \operatorname{sen} 3x) dx$$

$$6. \int x \cdot \sqrt[4]{1-x^2} dx$$

$$7. \int \frac{\operatorname{sen} x}{1 + \cos^2 x} dx$$

$$8. \int \frac{x+1}{\sqrt{x-1}} dx$$

$$9. \int \operatorname{sen} 2x dx$$

$$10. \int \frac{\cos \sqrt{x}}{\sqrt{x}} dx$$

$$11. \int \frac{\operatorname{arc} \operatorname{tg}^2 x}{1+x^2} dx$$

$$12. \int \frac{dx}{1+16x^2}$$

$$13. \int \frac{x^4}{x-1} dx$$

$$14. \int \frac{\operatorname{sen}^3 x}{\cos x} dx$$

$$15. \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$

$$16. \int \frac{\ln(\ln x)}{x} dx$$

$$17. \int \frac{\cos(\ln x)}{x} dx$$

$$18. \int e^{2x} \cdot \operatorname{sen} e^{2x} dx$$

$$19. \int \frac{4x^3 - 9x^2 + 5x - 1}{2x^4 - 6x^3 + 5x^2 - 2x + 7} dx$$

$$20. \int \frac{x-1}{\sqrt{1-x^2}} dx$$

SOLUCIONES

Integrales inmediatas

$$1. \int (3x^3 - 5x^2 + 3x + 4) dx = \frac{3x^4}{4} - \frac{5x^3}{3} + \frac{3x^2}{2} + 4x + C$$

$$2. \int (\sec x + 5 \cos x - 1) dx = -\cos x + 5 \sin x - x + C$$

$$3. \int \operatorname{tg}^2 x dx = \int (1 + \operatorname{tg}^2 x - 1) dx = \int (1 + \operatorname{tg}^2 x) dx - \int 1 dx = \\ = \operatorname{tg} x - x + C$$

$$4. \int (\sqrt{x} - 3) dx = \frac{x^{1/2}}{1/2} - 3x + C = 2\sqrt{x} - 3x + C$$

$$5. \int \frac{2}{\sqrt{x}} dx = 2 \cdot \frac{x^{1/2}}{1/2} + C = 4\sqrt{x} + C$$

$$6. \int \frac{x^3 - 3x^2 + 5x}{x} dx = \int (x^2 - 3x + 5) dx = \frac{x^3}{3} - \frac{3x^2}{2} + 5x + C$$

$$7. \int (5x-2)^2 dx = \frac{1}{5} \int 5(5x-2)^2 dx = \frac{1}{5} \cdot \frac{(5x-2)^3}{3} + C = \frac{(5x-2)^3}{15} + C$$

$$\int (5x-2)^2 dx = \int (25x^2 - 20x + 4) dx = \frac{25x^3}{3} - 10x^2 + 4x + C$$

$$8. \int \frac{(3x-1)^2}{3x} dx = \int \frac{9x^2 - 6x + 1}{3x} dx = \int \left(3x - 2 + \frac{1}{3x} \right) dx =$$

$$= \frac{3x^2}{2} - 2x + \frac{1}{3} \ln|x| + C$$

$$9. \int (2\sqrt{x} - \sqrt[3]{x} - x^5) dx = \int (2x^{1/2} - x^{1/3} - x^5) dx =$$

$$= 2 \cdot \frac{x^{3/2}}{3/2} - \frac{x^{4/3}}{4/3} - \frac{x^6}{6} + C = \frac{4}{3} \sqrt{x^3} - \frac{3}{4} \sqrt[3]{x^4} - \frac{x^6}{6} + C$$

$$10. \int \left(\frac{5}{x} - \frac{x}{5} \right) dx = 5 \ln|x| - \frac{1}{5} \cdot \frac{x^2}{2} + C = 5 \ln|x| - \frac{x^2}{10} + C$$

$$11. \int \frac{2}{1+x^2} dx = 2 \operatorname{arctg} x + C$$

$$12. \int (x-5)(x+2) dx = \int (x^2 - 3x - 10) dx = \frac{x^3}{3} - \frac{3x^2}{2} - 10x + C$$

$$13. \int \frac{5e^x + e^{2x}}{e^x} dx = \int \frac{5e^x}{e^x} dx + \int \frac{e^{2x}}{e^x} dx = \int 5 dx + \int e^x dx =$$

