

Polinomios

Ejercicios resueltos

2.1-1 Realiza la suma de los siguientes polinomios:

$$\begin{aligned} a) \quad p(x) &= x^5 + x^4 - 4x^3 + 6x^2 + x - 7 \\ q(x) &= x^6 + 2x^4 + x^2 + 5 \end{aligned}$$

$$\begin{aligned} b) \quad p(x) &= 9x^5 - 2x^4 + 12x^3 + x^2 - x + 10 \\ q(x) &= -x^5 + 5x^4 - 12x^3 - 2x^2 + x - 15 \end{aligned}$$

$$\begin{aligned} c) \quad p(x) &= -5x^4 + 6x^3 - 2x^2 + 3x + 8 \\ q(x) &= 2x^4 - 3x^3 + 2x^2 - 4 \end{aligned}$$

$$\begin{aligned} d) \quad p(x) &= 3x^4 + x^3 - 2x^2 + x - 14 \\ q(x) &= 6x^4 - 8x^3 + 2x^2 - 3x \\ r(x) &= 2x + 14 \end{aligned}$$

$$\begin{aligned} e) \quad p(x) &= -x^6 + 4x^5 - 2x^4 - 7x^3 + 6x^2 + x - 2 \\ q(x) &= 3x^6 + 2x^5 - x^3 + 2x^2 - 2x + 5 \\ r(x) &= -2x^6 - 6x^5 + 2x^4 + 8x^3 - 8x^2 + x - 3 \end{aligned}$$

$$\begin{aligned} f) \quad p(x) &= x^4 - 3x^3 + x^2 - 7x + 11 \\ q(x) &= 2x^5 - 3x^4 + x^3 - x^2 - 7 \\ r(x) &= -3x^5 + 2x^4 - 5x^3 + 8x^2 + 3x - 4 \end{aligned}$$

Solución

$$\left. \begin{aligned} a) \quad p(x) &= x^5 + x^4 - 4x^3 + 6x^2 + x - 7 \\ q(x) &= x^6 + 2x^4 + x^2 + 5 \end{aligned} \right\}$$

$$p(x) + q(x) = x^6 + x^5 + 3x^4 - 4x^3 + 7x^2 + x - 2$$

$$\left. \begin{aligned} b) \quad p(x) &= 9x^5 - 2x^4 + 12x^3 + x^2 - x + 10 \\ q(x) &= -x^5 + 5x^4 - 12x^3 - 2x^2 + x - 15 \end{aligned} \right\}$$

$$p(x) + q(x) = 8x^5 + 3x^4 - x^2 - 5$$

$$c) \left. \begin{aligned} p(x) &= -5x^4 + 6x^3 - 2x^2 + 3x + 8 \\ q(x) &= 2x^4 - 3x^3 + 2x^2 - 4 \end{aligned} \right\}$$

$$p(x) + q(x) = -3x^4 + 3x^3 + 3x + 4$$

$$d) \left. \begin{aligned} p(x) &= 3x^4 + x^3 - 2x^2 + x - 14 \\ q(x) &= 6x^4 - 8x^3 + 2x^2 - 3x \\ r(x) &= 2x + 14 \end{aligned} \right\}$$

$$\begin{aligned} p(x) + q(x) + r(x) &= [p(x) + q(x)] + r(x) = \\ &= [9x^4 - 7x^3 - 2x - 14] + 2x + 14 = 9x^4 - 7x^3 \end{aligned}$$

$$e) \left. \begin{aligned} p(x) &= -x^6 + 4x^5 - 2x^4 - 7x^3 + 6x^2 + x - 2 \\ q(x) &= 3x^6 + 2x^5 - x^3 + 2x^2 - 2x + 5 \\ r(x) &= -2x^6 - 6x^5 + 2x^4 + 8x^3 - 8x^2 + x - 3 \end{aligned} \right\}$$

