

EJERCICIOS Resueltos de REGLA DE L' HÔPITAL

Indeterminaciones

$$\left[\begin{array}{c} 0 \\ 0 \end{array} \right]$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = \left[\begin{array}{c} 0 \\ 0 \end{array} \right] = \lim_{x \rightarrow 0} \frac{\cos x}{1} = \frac{\cos 0}{1} = \frac{1}{1} = 1$$

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = \left[\begin{array}{c} 0 \\ 0 \end{array} \right] = \lim_{x \rightarrow 0} \frac{e^x}{1} = \frac{e^0}{1} = \frac{1}{1} = 1$$

$$\lim_{x \rightarrow 1} \frac{\ln x}{x^2 + x - 2} = \left[\begin{array}{c} 0 \\ 0 \end{array} \right] = \lim_{x \rightarrow 1} \frac{\frac{1}{x}}{2x + 1} = \lim_{x \rightarrow 1} \frac{1}{2x^2 + x} = \frac{1}{2 \cdot 1^2 + 1} = \frac{1}{3}$$

$$\lim_{x \rightarrow 0} \frac{x - \sin x}{x^3} = \left[\begin{array}{c} 0 \\ 0 \end{array} \right] = \lim_{x \rightarrow 0} \frac{1 - \cos x}{3x^2} = \left[\begin{array}{c} 0 \\ 0 \end{array} \right] = \lim_{x \rightarrow 0} \frac{\sin x}{6x} = \left[\begin{array}{c} 0 \\ 0 \end{array} \right] = \lim_{x \rightarrow 0} \frac{\cos x}{6} = \frac{1}{6}$$

$$\lim_{x \rightarrow 0} \frac{\operatorname{tg} 3x}{\operatorname{tg} x} = \left[\begin{array}{c} 0 \\ 0 \end{array} \right] = \lim_{x \rightarrow 0} \frac{\frac{3}{\cos^2 3x}}{\frac{1}{\cos^2 x}} = \frac{3}{1} = 3$$

$$\lim_{x \rightarrow 0} \frac{\sin 3x}{x - \frac{3}{2} \sin 2x} = \left[\begin{array}{c} 0 \\ 0 \end{array} \right] = \lim_{x \rightarrow 0} \frac{3 \cdot \cos 3x}{1 - \frac{3}{2} \cdot \cos 2x} = \lim_{x \rightarrow 0} \frac{3 \cdot \cos 3x}{1 - 3 \cos 2x} = \frac{3 \cdot 1}{1 - 3 \cdot 1} = -\frac{3}{2}$$

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin x - x \cos x}{\sin^3 x} &= \left[\begin{array}{c} 0 \\ 0 \end{array} \right] = \lim_{x \rightarrow 0} \frac{\cos x - (\cos x - x \sin x)}{3 \sin^2 x \cos x} = \lim_{x \rightarrow 0} \frac{x \sin x}{3 \sin^2 x \cos x} \\ &= \lim_{x \rightarrow 0} \frac{x}{3 \sin x \cos x} \stackrel{\sin 2x = 2 \sin x \cos x}{=} \lim_{x \rightarrow 0} \frac{x}{\frac{3}{2} \sin 2x} = \left[\begin{array}{c} 0 \\ 0 \end{array} \right] = \\ &= \lim_{x \rightarrow 0} \frac{1}{\frac{3}{2} \cdot 2 \cos 2x} = \lim_{x \rightarrow 0} \frac{x}{3 \cos 2x} = \frac{2}{3 \cdot \cos 0} = \frac{2}{3 \cdot 1} = \frac{2}{3} \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{x \operatorname{arc} \operatorname{sen} x}{\sin x \cos x} &= \left[\begin{array}{c} 0 \\ 0 \end{array} \right] \stackrel{\sin 2x = 2 \sin x \cos x}{=} \lim_{x \rightarrow 0} \frac{x \operatorname{arc} \operatorname{sen} x}{\frac{1}{2} \sin 2x} = \lim_{x \rightarrow 0} \frac{2x \operatorname{arc} \operatorname{sen} x}{\sin 2x} = \left[\begin{array}{c} 0 \\ 0 \end{array} \right] = \\ &= \lim_{x \rightarrow 0} \frac{2 \cdot \operatorname{arc} \operatorname{sen} x + \frac{2x}{1+x^2}}{2 \cdot \cos 2x} = \frac{2 \cdot \operatorname{arc} \operatorname{sen} 0 + \frac{2 \cdot 0}{1+0^2}}{2 \cdot \cos 0} = \frac{2 \cdot 0 + \frac{2 \cdot 0}{1+0^2}}{2 \cdot 1} = 0 \end{aligned}$$

