1º- Calcular:

a) $\int\left(\frac{arsenx}{2\sqrt{1-x^2}}\right)dx$

Solución:

$$\int\left(\frac{arsenx}{2\sqrt{1-x^2}}\right)dx=\frac{1}{2}\int\left(\frac{arsenx}{\sqrt{1-x^2}}\right)dx=\frac{1}{4}arsen^2x+k$$

b) $\int\left(\frac{x^5+x^4-8}{x^3-4x}\right)dx$

Solución:

$$\int\left(\frac{x^5+x^4-8}{x^3-4x}\right)dx=\int\left(x^2+x+4+\frac{2}{x}-\frac{3}{x+2}+\frac{5}{x-2}\right)dx=$$

$$=\int x^2\ dx+\int x\ dx+\int 4\ dx+\int\frac{2}{x}\ dx-\int\frac{3}{x+2}\ dx+\int\frac{5}{x-2}\ dx=$$

$$=\frac{x^3}{3}+\frac{x^2}{2}+4x+2\ln x-3\ln|x+2|+5\ln|x-2|+k=$$

$$=\frac{x^3}{3}+\frac{x^2}{2}+4x+\ln\left(k\frac{x^2(x-2)^5}{(x+2)^3}\right)$$

2º.- Calcular:

a) $\int_1^e \ln xdx$

Solución:

$$\int_1^e \ln xdx\Leftrightarrow\begin{cases}u=\ln x\to du=\frac{1}{x}dx\\ dv=dx\to v=\int\ dx=x\end{cases}\Rightarrow x\ln x-\int x\bullet\frac{1}{x}\ dx\Big|_1^e=x\ln x-x\Big|_1^e=$$

$$=\left(\mathrm{e}\cdot\ln(\mathrm{e})\text{-}\mathrm{e}\right)-(1\cdot\ln 1-1)=1$$

b) $\int_0^1 \frac{2}{\sqrt{4-x^2}}\,dx$

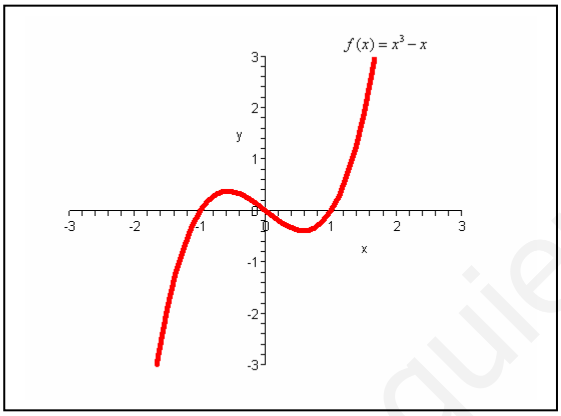
Solución:

Haciendo el cambio $x = 2sent$, $dx = 2\cos t$, luego

$$\int_0^1 \frac{2}{\sqrt{4-x^2}}\,dx = \int_0^{\frac{\pi}{6}} \frac{2}{\sqrt{4-4sen^2 t}}\,2\text{costdt}=\int_0^{\frac{\pi}{6}} \frac{2}{2\sqrt{1-sen^2 t}}\,2\text{costdt}=$$

$$=\int_0^{\frac{\pi}{6}} \frac{2}{2\cos t}\,2\text{costdt} = \int_0^{\frac{\pi}{6}} 2\text{dt}= 2\text{t}\Big|_0^{\frac{\pi}{6}} = \frac{\pi}{3}$$

3º.- Calcular el área limitada por la gráfica de la función $f(x) = x^3 - x$, el eje de abscisas y las rectas $x = -2$ y $x = 2$



Solución:

$$S = -\int_{-2}^{-1} \left(x^3 - x\right)\,dx + \int_{-1}^{0} \left(x^3 - x\right)\,dx -$$

$$-\int_0^1 \left(x^3 - x\right)\,dx + \int_1^2 \left(x^3 - x\right)\,dx =$$

$$= \frac{9}{4} + \frac{1}{4} + \frac{1}{4} + \frac{9}{4} = 5 \text{ un. de área}$$