$$= 5x + e^x + C$$

$$14. \int e^{ax+b} dx = \frac{1}{a} \int a \cdot e^{ax+b} dx = \frac{1}{a} e^{ax+b} + C$$

$$15. \int \cos(ax+b) dx = \frac{1}{a} \int a \cdot \cos(ax+b) dx = \frac{1}{a} \operatorname{sen}(ax+b) + C$$

$$16. \int \left(\frac{a}{x} + b \right) dx = a \cdot \ln|x| - bx + C$$

Integrales por cambio de variable

$$1. \int \operatorname{sen} x \cos x dx = \int t dt = \frac{t^2}{2} + C = \frac{\operatorname{sen}^2 x}{2} + C$$

$$t = \operatorname{sen} x \Rightarrow dt = \cos x dx$$

$$2. \int \frac{\operatorname{arctg} x}{1+x^2} dx = \int t dt = \frac{t^2}{2} + C = \frac{\operatorname{arctg}^2 x}{2} + C$$

$$t = \operatorname{arctg} x \Rightarrow dt = \frac{1}{1+x^2} dx$$

$$3. \int (x^2 - x - 1)^5 \cdot (2x - 1) dx = \int t^5 dt = \frac{t^6}{6} + C = \frac{(x^2 - x - 1)^6}{6} + C$$

$$t = x^2 - x - 1 \Rightarrow dt = 2x - 1$$

$$4. \int \frac{x}{x^2-1} dx = \frac{1}{2} \int \frac{2x}{x^2-1} dx = \frac{1}{2} \int \frac{1}{t} dt = \frac{1}{2} \ln|t| + C =$$

$$t = x^2 - 1 \Rightarrow dt = 2x dx$$

$$= \frac{1}{2} \ln|x^2-1| + C$$

$$5. \int \frac{4x-3}{2x^2-3x-14} dx = \int \frac{1}{t} dt = \ln|t| + C = \ln|2x^2-3x-14| + C$$

$$t = 2x^2 - 3x - 14 \Rightarrow dt = 4x - 3$$

$$6. \int \frac{\operatorname{tg} x}{\cos^2 x} dx = \int t dt = \frac{t^2}{2} + C = \frac{\operatorname{tg}^2 x}{2} + C$$

$$t = \operatorname{tg} x \Rightarrow dt = \frac{1}{\cos^2 x} dx$$

$$7. \int 2x \cos x^2 dx = \int \cos t dt = \operatorname{sen} t + C = \operatorname{sen} x^2 + C$$

$$t = x^2 \Rightarrow dt = 2x dx$$

$$8. \int (1 - \cos x)^3 \cdot \operatorname{sen} x dx = \int t^3 dt = \frac{t^4}{4} + C = \frac{(1 - \cos x)^4}{4} + C$$

$$t = 1 - \cos x \Rightarrow dt = \operatorname{sen} x dx$$

$$9. \int \frac{1}{x \ln x} dx = \int \frac{1}{t} dt = \ln|t| + C = \ln|\ln x| + C$$

$$t = \ln x \Rightarrow dt = \frac{1}{x} dx$$

$$10. \int \operatorname{tg} x dx = \int \frac{\operatorname{sen} x}{\cos x} dx = - \int \frac{1}{t} dt = -\ln|t| + C =$$

$$t = \cos x \Rightarrow dt = -\operatorname{sen} x dx$$

$$= -\ln|\cos x| + C$$

$$11. \int x^2 e^{x^3} dx = \frac{1}{3} \int e^t dt = \frac{1}{3} e^t + C = \frac{1}{3} e^{x^3} + C$$

$$t = x^3 \Rightarrow dt = 3x^2 dx$$

$$12. \int \frac{2x}{1+x^4} dx = \int \frac{2x}{1+(x^2)^2} dx = \int \frac{1}{1+t^2} dt = \operatorname{arctg} t + C =$$

$$t = x^2 \Rightarrow dt = 2x dx$$

$$= \operatorname{arctg} (x^2) + C$$

$$13. \int \frac{e^x}{5+e^x} dx = \int \frac{1}{t} dt = \ln |t| + C = \ln (5+e^x) + C$$

$$t = 5+e^x \Rightarrow dt = e^x dx$$

$$14. \int \frac{\operatorname{sen} x}{e^{\cos x}} dx = - \int \frac{1}{e^t} dt = - \int e^{-t} dt = e^{-t} + C =$$

