Calcula la primitiva de $\int \frac{x+\sqrt{x}}{x^2}\,dx$

$$\int \frac{x+\sqrt{x}}{x^2}\,dx = \int \frac{x}{x^2}\,dx + \int \frac{\sqrt{x}}{x^2}\,dx = \int \frac{1}{x}\,dx + \int \frac{x^{1/2}}{x^2}\,dx =$$

$$= \ln|x| + \int x^{-3/2}\,dx = \ln|x| + \frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} = \ln|x| + \frac{x^{-1/2}}{-1/2} =$$

$$\boxed{= \ln|x| - \frac{2}{\sqrt{x}} + C}$$

Página 158 → Junio de 2006 → 2º Bl. → A)

Calcula la integral indefinida $\int \frac{x+2}{x^2-2x+1}\,dx$

Factorizamos el denominador:

$x^2-2x+1 = (x-1)^2$

	1	−2	1
1		1	−1
	1	−1	0

Descomponemos en fracciones simples:

$$\frac{x+2}{x^2-2x+1} = \frac{x+2}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2} = \frac{A(x-1)+B}{(x-1)^2}$$

$$x+2 = A(x-1)+B \Rightarrow \begin{cases} x=1 \Rightarrow 3 = B \\ x=0 \Rightarrow 2 = -A+B \Rightarrow A = 1 \end{cases}$$

$$\frac{x+2}{x^2-2x+1} = \frac{5}{x-1} + \frac{3}{(x-1)^2}$$

Resolvemos:

$$\int \frac{x+2}{x^2-2x+1}\,dx = \int \frac{1}{x-1}\,dx + \int \frac{3}{(x-1)^2}\,dx = \boxed{\ln|x-1| - \frac{3}{x-1} + C}$$

Calcula la siguiente integral $\int \frac{x^3+1}{x^2+4}\,dx$

Efectuamos la división:

$$\begin{array}{rl|l} x^3 & \quad +1 & x^2+4 \\ \cline{3-3} -x^3 & -4x & x \\ \hline & -4x+1 & \end{array}$$

$$\frac{x^3+1}{x^2+4} = x + \frac{-4x+1}{x^2+4}$$

Resolvemos:

$$\int \frac{x^3+1}{x^2+4}\,dx = \int x\,dx + \int \frac{-4x+1}{x^2+4}\,dx$$

Calculamos $\int \frac{-4x+1}{x^2+4}\,dx = \int \frac{1}{x^2+4}\,dx - 4\int \frac{x}{x^2+4}\,dx =$

$$= \int \frac{1}{4\left(\frac{x^2}{4}+1\right)}\,dx - 4\frac{1}{2}\int \frac{2x}{x^2+4}\,dx =$$

$$= \frac{1}{4}\int \frac{1}{\left(\frac{x}{2}\right)^2+1}\,dx - 2\ln|x^2+4| = 2\frac{1}{4}\int \frac{\frac{1}{2}}{\left(\frac{x}{2}\right)^2+1}\,dx -$$

$$- 2\ln|x^2+4| = \frac{1}{2}\,\mathrm{arctg}\left(\frac{x}{2}\right) - 2\ln|x^2+4| + C$$

Por tanto:

$$\int \frac{x^3+1}{x^2+4}\,dx = \boxed{\frac{x^2}{2} + \frac{1}{2}\,\mathrm{arctg}\left(\frac{x}{2}\right) - 2\ln|x^2+4| + C}$$

Calcula la siguiente integral $\int \frac{2}{1+\sqrt{x}}dx$

$$\int \frac{2}{1+\sqrt{x}}dx = 2\int \frac{1}{1+\sqrt{x}}dx = \left[t=\sqrt{x} \rightarrow t^2 = x \quad dx = 2t\,dt \right] =$$

$$= 2\int \frac{1}{1+t}2t\,dt = 4\int \frac{t}{1+t}dt \underset{①}{=}$$

$$\begin{array}{r|l} t & t+1 \\ \hline -t-1 & 1 \\ -1 & \end{array} \qquad \frac{t}{1+t} = 1 + \frac{-1}{t+1}$$

$$\int \frac{t}{1+t}dt = \int 1\,dt - \int \frac{1}{t+1}dt = t - \ln|t+1|$$

$$\underset{①}{=} 4\left(t - \ln|t+1|\right) = \boxed{4\left(\sqrt{x} - \ln\left|\sqrt{x}+1\right|\right) + C}$$