$$\begin{aligned} p(x) + q(x) + r(x) &= [p(x) + q(x)] + r(x) = \\ &= [2x^6 + 6x^5 - 2x^4 - 8x^3 + 8x^2 - x + 3] + \\ &+ [-2x^6 - 6x^5 + 2x^4 + 8x^3 - 8x^2 + x - 3] = 0 \end{aligned}$$

$$f) \left. \begin{aligned} p(x) &= x^4 - 3x^3 + x^2 - 7x + 11 \\ q(x) &= 2x^5 - 3x^4 + x^3 - x^2 - 7 \\ r(x) &= -3x^5 + 2x^4 - 5x^3 + 8x^2 + 3x - 4 \end{aligned} \right\}$$

$$\begin{aligned} p(x) + q(x) + r(x) &= [p(x) + q(x)] + r(x) = \\ &= [2x^5 - 2x^4 - 2x^3 - 7x + 4] + [-3x^5 + 2x^4 - 5x^3 + 8x^2 + 3x - 4] = \\ &= -x^5 - 7x^3 + 8x^2 - 4x \end{aligned}$$

2.1-2 Realiza la resta de los siguientes polinomios:

$$a) \quad p(x) = x^6 + 2x^5 - 3x^4 + x^3 + 4x^2 + 4x - 4$$

$$q(x) = -x^6 + 2x^5 - 5x^4 + x^3 + 2x^2 + 3x - 8$$

$$b) \quad p(x) = -3x^3 + 7x^2 - 3x - 2$$

$$q(x) = 5x^3 + 5x^2 + 5x + 5$$

$$c) \quad p(x) = x^4 + 4x^3 - 2x^2 + 7x + 10$$

$$q(x) = -2x^4 + 5x^3 - 8x^2 + 3x + 11$$

$$d) \quad p(x) = -x^5 + 5x^3 + 4x^2 - x + 1$$

$$q(x) = x^4 + 9x^3 - 3x^2 + x - 1$$

$$e) \quad p(x) = -7x^3 + x^2 - 12x - 2$$

$$q(x) = -6x^3 + 3x^2 - 13x + 15$$

$$f) \quad p(x) = x^4 + 3x^3 - 3x^2 + 2x + 14$$

$$q(x) = -x^5 - 2x^4 + 3x^3 - 3x + 14$$

Solución

$$a) \quad \left. \begin{array}{l} p(x) = x^6 + 2x^5 - 3x^4 + x^3 + 4x^2 + 4x - 4 \\ q(x) = -x^6 + 2x^5 - 5x^4 + x^3 + 2x^2 + 3x - 8 \end{array} \right\}$$

$$\begin{aligned} p(x) - q(x) &= p(x) + [-q(x)] = \\ &= x^6 + 2x^5 - 3x^4 + x^3 + 4x^2 + 4x - 4 - \\ &\quad -[-x^6 + 2x^5 - 5x^4 + x^3 + 2x^2 + 3x - 8] \end{aligned}$$

$$p(x) - q(x) = 2x^6 + 2x^4 + 2x^2 + x + 4$$

$$b) \quad \left. \begin{array}{l} p(x) = -3x^3 + 7x^2 - 3x - 2 \\ q(x) = 5x^3 + 5x^2 + 5x + 5 \end{array} \right\}$$

$$\begin{aligned} p(x) - q(x) &= p(x) + [-q(x)] = \\ &= -3x^3 + 7x^2 - 3x - 2 - [5x^3 + 5x^2 + 5x + 5] \end{aligned}$$

$$p(x) - q(x) = -8x^3 + 2x^2 - 8x - 7$$

$$c) \left. \begin{aligned} p(x) &= x^4 + 4x^3 - 2x^2 + 7x + 10 \\ q(x) &= -2x^4 + 5x^3 - 8x^2 + 3x + 11 \end{aligned} \right\}$$