$$t = \cos x \Rightarrow dt = -\operatorname{sen} x dx$$

$$= e^{-\cos x} + C = \frac{1}{e^{\cos x}} + C$$

Integrales por partes:

$$1. \int e^x \operatorname{sen} x dx = -e^x \cos x + \int e^x \cos x dx =$$

$$u = e^x \Rightarrow du = e^x dx \quad v = \cos x dx \Rightarrow v = -\operatorname{sen} x$$

$$dv = \operatorname{sen} x dx \Rightarrow v = -\cos x$$

$$= -e^x \cos x + e^x \operatorname{sen} x - \int e^x \operatorname{sen} x dx$$

Llamando $I = \int e^x \operatorname{sen} x dx$, hemos obtenido:

$$I = -e^x \cos x + e^x \operatorname{sen} x - I$$

$$2I = -e^x \cos x + e^x \operatorname{sen} x$$

$$I = \frac{-e^x \cos x + e^x \operatorname{sen} x}{2}$$

$$\text{Por tanto, } \int e^x \operatorname{sen} x dx = \frac{e^x (\operatorname{sen} x - \cos x)}{2} + C$$

$$2. \int (5x-2) \sin x \, dx = (5x-2) \cos x - \int 5 \cos x \, dx =$$

$$u = 5x-2 \Rightarrow du = 5 \, dx$$

$$dv = \sin x \, dx \Rightarrow v = -\cos x$$

$$= (5x-2) \cos x - 5 \sin x + C$$

$$3. \int \ln x \, dx = x \ln x - \int \frac{1}{x} \cdot x \, dx = x \ln x - x + C$$

$$u = \ln x \Rightarrow du = \frac{1}{x} \, dx$$

$$dv = dx \Rightarrow v = x$$

$$4. \int x \cos x \, dx = x \sin x - \int \sin x \, dx = x \sin x + \cos x + C$$

$$u = x \Rightarrow du = dx$$

$$dv = \cos x \, dx \Rightarrow v = \sin x$$

$$5. \int x e^x \, dx = x e^x - \int e^x \, dx = x e^x - e^x + C$$

$$u = x \Rightarrow du = dx$$

$$dv = e^x \Rightarrow v = e^x$$

$$6. \int x^2 e^x \, dx = x^2 e^x - \int 2x e^x \, dx = x^2 e^x - 2x e^x + \int 2e^x \, dx =$$

$$u = x^2 \Rightarrow du = 2x \, dx$$

$$dv = e^x \, dx \Rightarrow v = e^x$$

$$u = 2x \Rightarrow du = 2 \, dx$$

$$dv = e^x \, dx \Rightarrow v = e^x$$

$$= x^2 e^x - 2x e^x + 2e^x + C$$

$$7. \int (2x+4) e^{2x+4} \, dx = \frac{2x+4}{2} e^{2x+4} - \int \frac{1}{2} \cdot 2 \cdot e^{2x+4} \, dx =$$

$$u = 2x+4 \Rightarrow du = 2 \, dx$$

$$dv = e^{2x+4} \, dx \Rightarrow v = \frac{1}{2} e^{2x+4}$$

$$= (x+2) e^{2x+4} - \frac{1}{2} e^{2x+4} + C = \left(x + \frac{3}{2}\right) e^{2x+4} + C$$

$$8. \int \operatorname{arctg} x \, dx = x \operatorname{arctg} x - \int \frac{x}{1+x^2} \, dx = x \operatorname{arctg} x - \frac{1}{2} \ln(1+x^2) + C$$

$$u = \operatorname{arctg} x \Rightarrow du = \frac{1}{1+x^2} \, dx$$

$$dv = dx \Rightarrow v = x$$

$$9. \int \operatorname{arcsen} x \, dx = x \operatorname{arcsen} x - \int \frac{x}{\sqrt{1-x^2}} \, dx = x \operatorname{arcsen} x - \frac{1}{2} \cdot \frac{(1-x^2)^{1/2}}{1/2} + C =$$

$$u = \operatorname{arcsen} x \Rightarrow du = \frac{1}{\sqrt{1-x^2}} \, dx$$

$$dv = dx \Rightarrow v = x$$

$$= x \operatorname{arcsen} x - \sqrt{1-x^2} + C$$

$$10. \int \frac{x}{e^x} \, dx = \int x e^{-x} \, dx = -x e^{-x} + \int e^{-x} \, dx =$$