Página 165 $\rightarrow$ Septiembre de 2007 $\rightarrow$ 2º Bl. $\rightarrow$ A)

Calcula la siguiente integral $\int \frac{x}{(x+1)^3}dx$

Descomponemos en fracciones simples:

$$\frac{x}{(x+1)^3} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3} = \frac{A(x+1)^2 + B(x+1) + C}{(x+1)^3}$$

$$x = A(x+1)^2 + B(x+1) + C = Ax^2 + (2A+B)x + (A+B+C) \Rightarrow$$

$$\Rightarrow \left\{ \begin{array}{l} A = 0 \\ 2A+B = 1 \\ A+B+C = 0 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} A = 0 \\ B = 1 \\ C = -1 \end{array} \right.$$

$$\frac{x}{(x+1)^3} = \frac{0}{x+1} + \frac{1}{(x+1)^2} + \frac{-1}{(x+1)^3}$$

Resolvemos:

$$\int \frac{x}{(x+1)^3}dx = \int \frac{1}{(x+1)^2}dx - \int \frac{1}{(x+1)^3}dx = \boxed{-\frac{1}{x+1} + \frac{1}{2(x+1)^2} + C}$$

- También se puede resolver haciendo el cambio de variable $t = x+1$.

Encuentra una primitiva de $f(x) = x^2 \operatorname{sen} x$ que pase por el origen de coordenadas.

Primitiva de $f(x)$:

$$\int x^2 \operatorname{sen} x \, dx = \left[\begin{array}{l} u = x^2 \rightarrow du = 2x\,dx \\ dv = \operatorname{sen} x\,dx \rightarrow v = -\cos x \end{array}\right] = -x^2 \cos x - \int -\cos x \,(2x)\, dx =$$

$$= -x^2 \cos x + 2\int x \cos x \, dx \underset{1}{=}$$

Calculamos $\int x \cos x \, dx$ por partes:

$$\int x \cos x \, dx = \left[\begin{array}{l} u = x \longrightarrow du = dx \\ dv = \cos x\,dx \rightarrow v = \operatorname{sen} x \end{array}\right] = x \operatorname{sen} x - \int \operatorname{sen} x \, dx =$$

$$= x \operatorname{sen} x + \cos x$$

$$\underset{1}{=} -x^2 \cos x + 2\left(x \operatorname{sen} x + \cos x\right) + C = F(x)$$

Imponemos que pase por el origen de coordenadas

Eso quiere decir que $F(0) = 0 \iff -0^2 \cos 0 + 2(0 \cdot \operatorname{sen} 0 + \cos 0) + C = 0$

$\iff 2 + C = 0 \iff C = -2$

Por tanto, la primitiva que nos piden es:

$$\boxed{\int x^2 \operatorname{sen} x \, dx = -x^2 \cos x + 2\left(x \operatorname{sen} x + \cos x\right) - 2}$$

Calcula la siguiente integral: $\int \frac{-x+3}{4x^2+9}\,dx$

Factorizamos el denominador:

$4x^2+9=0 \Rightarrow x=\pm\sqrt{\frac{-9}{4}}=\pm\frac{3}{2}i \in \mathbb{C}$

Resolvemos:

$$\int \frac{-x+3}{4x^2+9}\,dx = -\int \frac{x}{4x^2+9}\,dx + 3\int \frac{1}{4x^2+9}\,dx \overset{①}{=}$$

Resolvemos $\int \frac{1}{4x^2+9}\,dx = \int \frac{1}{9\left(\frac{4x^2}{9}+1\right)}\,dx = \frac{1}{9}\int \frac{1}{\left(\frac{2x}{3}\right)^2+1}\,dx =$

$$= \frac{1}{9}\,\frac{3}{2}\int \frac{\frac{2}{3}}{\left(\frac{2x}{3}\right)^2+1}\,dx = \frac{1}{6}\,\mathrm{arctg}\left(\frac{2x}{3}\right)$$

$$\overset{①}{=} -\frac{1}{8}\ln|4x^2+9| + 3\,\frac{1}{6}\,\mathrm{arctg}\left(\frac{2x}{3}\right) = \boxed{-\frac{1}{8}\ln|4x^2+9| + \frac{1}{2}\,\mathrm{arctg}\left(\frac{2x}{3}\right)+C}$$