$$\begin{aligned} p(x) - q(x) &= p(x) + [-q(x)] = \\ &= x^4 + 4x^3 - 2x^2 + 7x + 10 - [-2x^4 + 5x^3 - 8x^2 + 3x + 11] \end{aligned}$$

$$p(x) - q(x) = 3x^4 - x^3 + 6x^2 + 4x - 1$$

$$d) \left. \begin{aligned} p(x) &= -x^5 + 5x^3 + 4x^2 - x + 1 \\ q(x) &= x^4 + 9x^3 - 3x^2 + x - 1 \end{aligned} \right\}$$

$$\begin{aligned} p(x) - q(x) &= p(x) + [-q(x)] = \\ &= -x^5 + 5x^3 + 4x^2 - x + 1 - [x^4 + 9x^3 - 3x^2 + x - 1] \end{aligned}$$

$$p(x) - q(x) = -x^5 - x^4 - 4x^3 + 7x^2 - 2x + 2$$

$$e) \left. \begin{aligned} p(x) &= -7x^3 + x^2 - 12x - 2 \\ q(x) &= -6x^3 + 3x^2 - 13x + 15 \end{aligned} \right\}$$

$$\begin{aligned} p(x) - q(x) &= p(x) + [-q(x)] = \\ &= -7x^3 + x^2 - 12x - 2 - [-6x^3 + 3x^2 - 13x + 15] \end{aligned}$$

$$p(x) - q(x) = -x^3 - 2x^2 + x - 17$$

$$f) \begin{aligned} p(x) &= x^4 + 3x^3 - 3x^2 + 2x + 14 \\ q(x) &= -x^5 - 2x^4 + 3x^3 - 3x + 14 \end{aligned}$$

$$\begin{aligned} p(x) - q(x) &= p(x) + [-q(x)] = \\ &= x^4 + 3x^3 - 3x^2 + 2x + 14 - [-x^5 - 2x^4 + 3x^3 - 3x + 14] \end{aligned}$$

$$p(x) - q(x) = x^5 + 3x^4 - 3x^2 + 5x$$

2.1-3 Realiza el producto de los siguientes polinomios:

$$\begin{aligned} a) \quad p(x) &= x^4 + 2x^3 - x^2 + 3x + 1 \\ q(x) &= 2x \end{aligned}$$

$$\begin{aligned} b) \quad p(x) &= -x^5 + x^4 - x^3 + x^2 - x + 1 \\ q(x) &= -5x^4 \end{aligned}$$

$$\begin{aligned} c) \quad p(x) &= 2x^6 + 3x^4 + x^2 - 6 \\ q(x) &= x^3 + x \end{aligned}$$

$$\begin{aligned} d) \quad p(x) &= x^2 + 2x + 3 \\ q(x) &= -x^2 + x + 4 \end{aligned}$$

$$\begin{aligned} e) \quad p(x) &= x^5 + 3x^3 + 6x \\ q(x) &= -x^2 + 2x - 2 \\ r(x) &= 2x^3 + 5x^2 - 2x + 3 \end{aligned}$$

$$\begin{aligned} f) \quad p(x) &= -x^2 + 2x + 3 \\ q(x) &= x^2 + x + 1 \\ r(x) &= x^4 + x^3 + x^2 + x + 4 \end{aligned}$$

Solución

$$\begin{aligned} a) \quad p(x) &= x^4 + 2x^3 - x^2 + 3x + 1 \\ q(x) &= 2x \end{aligned} \quad \left. \vphantom{\begin{aligned} p(x) &= x^4 + 2x^3 - x^2 + 3x + 1 \\ q(x) &= 2x \end{aligned}} \right\}$$

$$\begin{aligned} p(x) \cdot q(x) &= [x^4 + 2x^3 - x^2 + 3x + 1] \cdot 2x = \\ &= x^4 \cdot 2x + 2x^3 \cdot 2x - x^2 \cdot 2x + 3x \cdot 2x + 1 \cdot 2x = \\ &= 2x^5 + 4x^4 - 2x^3 + 6x^2 + 2x \end{aligned}$$