$$u = x \Rightarrow du = dx$$

$$dv = e^{-x} \, dx \Rightarrow v = -e^{-x}$$

$$= -x e^{-x} - e^{-x} + C = \frac{-x-1}{e^x} + C$$

Integrales Racionales:

$$1. \int \frac{1}{x^2-1} \, dx$$

$$\frac{1}{x^2-1} = \frac{1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1} = \frac{A(x-1) + B(x+1)}{x^2-1}$$

$$1 = A(x-1) + B(x+1) \Rightarrow \left. \begin{array}{l} A = -1/2 \\ B = 1/2 \end{array} \right\}$$

$$\int \frac{1}{x^2-1} \, dx = \int \frac{-1/2}{x+1} \, dx + \int \frac{1/2}{x-1} \, dx = -\frac{1}{2} \ln|x+1| + \frac{1}{2} \ln|x-1| + C =$$

$$= \frac{1}{2} \ln \left| \frac{x-1}{x+1} \right| + C = \ln \sqrt{\left| \frac{x-1}{x+1} \right|} + C$$

$$2. \int \frac{x-1}{x^2+x-6} dx = \int \frac{x-1}{(x-2)(x+3)} dx$$

$$\frac{x-1}{x^2+x-6} = \frac{x-1}{(x-2)(x+3)} = \frac{A}{x-2} + \frac{B}{x+3} = \frac{A(x+3) + B(x-2)}{(x-2)(x+3)}$$

$$x-1 = A(x+3) + B(x-2) \Rightarrow \begin{cases} A = \frac{1}{5} \\ B = \frac{2}{3} \end{cases}$$

$$\int \frac{x-1}{x^2+x-6} dx = \int \frac{1/5}{x-2} dx + \int \frac{2/3}{x+3} dx = \frac{1}{5} \ln|x-2| + \frac{2}{3} \ln|x+3| + C$$

$$3. \int \frac{x-5}{(x-1)^3} dx$$

$$\frac{x-5}{(x-1)^3} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} = \frac{A(x-1)^2 + B(x-1) + C}{(x-1)^3}$$

$$x-5 = A(x-1)^2 + B(x-1) + C \Rightarrow \begin{cases} A = -4 \\ B = 1 \\ C = -4 \end{cases}$$

$$\int \frac{x-5}{(x-1)^3} dx = \int \frac{-4}{x-1} dx + \int \frac{1}{(x-1)^2} dx + \int \frac{-4}{(x-1)^3} dx =$$

$$= -4 \ln|x-1| + \frac{(x-1)^{-1}}{-1} - \frac{4(x-1)^{-2}}{-2} + C = -4 \ln|x-1| - \frac{1}{x-1} + \frac{2}{(x-1)^2} + C$$

$$4. \int \frac{x+2}{x^3-2x^2} dx = \int \frac{x+2}{x^2(x-2)} dx$$

$$\frac{x+2}{x^3-2x^2} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x^2} = \frac{Ax(x-2) + B(x-2) + Cx^2}{x^2(x-2)}$$

$$x+2 = Ax(x-2) + B(x-2) + Cx^2 \Rightarrow \begin{cases} A = -1 \\ B = -1 \\ C = 1 \end{cases}$$

$$\int \frac{x+2}{x^3-2x^2} dx = \int \frac{-1}{x} dx + \int \frac{-1}{x^2} dx + \int \frac{1}{x-2} dx =$$

$$= -\ln|x| - \frac{x^{-1}}{-1} + \ln|x-2| = \ln \left| \frac{x-2}{x} \right| + \frac{1}{x} + C$$

$$5. \int \frac{1}{(x-1)(x+2)^2} dx$$

$$\frac{1}{(x-1)(x+2)^2} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2} = \frac{A(x+2)^2 + B(x-1)(x+2) + C(x-1)}{(x-1)(x+2)^2}$$

$$1 = A(x+2)^2 + B(x-1)(x+2) + C(x-1) \Rightarrow \begin{cases} A = 1/9 \\ B = -1/9 \\ C = -1/3 \end{cases}$$

$$\int \frac{1}{(x-1)(x+2)^2} dx = \int \frac{1/9}{x-1} dx - \int \frac{1/9}{x+2} dx - \int \frac{1/3}{(x+2)^2} dx =$$