Página 169 → Junio de 2008 → 2° Bl. → A)

Calcula la integral $\int \frac{2x^3-9x^2+9x+6}{x^2-5x+6}\,dx$

$$\begin{array}{rl|l}
 & 2x^3-9x^2+9x+6 & x^2-5x+6 \\ \cline{3-3}
 & -2x^3+10x^2-12x & 2x+1 \\ \hline
 & x^2-3x+6 & \\
 & -x^2+5x-6 & \\ \hline
 & 2x &
\end{array}$$

$$\frac{2x^3-9x^2+9x+6}{x^2-5x+6} = 2x+1+\frac{2x}{x^2-5x+6}$$

$$\int \frac{2x^3-9x^2+9x+6}{x^2-5x+6}\,dx = \int (2x+1)\,dx + \int \frac{2x}{x^2-5x+6}\,dx \overset{①}{=}$$

$$\int \frac{2x}{x^2-5x+6}\,dx \overset{②}{=} \int \frac{-4}{x-2}\,dx + \int \frac{6}{x-3}\,dx =$$

$$= -4\ln|x-2| + 6\ln|x-3|$$

$$\overset{①}{=} \boxed{x^2+x-4\ln|x-2|+6\ln|x-3|+C}$$

donde en ② hemos descompuesto en fracciones simples:

$$\frac{2x}{x^2-5x+6} = \frac{2x}{(x-2)(x-3)} = \frac{-4}{x-2}+\frac{6}{x-3}$$

Calcula las siguientes integrales: a) $\int \ln x \, dx$; b) $\int tg\, x \, dx$

a) $\int \ln x\, dx = \begin{bmatrix} u = \ln x \rightarrow du = \frac{1}{x}dx \\ dv = dx \rightarrow v = x \end{bmatrix} = x\ln x - \int x \frac{1}{x} dx =$

$= \boxed{x\ln x - x + C}$

b) $\int tg\, x\, dx = \int \frac{sen\, x}{cos\, x} dx = -\int \frac{-sen\, x}{cos\, x} dx = \boxed{-\ln|cos\, x| + C}$

Página 178 → Reserva 1 de 2009 → 2º Bl. → A)

Fórmula de integración por partes

Sean $D \subseteq \mathbb{R}$ un intervalo y f y g dos funciones con derivadas continuas en D. Entonces: $\int fg' = fg - \int f'g$

Calcular $\int \left(1 - \frac{1}{x^2}\right) \ln x\, dx$

$\int \left(1 - \frac{1}{x^2}\right) \ln x\, dx = \begin{bmatrix} u = \ln x \rightarrow du = \frac{1}{x}dx \\ dv = \left(1 - \frac{1}{x^2}\right)dx \rightarrow v = x + \frac{1}{x} \end{bmatrix} =$

$= \left(x + \frac{1}{x}\right)\ln x - \int \left(x + \frac{1}{x}\right)\frac{1}{x} dx = \left(x + \frac{1}{x}\right)\ln x - \int \left(1 + \frac{1}{x^2}\right)dx =$

$= \boxed{\left(x + \frac{1}{x}\right)\ln x - x + \frac{1}{x} + C}$

Página 182 → Junio de 2010 → Propuesta B → B)

Calcula la integral indefinida $\int \frac{cos\, x}{1 + sen^2 x} dx$

$\int \frac{cos\, x}{1 + sen^2 x} dx = \boxed{arctg\,(sen\, x) + C}$

• También se puede hacer usando el cambio de variable $t = sen\, x$

$\int \frac{cos\, x}{1 + sen^2 x} dx = \begin{bmatrix} t = sen\, x \\ dt = cos\, x\, dx \rightarrow dx = \frac{1}{cos\, x} dt \end{bmatrix} =$

