Problema 1 Sea la matriz

$$A = \begin{pmatrix} -1 & 2 & m \\ m+1 & 1 & 3 \\ m-1 & 5 & 5 \end{pmatrix}$$

1. Calcular los valores de m para los que la matriz A es inversible.
2. Calcular A^{-1} para $m = 0$.

Solución:

1.

$$\begin{vmatrix} -1 & 2 & m \\ m+1 & 1 & 3 \\ m-1 & 5 & 5 \end{vmatrix} = 2(2m^2 + m - 3) = 0 \Longrightarrow m = 1, \ \ m = -3/2$$

Si $m = 1$ o $m = -3/2 \Longrightarrow |A| = 0 \Longrightarrow$ no existe A^{-1}.

Si $m \neq 1$ y $m \neq -3/2 \Longrightarrow |A| \neq 0 \Longrightarrow$ existe A^{-1}.

2.

$$A = \begin{pmatrix} -1 & 2 & 0 \\ 1 & 1 & 3 \\ -1 & 5 & 5 \end{pmatrix} \Longrightarrow A^{-1} = \begin{pmatrix} 5/3 & 5/3 & -1 \\ 4/3 & 5/6 & -1/2 \\ -1 & -1/2 & 1/2 \end{pmatrix}$$

Problema 2 Resolver la ecuación matricial $AX - I = C - BX$. Donde

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}; \quad B = \begin{pmatrix} -1 & 2 \\ 1 & -1 \end{pmatrix}; \quad C = \begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}$$

Solución:

$$AX + BX = C + I \Longrightarrow (A+B)X = C + I \Longrightarrow X = (A+B)^{-1}(C+I)$$

$$(A+B)^{-1} = \begin{pmatrix} -3/16 & 4/16 \\ 4/16 & 0 \end{pmatrix}, \quad C + I = \begin{pmatrix} 4 & 1 \\ 0 & 2 \end{pmatrix}$$

$$X = (A+B)^{-1}(C+I) = \begin{pmatrix} -3/4 & 5/16 \\ 1 & 1/4 \end{pmatrix}$$

Problema 3 Resolver, utilizando las propiedades de los determinantes, la ecuación:

$$\left|\begin{array}{cccc} 1 & x & 1 & 0 \\ 1 & 1 & 0 & x \\ x & 1 & 1 & 0 \\ 1 & 0 & x & 1 \end{array}\right| = 0$$

Solución:

$$\left|\begin{array}{cccc} 1 & x & 1 & 0 \\ 1 & 1 & 0 & x \\ x & 1 & 1 & 0 \\ 1 & 0 & x & 1 \end{array}\right| = \left[\begin{array}{c} \mathrm{C}_1 + \mathrm{C}_2 + \mathrm{C}_3 + \mathrm{C}_4 \\ \mathrm{C}_2 \\ \mathrm{C}_3 \\ \mathrm{C}_4 \end{array}\right] = \left|\begin{array}{cccc} x+2 & x & 1 & 0 \\ x+2 & 1 & 0 & x \\ x+2 & 1 & 1 & 0 \\ x+2 & 0 & x & 1 \end{array}\right| =$$

$$(x+2)\left|\begin{array}{cccc} 1 & x & 1 & 0 \\ 1 & 1 & 0 & x \\ 1 & 1 & 1 & 0 \\ 1 & 0 & x & 1 \end{array}\right| = \left[\begin{array}{c} \mathrm{F}_1 \\ \mathrm{F}_2 - \mathrm{F}_1 \\ \mathrm{F}_3 - \mathrm{F}_1 \\ \mathrm{F}_4 - \mathrm{F}_1 \end{array}\right] = (x+2)\left|\begin{array}{cccc} 1 & x & 1 & 0 \\ 0 & 1-x & -1 & x \\ 0 & 1-x & 0 & 0 \\ 0 & -x & x-1 & 1 \end{array}\right| =$$

$$(x+2)\left|\begin{array}{ccc} 1-x & -1 & x \\ -1-x & 0 & 0 \\ -x & x-1 & 1 \end{array}\right| = (x-1)(x+2)\left|\begin{array}{cc} -1 & x \\ x-1 & 1 \end{array}\right| =$$

$$(x-1)(x+2)(-x^2+x-1) = 0 \Longrightarrow x = 1, \;\; x = -2$$