$$\begin{aligned} b) \quad p(x) &= -x^5 + x^4 - x^3 + x^2 - x + 1 \\ q(x) &= -5x^4 \end{aligned} \quad \left. \vphantom{\begin{aligned} p(x) &= -x^5 + x^4 - x^3 + x^2 - x + 1 \\ q(x) &= -5x^4 \end{aligned}} \right\}$$

$$\begin{aligned} p(x) \cdot q(x) &= [-x^5 + x^4 - x^3 + x^2 - x + 1] \cdot (-5x^4) = \\ &= 5x^9 - 5x^8 + 5x^7 - 5x^6 + 5x^5 - 5x^4 \end{aligned}$$

$$c) \left. \begin{aligned} p(x) &= 2x^6 + 3x^4 + x^2 - 6 \\ q(x) &= x^3 + x \end{aligned} \right\}$$

$$\begin{aligned} p(x) \cdot q(x) &= [2x^6 + 3x^4 + x^2 - 6] \cdot [x^3 + x] = \\ &= [2x^6 + 3x^4 + x^2 - 6] \cdot x^3 + [2x^6 + 3x^4 + x^2 - 6] \cdot x = \\ &= [2x^9 + 3x^7 + x^5 - 6x^3] + [2x^7 + 3x^5 + x^3 - 6x] = \\ &= 2x^9 + 5x^7 + 4x^5 - 5x^3 - 6x \end{aligned}$$

$$d) \left. \begin{aligned} p(x) &= x^2 + 2x + 3 \\ q(x) &= -x^2 + x + 4 \end{aligned} \right\}$$

$$\begin{aligned} p(x) \cdot q(x) &= [x^2 + 2x + 3] \cdot [-x^2 + x + 4] = \\ &= [x^2 + 2x + 3] \cdot (-x^2) + [x^2 + 2x + 3] \cdot x + [x^2 + 2x + 3] \cdot 4 = \\ &= -x^4 - 2x^3 - 3x^2 + x^3 + 2x^2 + 3x + 4x^2 + 8x + 12 = \\ &= -x^4 - x^3 + 3x^2 + 11x + 12 \end{aligned}$$

$$e) \left. \begin{aligned} p(x) &= x^5 + 3x^3 + 6x \\ q(x) &= -x^2 + 2x - 2 \\ r(x) &= 2x^3 + 5x^2 - 2x + 3 \end{aligned} \right\}$$

$$\begin{aligned} p(x) \cdot q(x) \cdot r(x) &= [p(x) \cdot q(x)] \cdot r(x) = \\ &= [(x^5 + 3x^3 + 6x) \cdot (-x^2 + 2x - 2)] \cdot (2x^3 + 5x^2 - 2x + 3) = \\ &= (-x^7 - 3x^5 - 6x^3 + 2x^6 + 6x^4 + 12x^2 - 2x^5 - 6x^3 - 12x) \cdot \\ &\quad \cdot (2x^3 + 5x^2 - 2x + 3) \end{aligned}$$

$$\begin{aligned} p(x) \cdot q(x) \cdot r(x) &= (-x^7 + 2x^6 - 5x^5 + 6x^4 - 12x^3 + 12x^2 - 12x) \cdot \\ &\quad \cdot (2x^3 + 5x^2 - 2x + 3) \end{aligned}$$

$$\begin{aligned} p(x) \cdot q(x) \cdot r(x) &= \\ &= -2x^{10} + 4x^9 - 10x^8 + 12x^7 - 24x^6 + 24x^5 - 24x^4 - \\ &\quad - 5x^9 + 10x^8 - 25x^7 + 30x^6 - 60x^5 + 60x^4 - 60x^3 + \\ &\quad + 2x^8 - 4x^7 + 10x^6 - 12x^5 + 24x^4 - 24x^3 + 24x^2 - \\ &\quad - 3x^7 + 6x^6 - 15x^5 + 18x^4 - 36x^3 + 36x^2 - 36x \end{aligned}$$

