Problema 1 Calcular los siguientes límites:

1. $\lim\limits_{x\longrightarrow\infty} \dfrac{-2x^3+2x-1}{3x^3-5}$

2. $\lim\limits_{x\longrightarrow\infty} \left(\dfrac{x^2+2x+1}{x^2+3}\right)^{2x}$

3. $\lim\limits_{x\longrightarrow\infty} \dfrac{\sqrt{16x^2-x+1}}{-x^2+3}$

4. $\lim\limits_{x\longrightarrow 1} \dfrac{8x^4+2x^3-9x^2-2x+1}{2x^3+x^2-4x+1}$

5. $\lim\limits_{x\longrightarrow 5} \dfrac{\sqrt{x^2-2}-\sqrt{4x+3}}{x-5}$

Solución:

1. $\lim\limits_{x\longrightarrow\infty} \dfrac{-2x^3+2x-1}{3x^3-5} = -\dfrac{2}{3}$

2. $\lim\limits_{x\longrightarrow\infty} \left(\dfrac{x^2+2x+1}{x^2+3}\right)^{2x} = e^4$

3. $\lim\limits_{x\longrightarrow\infty} \dfrac{\sqrt{16x^2-x+1}}{-x^2+3} = 0$

4. $\lim\limits_{x\longrightarrow 1} \dfrac{8x^4+2x^3-9x^2-2x+1}{2x^3+x^2-4x+1} = \dfrac{9}{2}$

5. $\lim\limits_{x\longrightarrow 5} \dfrac{\sqrt{x^2-2}-\sqrt{4x+3}}{x-5} = \dfrac{3\sqrt{23}}{23}$

Problema 2 Calcular las siguientes derivadas:

1. $y = e^{5x^3+2x^2-x+3}$

2. $y = \ln(3x^5+2)$

3. $y = (3x^2+2)^{12}$

4. $y = (x^2-x+1)(2x^3+2x^2+1)$

5. $y = \frac{3x^2-2x+1}{2x-6}$

6. $y = x^7 \ln x$

Solución:

1. $y = e^{5x^3+2x^2-x+3} \Longrightarrow y' = (15x^2+4x-1)e^{5x^3+2x^2-x+3}$
2. $y = \ln(3x^5+2) \Longrightarrow y' = \dfrac{15x^4}{3x^5+2}$
3. $y = (3x^2+2)^{12} \Longrightarrow y' = 12(3x^2+2)^{11}(6x)$
4. $y = (x^2-x+1)(2x^3+2x^2+1) \Longrightarrow y' = (2x-1)(2x^3+2x^2+1) + (x^2-x+1)(6x^2+2x)$
5. $y = \dfrac{3x^2-2x+1}{2x-6} \Longrightarrow y' = \dfrac{(6x-2)(2x-6)-(3x^2-2x+1)2}{(2x-6)^2}$
6. $y = x^7 \ln x \Longrightarrow y' = 7x^6 \ln x + x^7 \dfrac{1}{x}$

Problema 3 Calcular las rectas tangente y normal de las siguientes funciones:

1. $f(x) = \dfrac{3x^2-5}{x^2+1}$ en el punto $x = 1$.
2. $f(x) = \dfrac{2x+1}{x+5}$ en el punto $x = 0$.

Solución:

1. $b = f(a) \Longrightarrow b = f(1) = -1$ e $y - b = m(x-a)$

   $$f'(x) = -\frac{16x}{(x^2+1)^2} \Longrightarrow m = f'(1) = 4$$

   Recta Tangente: $y + 1 = 4(x-1)$

   Recta Normal: $y + 1 = -\frac{1}{4}(x-1)$

2. $b = f(a) \Longrightarrow b = f(0) = \dfrac{1}{5}$ e $y - b = m(x-a)$

   $$f'(x) = \frac{9}{(x+5)^2} \Longrightarrow m = f'(0) = \frac{9}{25}$$

   Recta Tangente: $y - \dfrac{1}{5} = \dfrac{9}{25}x$

   Recta Normal: $y - \dfrac{1}{5} = -\dfrac{25}{9}x$

Problema 4 Calcular las siguientes integrales:

1. $\int (5x^2 - 3x + 1)\, dx$

2. $\int \left(\frac{3x^2 - 2\sqrt[4]{x} - 4}{x} - 5e^x \right) dx$

3. $\int (x^2 + 3x + 1)\, dx$ sabiendo que la primitiva $F(1) = 1$.

4. $\int 5xe^{3x^2-2}\, dx$

5. $\int \frac{5x}{7x^2 - 1}\, dx$

Solución:

1. $\int (5x^2 - 3x + 1)\, dx = \frac{5x^3}{3} - \frac{3x^2}{2} + x + C$

2. $\int \left(\frac{3x^2 - 2\sqrt[4]{x} - 4}{x} - 5e^x \right) dx = \frac{3x^2}{2} - 8x^{1/4} - 4\ln|x| - 5e^x + C$

3. $$F(x) = \int (x^2 + 3x + 1)\, dx = \frac{x^3}{3} + \frac{3x^2}{2} + x + C$$

   como
   $$F(1) = 1 \Longrightarrow \frac{1}{3} + \frac{3}{2} + 1 + C = 1 \Longrightarrow C = -\frac{11}{6}$$
   $$F(x) = \frac{x^3}{3} + \frac{3x^2}{2} + x - \frac{11}{6}$$

4. $\int 5xe^{3x^2-2}\, dx = \frac{5}{6}e^{3x^2-2} + C$

5. $\int \frac{5x}{7x^2 - 1}\, dx = \frac{5}{14}\ln|7x^2 - 1| + C$