$$= \frac{1}{9} \ln|x-1| - \frac{1}{9} \ln|x+2| - \frac{1}{3} \cdot \frac{(x+2)^{-1}}{-1} + C =$$

$$= \frac{1}{9} \ln \left| \frac{x-1}{x+2} \right| + \frac{1}{3(x+2)} + C$$

$$6. \int \frac{x^3}{x-2} dx = \int \left(x^2 + 2x + 4 + \frac{8}{x-2} \right) dx = \frac{x^3}{3} + x^2 + 4x + 8 \ln|x-2| + C$$

$$\begin{array}{r} x^3 \\ -x^3 + 2x^2 \\ \hline 2x^2 \\ -2x^2 + 4x \\ \hline 4x \\ -4x + 8 \\ \hline 8 \end{array} \quad \begin{array}{l} \frac{x-2}{x^2+2x+4} \end{array}$$

Integrales Varias:

$$1. \int \frac{x^2-1}{x^2+1} dx = \int \left(1 - \frac{2}{x^2+1}\right) dx = x - 2 \operatorname{arctg} x + C$$

$$\frac{x^2-1}{-x^2-1} \quad \frac{x^2+1}{1}$$

$$2. \int \frac{\ln x}{x} dx = \int t dt = \frac{t^2}{2} + C = \frac{\ln^2 x}{2} + C$$

$$t = \ln x \Rightarrow dt = \frac{1}{x} dx$$

$$3. \int \frac{x^2}{1+x^6} dx = \frac{1}{3} \int \frac{3x^2}{1+(x^3)^2} dx = \frac{1}{3} \int \frac{1}{1+t^2} dt = \frac{1}{3} \operatorname{arctg} t + C =$$

$$t = x^3 \Rightarrow dt = 3x^2 dx$$

$$= \frac{1}{3} \operatorname{arctg} x^3 + C$$

$$4. \int \frac{\ln x}{x^2} dx = -\frac{\ln x}{x} + \int \frac{1}{x} \cdot \frac{1}{x} dx = -\frac{\ln x}{x} + \int \frac{1}{x^2} dx =$$

$$u = \ln x \Rightarrow du = \frac{1}{x} dx$$

$$dv = \frac{1}{x^2} dx \Rightarrow v = -\frac{1}{x}$$

$$= -\frac{\ln x}{x} - \frac{1}{x} + C = -\frac{\ln x - 1}{x} + C$$

$$5. \int (\cos 2x - \operatorname{sen} 3x) dx = \frac{1}{2} \int 2 \cos 2x - \frac{1}{3} \int 3 \operatorname{sen} 3x dx =$$

$$= \frac{1}{2} \operatorname{sen} 2x + \frac{1}{3} \cos 3x + C$$

$$6. \int x \cdot \sqrt[4]{1-x^2} dx = -\frac{1}{2} \int \sqrt[4]{t} dt = -\frac{1}{2} \cdot \frac{t^{5/4}}{5/4} + C = -\frac{2}{5} \sqrt[4]{t^5} + C =$$

$$t = 1-x^2 \Rightarrow dt = -2x dx$$

$$= -\frac{2}{5} \sqrt[4]{(1-x^2)^5} + C$$

$$7. \int \frac{\sin x}{1 + \cos x} dx = - \int \frac{1}{t} dt = -\ln|t| + C = -\ln|1 + \cos x| + C$$

$$t = 1 + \cos x \Rightarrow dt = -\sin x dx$$

$$8. \int \frac{x+1}{\sqrt{x-1}} dx = \int \frac{t^2+2}{\sqrt{t^2}} \cdot 2t dt = \int (2t^2+4) dt =$$

$$t^2 = x+1 \Rightarrow 2t dt = dx$$

$$= \frac{2t^3}{3} + 4t + C = \frac{2(\sqrt{x-1})^3}{3} + 4\sqrt{x-1} + C$$

$$9. \int \sin 2x dx = \frac{1}{2} \int 2 \sin 2x dx = -\frac{1}{2} \cos 2x + C$$

$$10. \int \frac{\cos \sqrt{x}}{\sqrt{x}} = 2 \int \frac{\cos \sqrt{x}}{2\sqrt{x}} dx = 2 \int \cos t dt = -2 \sin t + C =$$