$$\begin{aligned} p(x) \cdot q(x) \cdot r(x) &= \\ &= -2x^{10} - x^9 + 2x^8 - 20x^7 + 22x^6 - 63x^5 + 78x^4 - 120x^3 + 60x^2 - 36x \end{aligned}$$

$$f) \left. \begin{aligned} p(x) &= -x^2 + 2x + 3 \\ q(x) &= x^2 + x + 1 \\ r(x) &= x^4 + x^3 + x^2 + x + 4 \end{aligned} \right\}$$

$$\begin{aligned} p(x) \cdot q(x) \cdot r(x) &= [p(x) \cdot q(x)] \cdot r(x) = \\ &= [(-x^2 + 2x + 3) \cdot (x^2 + x + 1)] \cdot (x^4 + x^3 + x^2 + x + 4) \end{aligned}$$

$$\begin{aligned} p(x) \cdot q(x) \cdot r(x) &= [p(x) \cdot q(x)] \cdot r(x) = \\ &= (-x^4 + 2x^3 + 3x^2 - x^3 + 2x^2 + 3x - x^2 + 2x + 3) \cdot \\ &\quad \cdot (x^4 + x^3 + x^2 + x + 4) \end{aligned}$$

$$\begin{aligned} p(x) \cdot q(x) \cdot r(x) &= [p(x) \cdot q(x)] \cdot r(x) = \\ &= (-x^4 + x^3 + 4x^2 + 5x + 3) \cdot (x^4 + x^3 + x^2 + x + 4) \end{aligned}$$

$$\begin{aligned} p(x) \cdot q(x) \cdot r(x) &= [p(x) \cdot q(x)] \cdot r(x) = \\ &= -x^8 + x^7 + 4x^6 + 5x^5 + 3x^4 - \\ &\quad -x^7 + x^6 + 4x^5 + 5x^4 + 3x^3 - \\ &\quad -x^6 + x^5 + 4x^4 + 5x^3 + 3x^2 - \\ &\quad -x^5 + x^4 + 4x^3 + 5x^2 + 3x - \\ &\quad -4x^4 + 4x^3 + 16x^2 + 20x + 12 \end{aligned}$$

$$\begin{aligned} p(x) \cdot q(x) \cdot r(x) &= \\ &= -x^8 + 4x^6 + 9x^5 + 9x^4 + 16x^3 + 24x^2 + 23x + 12 \end{aligned}$$

2.1-4 Dados los siguientes polinomios, realiza la operación que se indica:

$$a) \left. \begin{array}{l} p(x) = x^3 + 6x^2 - x + 2 \\ q(x) = 2x^2 + 4x - 3 \end{array} \right\} (p(x) + q(x)) \cdot (p(x) - q(x))$$

$$b) \left. \begin{array}{l} p(x) = x^2 - 5x + 2 \\ q(x) = -x^2 + 3x - 4 \end{array} \right\} (3 \cdot p(x) + q(x)) \cdot (p(x) - 2 \cdot q(x))$$

Solución

$$a) \left. \begin{array}{l} p(x) = x^3 + 6x^2 - x + 2 \\ q(x) = 2x^2 + 4x - 3 \end{array} \right\} (p(x) + q(x)) \cdot (p(x) - q(x))$$

$$\left. \begin{array}{l} p(x) + q(x) = x^3 + 8x^2 + 3x - 1 \\ p(x) - q(x) = x^3 + 4x^2 - 5x + 5 \end{array} \right\}$$

$$(p(x) + q(x)) \cdot (p(x) - q(x)) = (x^3 + 8x^2 + 3x - 1) \cdot (x^3 + 4x^2 - 5x + 5)$$

