

Problema 1 Calcular los siguientes límites:

1. $\lim_{x \rightarrow \infty} (x^2 + x - 1)^{1/x^2}$
2. $\lim_{x \rightarrow \infty} \left(\frac{x^2 - x - 1}{x^2 + 2} \right)^{2x}$
3. $\lim_{x \rightarrow \infty} (\sqrt{3x^2 - 2} - \sqrt{3x^2 + x - 1})$
4. $\lim_{x \rightarrow 2} \frac{\sqrt{5x^2 - 1} - \sqrt{8x + 3}}{x - 2}$
5. $\lim_{x \rightarrow 0} \frac{\cos x - e^{2x}}{\sin^2 x - xe^x}$

Solución:

1. $\lim_{x \rightarrow \infty} (x^2 + x - 1)^{1/x^2} = 1$
2. $\lim_{x \rightarrow \infty} \left(\frac{x^2 - x - 1}{x^2 + 2} \right)^{2x} = e^{-2}$
3. $\lim_{x \rightarrow \infty} (\sqrt{3x^2 - 2} - \sqrt{3x^2 + x - 1}) = -\frac{\sqrt{3}}{6}$
4. $\lim_{x \rightarrow 2} \frac{\sqrt{5x^2 - 1} - \sqrt{8x + 3}}{x - 2} = \frac{6\sqrt{19}}{19}$
5. $\lim_{x \rightarrow 0} \frac{\cos x - e^{2x}}{\sin^2 x - xe^x} = -2$

Problema 2 Calcular las siguientes integrales:

1. $\int xe^{x-1} dx$
2. $\int 5xe^{4x^2+3} dx$
3. $\int \frac{3x^2 + 5}{x^2 + 1} dx$
4. $\int \frac{x^3 + 2}{x^2 - 5x - 14} dx$

$$5. \int \frac{5x}{4x^2 + 1} dx$$

Solución:

$$1. \int xe^{x-1} dx = e^{x-1}(x-1) + C$$

$$2. \int 5xe^{4x^2+3} dx = \frac{5}{8}e^{4x^2+3} + C$$

$$3. \int \frac{3x^2 + 5}{x^2 + 1} dx = 3x + 2 \arctan x + C$$

$$4. \int \frac{x^3 + 2}{x^2 - 5x - 14} dx = \frac{x^2}{2} + 5x + \frac{115 \ln(x-7)}{3} + \frac{2 \ln(x+2)}{3}$$

$$5. \int \frac{5x}{4x^2 + 1} dx = \frac{5}{8} \ln(4x^2 + 1)$$