$$\lim_{x \rightarrow 0} \frac{a^x - b^x}{x} = \left[\begin{array}{c} 0 \\ 0 \end{array} \right] = \lim_{x \rightarrow 0} \frac{a^x \cdot \ln a - b^x \cdot \ln b}{1} = \ln a - \ln b$$

$$\lim_{x \rightarrow 0^+} \frac{x - 1}{x^n - 1} = \left[\begin{array}{c} 0 \\ 0 \end{array} \right] = \lim_{x \rightarrow 0^+} \frac{x - 1}{x^n - 1} = \lim_{x \rightarrow 0^+} \frac{1}{n \cdot x^{n-1}} = \left[\begin{array}{c} 1 \\ 0^+ \end{array} \right] = +\infty$$

Indeterminaciones

$$\left[\begin{array}{c} \infty \\ - \\ \infty \end{array} \right]$$

e Inteterminaciones

$$0 \cdot \infty$$

$$\lim_{x \rightarrow +\infty} \frac{3x^2 + 2x - 1}{4x^2 + 6} = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow +\infty} \frac{6x + 2}{8x} = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow +\infty} \frac{6}{8} = \frac{3}{4}$$

$$\lim_{x \rightarrow +\infty} \frac{e^x}{x^2} = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow +\infty} \frac{e^x}{2x} = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow +\infty} \frac{e^x}{2} = \left[\frac{+\infty}{2} \right] = +\infty$$

$$\lim_{x \rightarrow +\infty} \frac{2^x}{x} = \lim_{x \rightarrow +\infty} \frac{2^x \cdot \ln 2}{1} = \left[\frac{+\infty}{1} \right] = +\infty$$