$$\begin{aligned} (p(x) + q(x)) \cdot (p(x) - q(x)) &= x^6 + 8x^5 + 3x^4 - x^3 + \\ &\quad + 4x^5 + 32x^4 + 12x^3 - 4x^2 - \\ &\quad - 5x^4 - 40x^3 - 15x^2 + 5x + \\ &\quad + 5x^3 + 40x^2 + 15x - 5 \end{aligned}$$

$$(p(x) + q(x)) \cdot (p(x) - q(x)) = x^6 + 12x^5 + 30x^4 - 24x^3 + 21x^2 + 20x - 5$$

$$b) \left. \begin{array}{l} p(x) = x^2 - 5x + 2 \\ q(x) = -x^2 + 3x - 4 \end{array} \right\} (3 \cdot p(x) + q(x)) \cdot (p(x) - 2 \cdot q(x))$$

$$\left. \begin{array}{l} 3 \cdot p(x) + q(x) = 3x^2 - 15x + 6 - x^2 + 3x - 4 = 2x^2 - 12x + 2 \\ p(x) - 2 \cdot q(x) = x^2 - 5x + 2 + 2x^2 - 6x + 8 = 3x^2 - 11x + 10 \end{array} \right\}$$

$$(3 \cdot p(x) + q(x)) \cdot (p(x) - 2 \cdot q(x)) = (2x^2 - 12x + 2) \cdot (3x^2 - 11x + 10)$$

$$\begin{aligned} (3 \cdot p(x) + q(x)) \cdot (p(x) - 2 \cdot q(x)) &= 6x^4 - 36x^3 + 6x^2 - \\ &\quad - 22x^3 + 132x^2 - 22x + \\ &\quad + 20x^2 - 120x + 20 \end{aligned}$$

$$(3 \cdot p(x) + q(x)) \cdot (p(x) - 2 \cdot q(x)) = 6x^4 - 58x^3 + 158x^2 - 142x + 20$$

2.1-5 Calcula las siguientes potencias de binomios utilizando el desarrollo del binomio de Newton:

a) $(x+1)^4$

d) $(x^2+2)^3$

b) $(x-4)^3$

e) $(x^3+1)^5$

c) $(2x+3)^5$

f) $(3x^2-1)^4$

Solución

$$\begin{aligned} a) (x+1)^4 &= \binom{4}{0}x^4 + \binom{4}{1}x^3 \cdot 1^1 + \binom{4}{2}x^2 \cdot 1^2 + \binom{4}{3}x \cdot 1^3 + \binom{4}{4}1^4 = \\ &= \frac{4!}{4! \cdot 0!}x^4 + \frac{4!}{3! \cdot 1!}x^3 + \frac{4!}{2! \cdot 2!}x^2 + \frac{4!}{1! \cdot 3!}x + \frac{4!}{0! \cdot 4!} = \\ &= x^4 + 4x^3 + 6x^2 + 4x + 1 \end{aligned}$$

$$\begin{aligned} b) (x-4)^3 &= \binom{3}{0}x^3 + \binom{3}{1}x^2 \cdot (-4)^1 + \binom{3}{2}x \cdot (-4)^2 + \binom{3}{3}(-4)^3 = \\ &= \frac{3!}{3! \cdot 0!}x^3 - 4 \frac{3!}{2! \cdot 1!}x^2 + 16 \frac{3!}{1! \cdot 2!}x - 64 \frac{3!}{0! \cdot 3!} = \\ &= x^3 - 12x^2 + 48x - 64 \end{aligned}$$

