INTEGRACIÓN POR PARTES

Las integrales que podemos resolver con este método son:

a. Integrales de la forma: $\int x^n \sin x dx \qquad \int x^n \cos x dx \qquad n \in N, n \geq 1$

b. Integrales de la forma: $\int e^x \cos x dx \qquad \int e^x \sin x dx \qquad \int x^n e^x dx$

c. Integrales de la forma: $\int x^n \ln x dx \qquad n \in N$

d. Integrales de la forma: $\int \sin^n x dx \qquad \int \cos^n x dx \qquad n \in N, n > 1$

e. Integrales de la forma: $\int \arcsin x dx \qquad \int \arccos x dx \qquad \int \arctan x dx \qquad \int \operatorname{arccot} x dx$

En el ejercicio nº8, antes de integrar por partes considera que $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

En el ejercicio nº9 haz primero el cambio de variable:

$$y = \arcsin x \Rightarrow \begin{cases} x = \sin y \\ dx = \cos y dy \end{cases}$$

1. $\int \arcsin x dx = (\arcsin x)x - \int \frac{1}{\sqrt{(1-x^2)}} x\, dx = (\arcsin x)x + \sqrt{(1-x^2)} + C$

2. $\int x^2 \arctan x dx = \frac{1}{3}(\arctan x)x^3 - \int \frac{1}{3(1+x^2)} x^3\, dx = \frac{1}{3}(\arctan x)x^3 - \frac{1}{6}x^2 + \frac{1}{6}\ln(3+3x^2) + C$

3. $\int x \cos x dx = x \sin x - \int \sin x\, dx = x \sin x + \cos x + C$

4. $\int \cos x \ln(\sin x) dx = \ln(\sin x)\sin x - \int \cos x\, dx = \ln(\sin x)\sin x - \sin x + C$

5. $\int \arcsin 2x dx = (\arcsin 2x)x - \int \frac{2}{\sqrt{(1-4x^2)}} x\, dx = (\arcsin 2x)x + \frac{1}{2}\sqrt{(1-4x^2)} + C$

6. $\int x\sqrt{1+x}\, dx = \frac{2}{3}x\left(\sqrt{(1+x)}\right)^3 - \int \frac{2}{3}\left(\sqrt{(1+x)}\right)^3 dx = \frac{2}{3}x\left(\sqrt{(1+x)}\right)^3 - \frac{4}{15}\left(\sqrt{(1+x)}\right)^5 + C$

7. $\int x^2 e^{3x} dx = \frac{1}{3}x^2 e^{3x} - \int \frac{2}{3}xe^{3x}\, dx = \frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \int \frac{2}{9}e^{3x}\, dx = \frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{27}e^{3x} + C$

8. $\int e^{2x}\cos^2 x dx = \int \frac{1}{2}e^{2x}(1+\cos 2x)dx = \int \frac{1}{2}e^{2x}dx + \int \frac{1}{2}e^{2x}\cos 2x dx = \frac{1}{4}e^{2x} + \frac{1}{8}e^{2x}\cos 2x + \frac{1}{8}e^{2x}\sin 2x + C$

9. $\int e^{\arcsin x} dx = \int e^y \cos y dy = \frac{1}{2}e^y \cos y + \frac{1}{2}e^y \sin y = \frac{1}{2}e^{\arcsin x}\sqrt{1-x^2} + \frac{1}{2}e^{\arcsin x}x + C$

10. $\int x \sin x dx = -x\cos x - \int(-\cos x)\, dx = -x\cos x + \sin x + C$

11. $\int xe^x dx = xe^x - \int e^x\, dx = xe^x - e^x + C$

12. $\int x^2 \ln x dx = \frac{1}{3}(\ln x)x^3 - \int \frac{1}{3}x^2\, dx = \frac{1}{3}(\ln x)x^3 - \frac{1}{9}x^3 + C$

13. $\int x^2 \sin x dx = -x^2\cos x - \int(-2x\cos x)\, dx = -x^2\cos x + 2x\sin x + \int(-2\sin x)\, dx =$
$-x^2\cos x + 2x\sin x + 2\cos x + C$

14. $\int x^3 e^{2x} dx = \frac{1}{2}x^3e^{2x} - \int \frac{3}{2}x^2e^{2x}\, dx = \frac{1}{2}x^3e^{2x} - \frac{3}{4}x^2e^{2x} + \int \frac{3}{2}xe^{2x}\, dx =$
$\frac{1}{2}x^3e^{2x} - \frac{3}{4}x^2e^{2x} + \frac{3}{4}xe^{2x} - \int \frac{3}{4}e^{2x}\, dx = \frac{1}{2}x^3e^{2x} - \frac{3}{4}x^2e^{2x} + \frac{3}{4}xe^{2x} - \frac{3}{8}e^{2x} + C$

15. $\int x \arctan x dx = \frac{1}{2}(\arctan x)x^2 - \int \frac{x^2}{2(1+x^2)}\, dx = \frac{1}{2}(\arctan x)x^2 - \frac{1}{2}x + \frac{1}{2}\arctan x + C$

16. $\int \sin x \sin 3x dx = -\frac{1}{3}\sin x\cos 3x - \int\left(-\frac{1}{3}\cos x\cos 3x\right)dx =$
$-\frac{1}{3}\sin x\cos 3x + \frac{1}{9}\cos x\sin 3x + \int \frac{1}{9}\sin x\sin 3x\, dx$

Obtenemos la siguiente igualdad:

$\int \sin x\sin 3x dx = -\frac{1}{3}\sin x\cos 3x + \frac{1}{9}\cos x\sin 3x + \frac{1}{9}\int \sin x\sin 3x\, dx$

$\frac{8}{9}\int \sin x\sin 3x dx = -\frac{1}{3}\sin x\cos 3x + \frac{1}{9}\cos x\sin 3x$

$$\int \sin x \sin 3x dx = -\frac{9}{24}\sin x \cos 3x + \frac{1}{8}\cos x \sin 3x = -\frac{3}{8}\sin x \cos 3x + \frac{1}{8}\cos x \sin 3x$$

17. $\int x \arcsin x^2 dx = \frac{1}{2}(\arcsin x^2)x^2 - \int \frac{x^3}{\sqrt{(1-x^4)}}dx = \frac{1}{2}(\arcsin x^2)x^2 + \frac{1}{2}\sqrt{(1-x^4)} + C$

18. $\int e^{ax}\sin bx dx = -\frac{b}{a^2+b^2}e^{ax}\cos bx + \frac{a}{a^2+b^2}e^{ax}\sin bx + C$

19. $\int \sin 3x \cos 2x dx = -\frac{1}{10}\cos 5x - \frac{1}{2}\cos x + C$

20. $\int \sin^4 x dx = -\frac{1}{4}\sin^3 x \cos x - \frac{3}{8}\cos x \sin x + \frac{3}{8}x + C$

21. $\int \cos^5 x dx = \frac{1}{5}\cos^4 x \sin x + \frac{4}{15}\cos^2 x \sin x + \frac{8}{15}\sin x + C$

22. $\int \sin^3 x \cos^2 x dx = -\frac{1}{5}\cos^3 x \sin^2 x - \frac{2}{15}\cos^3 x + C$