$$\begin{aligned} \lim_{x \rightarrow +\infty} \frac{\ln^2 x}{\sqrt{x}} &= \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow +\infty} \frac{2 \cdot \ln x \cdot \frac{1}{x}}{\frac{1}{2\sqrt{x}}} = \lim_{x \rightarrow +\infty} \frac{4 \cdot \sqrt{x} \cdot \ln x}{x} = \left[\frac{\infty}{\infty} \right] = \\ &= \lim_{x \rightarrow +\infty} \frac{4 \cdot \frac{1}{2\sqrt{x}} \cdot \ln x + \frac{1}{x} \cdot \sqrt{x}}{1} = \lim_{x \rightarrow +\infty} \left(\frac{2 \cdot \ln x}{x} + \frac{\sqrt{x}}{x} \right) \quad \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}} \\ &\stackrel{\frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}}}{=} \lim_{x \rightarrow +\infty} \left(\frac{2 \cdot \ln x}{\sqrt{x}} + \frac{1}{\sqrt{x}} \right) = \lim_{x \rightarrow +\infty} \frac{2 \cdot \ln x + 1}{\sqrt{x}} = \left[\frac{+\infty}{\infty} \right] = \\ &= \lim_{x \rightarrow +\infty} \frac{2 \cdot \frac{1}{x}}{\frac{1}{2\sqrt{x}}} = \lim_{x \rightarrow +\infty} \frac{4 \cdot \sqrt{x}}{x} \quad \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}} = \lim_{x \rightarrow +\infty} \frac{4}{\sqrt{x}} = \left[\frac{4}{\infty} \right] = 0 \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow \pi} [(x - \pi)] \cdot \operatorname{tg} \frac{x}{2} &= [0 \cdot \infty] = \lim_{x \rightarrow \pi} \frac{\operatorname{tg} \frac{x}{2}}{\frac{1}{x - \pi}} = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow \pi} \frac{\frac{1}{2} \cdot \frac{1}{\cos^2 \frac{x}{2}}}{-1} = \\ &= \lim_{x \rightarrow \pi} \frac{-(x - \pi)^2}{2 \cdot \cos^2 \frac{x}{2}} = \left[\frac{0}{0} \right] = \lim_{x \rightarrow \pi} \frac{-2 \cdot (x - \pi)}{2 \cdot \cancel{\cos \frac{x}{2}} \cdot \cos \frac{x}{2} \cdot (-\operatorname{sen} \frac{x}{2}) \cdot \frac{1}{\cancel{\cos \frac{x}{2}}}} \quad 2 \operatorname{sen} \frac{x}{2} \cos \frac{x}{2} = \operatorname{sen} x \\ &\stackrel{2 \operatorname{sen} \frac{x}{2} \cos \frac{x}{2} = \operatorname{sen} x}{=} \lim_{x \rightarrow \pi} \frac{-2 \cdot (x - \pi)}{-\operatorname{sen} x} = \left[\frac{0}{0} \right] = \lim_{x \rightarrow \pi} \frac{-2}{\cos x} = \frac{-2}{-\cos \pi} = \frac{-2}{-1} = -2 \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow \frac{\pi}{2}} (\sec x - \operatorname{tg} x) &= [\infty - \infty] = \lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{1}{\cos x} - \frac{\operatorname{sen} x}{\cos x} \right) = \lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \operatorname{sen} x}{\cos x} = \left[\frac{\infty}{\infty} \right] = \\ &= \lim_{x \rightarrow \frac{\pi}{2}} \frac{-\cos x}{-\operatorname{sen} x} = \frac{-\cos \frac{\pi}{2}}{-\operatorname{sen} \frac{\pi}{2}} = \frac{0}{-1} = 0 \end{aligned}$$

$$\begin{aligned} \blacksquare \lim_{x \rightarrow 1} \left(\frac{e}{e^x - e} - \frac{1}{x - 1} \right) &= [\infty - \infty] = \lim_{x \rightarrow 1} \frac{e \cdot x - e}{(e^x - e) \cdot (x - 1)} = \lim_{x \rightarrow 1} \frac{e \cdot x - e}{x \cdot e^x - e \cdot x + e^x + e} = \\ &= \frac{e - e}{e + e - e - e} = \left[\frac{0}{0} \right] = \lim_{x \rightarrow 1} \frac{-e^x}{e^x + e^x + x \cdot e^x} = \left[\frac{0}{0} \right] = \\ &= \lim_{x \rightarrow 1} \frac{e - e^x}{e^x + x \cdot e^x - e - e^x} = \lim_{x \rightarrow 1} \frac{-e^x}{e^x + x \cdot e^x} = \frac{-e}{e + e} = \frac{-e}{2e} = -\frac{1}{2} \end{aligned}$$

Indeterminaciones $1^\infty, 0^0, \infty^0 \dots$

$$\blacksquare \lim_{x \rightarrow 0} \left(\frac{2^x + 3^x}{2} \right)^{\frac{1}{x}} = [\infty^0] = M$$

Tomando logaritmo neperiano en ambos miembros:

$$\begin{aligned} \ln M &= \ln \left[\lim_{x \rightarrow \infty} \left(\frac{2^x + 3^x}{2} \right)^{\frac{1}{x}} \right] = \lim_{x \rightarrow 0} \ln \left[\left(\frac{2^x + 3^x}{2} \right)^{\frac{1}{x}} \right] = \lim_{x \rightarrow 0} \frac{1}{x} \cdot \ln \left(\frac{2^x + 3^x}{2} \right) = \\ &= [0 \cdot \infty] = \lim_{x \rightarrow 0} \frac{\ln \left(\frac{2^x + 3^x}{2} \right)}{x} = \left[\frac{0}{0} \right] = \lim_{x \rightarrow 0} \frac{\frac{2^x \cdot \ln 2 + 3^x \cdot \ln 3}{2}}{\frac{2^x + 3^x}{2}} = \frac{\ln 2 + \ln 3}{2^0 + 3^0} = \\ &= \frac{\ln 2 + \ln 3}{4} \Rightarrow M = e^{\frac{\ln 2 + \ln 3}{4}} \end{aligned}$$