$$\begin{aligned} c) (2x+3)^5 &= \\ &= \binom{5}{0}(2x)^5 + \binom{5}{1}(2x)^4 \cdot (3)^1 + \binom{5}{2}(2x)^3 \cdot (3)^2 + \\ &+ \binom{5}{3}(2x)^2 \cdot (3)^3 + \binom{5}{4}(2x)^1 \cdot (3)^4 + \binom{5}{5}(3)^5 = \\ &= 32 \frac{5!}{5! \cdot 0!}x^5 + 48 \frac{5!}{4! \cdot 1!}x^4 + 72 \frac{5!}{3! \cdot 2!}x^3 + \\ &+ 108 \frac{5!}{2! \cdot 3!}x^2 + 162 \frac{5!}{1! \cdot 4!}x + 243 \frac{5!}{0! \cdot 5!} = \\ &= 32x^5 + 240x^4 + 720x^3 + 1080x^2 + 810x + 243 \end{aligned}$$

$$\begin{aligned}
 d) \quad (x^2 + 2)^3 &= \binom{3}{0}(x^2)^3 + \binom{3}{1}(x^2)^2 \cdot (2)^1 + \binom{3}{2}(x^2)^1 \cdot (2)^2 + \binom{3}{3}(2)^3 = \\
 &= \frac{3!}{3! \cdot 0!} x^6 + 2 \frac{3!}{2! \cdot 1!} x^4 + 4 \frac{3!}{1! \cdot 2!} x^2 + 8 \frac{3!}{0! \cdot 3!} = \\
 &= x^6 + 6x^4 + 12x^2 + 8
 \end{aligned}$$

$$\begin{aligned}
 e) \quad (x^3 + 1)^5 &= \\
 &= \binom{5}{0}(x^3)^5 + \binom{5}{1}(x^3)^4 \cdot (1)^1 + \binom{5}{2}(x^3)^3 \cdot (1)^2 + \\
 &+ \binom{5}{3}(x^3)^2 \cdot (1)^3 + \binom{5}{4}(x^3)^1 \cdot (1)^4 + \binom{5}{5}(1)^5 = \\
 &= \frac{5!}{5! \cdot 0!} x^{15} + \frac{5!}{4! \cdot 1!} x^{12} + \frac{5!}{3! \cdot 2!} x^9 + \frac{5!}{2! \cdot 3!} x^6 + \frac{5!}{1! \cdot 4!} x^3 + \frac{5!}{0! \cdot 5!} = \\
 &= x^{15} + 5x^{12} + 10x^9 + 10x^6 + 5x^3 + 1
 \end{aligned}$$

$$\begin{aligned}
 f) \quad (3x^2 - 1)^4 &= \\
 &= \binom{4}{0}(3x^2)^4 + \binom{4}{1}(3x^2)^3 \cdot (-1)^1 + \\
 &+ \binom{4}{2}(3x^2)^2 \cdot (-1)^2 + \binom{4}{3}(3x^2)^1 \cdot (-1)^3 + \binom{4}{4}(-1)^4 = \\
 &= \frac{4!}{4! \cdot 0!} 81x^8 - \frac{4!}{3! \cdot 1!} 27x^6 + \frac{4!}{2! \cdot 2!} 9x^4 - \frac{4!}{1! \cdot 3!} 3x^2 + \frac{4!}{0! \cdot 4!} = \\
 &= 81x^8 - 108x^6 + 54x^4 - 12x^2 + 1
 \end{aligned}$$

2.1-6 Realiza la división entera de los siguientes polinomios:

$$a) \begin{aligned} p(x) &= x^2 - 6x + 4 \\ q(x) &= x^2 - 2 \end{aligned}$$

$$b) \begin{aligned} p(x) &= x^5 + 3x^3 + 6x - 2 \\ q(x) &= x^3 + x \end{aligned}$$

$$c) \begin{aligned} p(x) &= x^6 + 3 \\ q(x) &= x^2 + 2x - 4 \end{aligned}$$

$$d) \begin{aligned} p(x) &= x^6 + 2x^5 - x^4 + 2x^3 + 6x^2 - x + 3 \\ q(x) &= x^3 + 10x^2 - 2x + 3 \end{aligned}$$