$= \int \frac{cos\, x}{1 + t^2} \frac{1}{cos\, x} dt = \int \frac{1}{1 + t^2} dt = arctg(t) = arctg\,(sen\, x) + C$

Calcula la integral indefinida $\int \frac{2x^3-9x^2+9x+6}{x^2-5x+6}\,dx$

Efectuamos la división:

$$\begin{array}{l|l} 2x^3-9x^2+9x+6 & x^2-5x+6 \\ \hline -2x^3+10x^2-12x & 2x+1 \\ \quad x^2-3x+6 & \\ \quad -x^2+5x-6 & \\ \quad\quad 2x & \end{array}$$

$$\frac{2x^3-9x^2+9x+6}{x^2-5x+6} = 2x+1+\frac{2x}{x^2-5x+6}$$

Descomponemos en fracciones simples:

$$\frac{2x}{x^2-5x+6} = \frac{2x}{(x-2)(x-3)} = \frac{A}{x-2}+\frac{B}{x-3} = \frac{A(x-3)+B(x-2)}{(x-2)(x-3)}$$

$$x^2-5x+6=0 \Rightarrow x=\begin{cases}2\\3\end{cases}$$

Igualamos los numeradores: $2x = A(x-3)+B(x-2)$

Le damos valores: $x=2 \Rightarrow 4=-A \Rightarrow A=-4$

$x=3 \Rightarrow 6=B$

$$\frac{2x}{x^2-5x+6} = \frac{-4}{x-2}+\frac{6}{x-3}$$

Resolvemos:

$$\int \frac{2x^3-9x^2+9x+6}{x^2-5x+6}\,dx = \int (2x+1)\,dx + \int \frac{2x}{x^2-5x+6}\,dx =$$

$$= x^2+x+\int \frac{-4}{x-2}\,dx + \int \frac{6}{x-3}\,dx =$$

$$\boxed{= x^2+x-4\ln|x-2|+6\ln|x-3|+C}$$

$$\int x \log x \, dx = \left[\begin{array}{l} u = \ln x \longrightarrow du = \frac{1}{x} dx \\ dv = x\,dx \longrightarrow v = \frac{x^2}{2} \end{array} \right] = \frac{1}{2} x^2 \ln x - \int \frac{x^2}{2} \frac{1}{x} dx =$$

$$= \boxed{\frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C}$$

Página 185 → Reserva 1 de 2010 → Propuesta B → B)

Calcula la integral indefinida $\int \frac{1}{x^3+x^2} dx$

Descomponemos en fracciones simples:

$$\frac{1}{x^3+x^2} = \frac{1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} = \frac{Ax(x+1)+B(x+1)+Cx^2}{x^2(x+1)}$$

Igualamos los numeradores: $1 = Ax(x+1) + B(x+1) + Cx^2$

y le damos valores:

$$\left. \begin{array}{l} x=0 \Rightarrow B = 1 \\ x=-1 \Rightarrow C = 1 \\ x=1 \Rightarrow 2A+2B+C = 1 \end{array} \right\} \begin{array}{l} A = -1 \\ B = 1 \\ C = 1 \end{array}$$

$$\frac{1}{x^3+x^2} = \frac{-1}{x} + \frac{1}{x^2} + \frac{1}{x+1}$$

Resolvemos:

$$\int \frac{1}{x^3+x^2} dx = \int \frac{-1}{x} dx + \int \frac{1}{x^2} dx + \int \frac{1}{x+1} dx =$$

$$= \boxed{-\ln|x| - \frac{1}{x} + \ln|x+1| + C}$$

Calcula la integral indefinida $\int \frac{x+2}{\sqrt{x+1}} dx$

$$\int \frac{x+2}{\sqrt{x+1}} dx = \left[\begin{array}{l} t = \sqrt{x+1} \longrightarrow t^2 = x+1 \\ \qquad\qquad 2t\,dt = dx \end{array} \right] = \int \frac{t^2+1}{t} 2t\,dt =$$

$$= 2\int (t^2+1)\,dt = 2\left(\frac{t^3}{3} + t\right) = \boxed{2\left(\frac{\sqrt{(x+1)^3}}{3} + \sqrt{x+1}\right) + C}$$