Problema 1 Sea la matriz

$$A = \begin{pmatrix} 3 & m & 2 \\ m & -1 & 0 \\ m & 3 & 2 \end{pmatrix}$$

1. Calcular los valores de m para los que la matriz A es inversible.
2. Calcular A^{-1} para $m = 0$.

Solución:

1.

$$\begin{vmatrix} 3 & m & 2 \\ m & -1 & 0 \\ m & 3 & 2 \end{vmatrix} = -2(m^2 - 4m + 3) = 0 \Longrightarrow m = 1, \;\; m = 3$$

Si $m = 1$ o $m = 3 \Longrightarrow |A| = 0 \Longrightarrow$ no existe A^{-1}.

Si $m \neq 1$ y $m \neq 3 \Longrightarrow |A| \neq 0 \Longrightarrow$ existe A^{-1}.

2.

$$A = \begin{pmatrix} 3 & 0 & 2 \\ 0 & -1 & 0 \\ 0 & 3 & 2 \end{pmatrix} \Longrightarrow A^{-1} = \begin{pmatrix} 1/3 & -1 & -1/3 \\ 0 & -1 & 0 \\ 0 & 3/2 & 1/2 \end{pmatrix}$$

Problema 2 Resolver la ecuación matricial $AX - B = C - X$. Donde

$$A = \begin{pmatrix} 5 & 1 \\ 0 & 3 \end{pmatrix}; \quad B = \begin{pmatrix} 1 & 6 \\ -2 & 3 \end{pmatrix}; \quad C = \begin{pmatrix} 3 & 5 \\ 1 & 0 \end{pmatrix}$$

Solución:

$$AX - B = C - X \Longrightarrow X = (A + I)^{-1}(C + B)$$

$$A+I = \begin{pmatrix} 5 & 1 \\ 0 & 3 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 6 & 1 \\ 0 & 4 \end{pmatrix}, \quad (A+I)^{-1} = \begin{pmatrix} 1/6 & -1/24 \\ 0 & 1/4 \end{pmatrix}$$

$$C + B = \begin{pmatrix} 3 & 5 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 6 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 11 \\ -1 & 3 \end{pmatrix}$$

$$X = (A+I)^{-1}(C+B) = \begin{pmatrix} 1/6 & -1/24 \\ 0 & 1/4 \end{pmatrix} \begin{pmatrix} 4 & 11 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 17/24 & 41/24 \\ -1/4 & 3/4 \end{pmatrix}$$

Problema 3 Resolver, utilizando las propiedades de los determinantes, calcular:

$$\begin{vmatrix} x & 1 & 1 & 0 \\ 1 & x & 0 & 1 \\ 1 & 0 & x & 1 \\ 0 & 1 & 1 & x \end{vmatrix}$$

Solución:

$$\begin{vmatrix} x & 1 & 1 & 0 \\ 1 & x & 0 & 1 \\ 1 & 0 & x & 1 \\ 0 & 1 & 1 & x \end{vmatrix} = \begin{bmatrix} \mathrm{F}_1+\mathrm{F}_2+\mathrm{F}_3+\mathrm{F}_4 \\ \mathrm{F}_2 \\ \mathrm{F}_3 \\ \mathrm{F}_4 \end{bmatrix} = \begin{vmatrix} x+2 & x+2 & x+2 & x+2 \\ 1 & x & 0 & 1 \\ 1 & 0 & x & 1 \\ 0 & 1 & 1 & x \end{vmatrix} =$$

$$(x+2)\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & x & 0 & 1 \\ 1 & 0 & x & 1 \\ 0 & 1 & 1 & x \end{vmatrix} = \begin{bmatrix} \mathrm{C}_1 \\ \mathrm{C}_2-\mathrm{C}_1 \\ \mathrm{C}_3-\mathrm{C}_1 \\ \mathrm{C}_4-\mathrm{C}_1 \end{bmatrix} = (x+2)\begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & x-1 & -1 & 0 \\ 1 & -1 & x-1 & 0 \\ 0 & 1 & 1 & x \end{vmatrix} =$$

$$(x+2)\begin{vmatrix} x-1 & -1 & 0 \\ -1 & x-1 & 0 \\ 1 & 1 & x \end{vmatrix} = x(x-2)\begin{vmatrix} x-1 & -1 \\ -1 & x-1 \end{vmatrix} = x^2(x+2)(x-2)$$