$$\blacksquare \lim_{x \rightarrow +\infty} (x + e^x + e^{2x})^{\frac{1}{x}} = [1^\infty] = M$$

Tomando logaritmo neperiano en ambos miembros:

$$\begin{aligned} \ln M &= \ln (x + e^x + e^{2x})^{\frac{1}{x}} = \lim_{x \rightarrow +\infty} \ln (x + e^x + e^{2x})^{\frac{1}{x}} = \lim_{x \rightarrow +\infty} \frac{\ln (x + e^x + e^{2x})}{x} = \\ &= \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow +\infty} \frac{1 + e^x + 2 \cdot e^{2x}}{x + e^x + e^{2x}} = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow +\infty} \frac{e^x + 4 \cdot e^{2x}}{1 + e^x + 2 \cdot e^{2x}} = \\ &= \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow +\infty} \frac{e^x + 8 \cdot e^{2x}}{e^x + 4 \cdot e^{2x}} = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow +\infty} \frac{\frac{e^x}{e^{2x}} + \frac{8 \cdot e^{2x}}{e^{2x}}}{\frac{e^x}{e^{2x}} + \frac{4 \cdot e^{2x}}{e^{2x}}} = \\ &= \lim_{x \rightarrow +\infty} \frac{e^{-x} + 8}{e^{-x} + 4} = \frac{0 + 8}{0 + 4} = 2 \Rightarrow M = e^2 \end{aligned}$$

- $\lim_{x \rightarrow \frac{\pi}{2}} (\sin x)^{\operatorname{tg} x} = [1^\infty] = M$

Tomando logaritmo neperiano en ambos miembros:

$$\ln M = \ln \lim_{x \rightarrow \frac{\pi}{2}} (\sin x)^{\operatorname{tg} x} = \lim_{x \rightarrow \frac{\pi}{2}} \ln (\sin x)^{\operatorname{tg} x} = \lim_{x \rightarrow \frac{\pi}{2}} \operatorname{tg} x \cdot \ln (\sin x) = \lim_{x \rightarrow \frac{\pi}{2}} \frac{\ln (\sin x)}{\frac{1}{\operatorname{tg} x}} =$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{\ln (\sin x)}{\cotg x} = \left[\frac{0}{0} \right] = \lim_{x \rightarrow \frac{\pi}{2}} \frac{\frac{\cos x}{\sin x}}{-1} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cancel{\sin} x \cdot \cos x}{-\cancel{\sin} x} =$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} -\frac{1}{2} \cdot \sin 2x = 0 \Rightarrow M = 1$$

- $\lim_{x \rightarrow +\infty} (e^{2x} + 1)^{\frac{1}{x}} = [\infty^0] = M$

Tomando logaritmo neperiano en ambos miembros:

$$\ln M = \ln \lim_{x \rightarrow +\infty} (e^{2x} + 1)^{\frac{1}{x}} = \lim_{x \rightarrow +\infty} \ln (e^{2x} + 1)^{\frac{1}{x}} = \lim_{x \rightarrow +\infty} \frac{1}{x} \cdot \ln (e^{2x} + 1) =$$

$$= \lim_{x \rightarrow +\infty} \frac{\ln (e^{2x} + 1)}{x} = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow +\infty} \frac{2 \cdot e^{2x}}{e^{2x} + 1} = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow +\infty} \frac{4 \cdot \cancel{e^{2x}}}{2 \cdot \cancel{e^{2x}}} =$$

$$\frac{4}{2} = 2 \Rightarrow M = e^2$$