$$t = \sqrt{x} \Rightarrow dt = \frac{1}{2\sqrt{x}} dx$$

$$= -2 \sin \sqrt{x} + C$$

$$11. \int \frac{\arctg^2 x}{1+x^2} dx = \int t^2 dt = \frac{t^3}{3} + C = \frac{\arctg^3 x}{3} + C$$

$$t = \arctg x \Rightarrow dt = \frac{1}{1+x^2} dx$$

$$12. \int \frac{dx}{1+16x^2} = \int \frac{dx}{1+(4x)^2} = \frac{1}{4} \int \frac{1}{1+t^2} dt = \frac{1}{4} \arctg t + C =$$

$$t = 4x \Rightarrow dt = 4 dx$$

$$= \frac{1}{4} \arctg (4x) + C$$

$$13. \int \frac{x^4}{x-1} dx = \int (x^3 + x^2 + x + 1 + \frac{1}{x-1}) dx = \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x + \ln|x-1| + C$$

$$\begin{array}{r} x^4 \\ -x^4 + x^3 \\ \hline x^3 \\ -x^3 + x^2 \\ \hline x^2 \\ -x^2 + x \\ \hline x \\ -x + 1 \\ \hline 1 \end{array}$$

$$\begin{aligned}
 14. \int \frac{\sin^3 x}{\cos x} dx &= \int \frac{\sin x \cdot \sin^2 x}{\cos x} dx = \int \frac{\sin x \cdot (1 - \cos^2 x)}{\cos x} dx = \\
 &= \int \frac{\sin x - \sin x \cos^2 x}{\cos x} dx = \int \frac{\sin x}{\cos x} dx - \int \sin x \cos x dx = \\
 &\quad t = \cos x \Rightarrow dt = -\sin x dx \\
 &= - \int \frac{1}{t} dt - \int t dt = -\ln|t| + \frac{t^2}{2} + C = \\
 &= -\ln|\cos x| + \frac{\cos^2 x}{2} + C
 \end{aligned}$$

$$\begin{aligned}
 15. \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx &= 2 \int e^t dt = 2e^t + C = 2e^{\sqrt{x}} + C \\
 t = \sqrt{x} &\Rightarrow dt = \frac{1}{2\sqrt{x}} dx
 \end{aligned}$$

$$\begin{aligned}
 16. \int \frac{\ln(\ln x)}{x} dx &= \int \ln t dt = t \cdot \ln t - \int \frac{1}{t} dt = \\
 u = \ln t &\Rightarrow du = \frac{1}{t} dt \\
 t = \ln x &\Rightarrow dt = \frac{1}{x} dx \quad dv = dt \Rightarrow v = t \\
 &= t \cdot \ln t - t + C = \ln(\ln x) - \ln x + C
 \end{aligned}$$

$$\begin{aligned}
 17. \int \frac{\cos(\ln x)}{x} dx &= \int \cos t dt = \sin t + C = \sin(\ln x) + C \\
 t = \ln x &\Rightarrow dt = \frac{1}{x} dx
 \end{aligned}$$

$$\begin{aligned}
 18. \int e^{2x} \cdot \sin e^{2x} dx &= \frac{1}{2} \int \sin t dt = -\frac{1}{2} \cos t + C = \\
 t = e^{2x} &\Rightarrow dt = 2e^{2x} dx \\
 &= -\frac{1}{2} \cos e^{2x} + C
 \end{aligned}$$

$$19. \int \frac{4x^3 - 9x^2 + 5x - 1}{2x^4 - 6x^3 + 5x^2 - 2x + 7} dx = \frac{1}{2} \int \frac{8x^3 - 18x^2 + 10x - 2}{2x^4 - 6x^3 + 5x^2 - 2x + 7} dx =$$

$$= \frac{1}{2} \ln |2x^4 - 6x^3 + 5x^2 - 2x + 7| + C$$

$$20. \int \frac{x-1}{\sqrt{1-x^2}} dx = \int \frac{x}{\sqrt{1-x^2}} dx - \int \frac{1}{\sqrt{1-x^2}} dx = -\sqrt{1-x^2} - \arcsin x + C$$

$$* \int \frac{x}{\sqrt{1-x^2}} dx = -\frac{1}{2} \int \frac{1}{\sqrt{t}} dt = -\frac{1}{2} \cdot \frac{t^{-1/2}}{-1/2} = -\sqrt{t} + C = -\sqrt{1-x^2} + C$$

$t = 1-x^2 \Rightarrow dt = -2x dx$

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