Solución

$$a) \begin{cases} p(x) = x^2 - 6x + 4 \\ q(x) = x^2 - 2 \end{cases} \Rightarrow p(x) = c(x) \cdot q(x) + r(x)$$

$$\begin{array}{r} x^2 - 6x + 4 \quad | \quad x^2 - 2 \\ \underline{-x^2 + 2} \quad 1 \\ -6x + 6 \end{array}$$

$$\begin{cases} c(x) = 1 \\ r(x) = -6x + 6 \end{cases} \Rightarrow x^2 - 6x + 4 = 1 \cdot (x^2 - 2) + (-6x + 6)$$

$$b) \begin{cases} p(x) = x^5 + 3x^3 + 6x - 2 \\ q(x) = x^3 + x \end{cases} \Rightarrow p(x) = c(x) \cdot q(x) + r(x)$$

$$\begin{array}{r} x^5 + 3x^3 + 6x - 2 \quad | \quad x^3 + x \\ \underline{-x^5 - x^3} \quad x^2 + 2 \\ 2x^3 + 6x - 2 \\ \underline{-2x^3 - 2x} \\ 4x - 2 \end{array}$$

$$\begin{cases} c(x) = x^2 + 2 \\ r(x) = 4x - 2 \end{cases} \Rightarrow x^5 + 3x^3 + 6x - 2 = (x^2 + 2) \cdot (x^3 + x) + (4x - 2)$$

$$c) \left. \begin{array}{l} p(x) = x^6 + 3 \\ q(x) = x^2 + 2x - 4 \end{array} \right\} \Rightarrow p(x) = c(x) \cdot q(x) + r(x)$$

$$\begin{array}{r} x^6 + 3 \\ \underline{-x^6 - 2x^5 + 4x^4} \\ -2x^5 + 4x^4 + 3 \\ \underline{2x^5 + 4x^4 - 8x^3} \\ 8x^4 - 8x^3 + 3 \\ \underline{-8x^4 - 16x^3 + 32x^2} \\ -24x^3 + 32x^2 + 3 \\ \underline{24x^3 + 48x^2 - 96x} \\ 80x^2 - 96x + 3 \\ \underline{-80x^2 - 160x + 320} \\ -256x + 323 \end{array} \quad \left| \begin{array}{l} x^2 + 2x - 4 \\ \hline x^4 - 2x^3 + 8x^2 - 24x + 80 \end{array} \right.$$

$$\left. \begin{array}{l} c(x) = x^4 - 2x^3 + 8x^2 - 24x + 80 \\ r(x) = -256x + 323 \end{array} \right\}$$

$$\Rightarrow x^6 + 3 = (x^4 - 2x^3 + 8x^2 - 24x + 80) \cdot (x^2 + 2x - 4) + (-256x + 323)$$

$$d) \left. \begin{array}{l} p(x) = x^6 + 2x^5 - x^4 + 2x^3 + 6x^2 - x + 3 \\ q(x) = x^3 + 10x^2 - 2x + 3 \end{array} \right\} \Rightarrow p(x) = c(x) \cdot q(x) + r(x)$$

$$\begin{array}{r} x^6 + 2x^5 - x^4 + 2x^3 + 6x^2 - x + 3 \\ \underline{-x^6 - 10x^5 + 2x^4 - 3x^3} \\ -8x^5 + x^4 - x^3 + 6x^2 - x + 3 \\ \underline{8x^5 + 80x^4 - 16x^3 + 24x^2} \\ 81x^4 - 17x^3 + 30x^2 - x + 3 \\ \underline{-81x^4 - 810x^3 + 162x^2 - 243x} \\ -827x^3 + 192x^2 - 244x + 3 \\ \underline{827x^3 + 8270x^2 - 1654x + 2481} \\ 8462x^2 - 1898x + 2484 \end{array} \quad \left| \begin{array}{l} x^3 + 10x^2 - 2x + 3 \\ \hline x^3 - 8x^2 + 81x - 827 \end{array} \right.$$

$$\left. \begin{array}{l} c(x) = x^3 - 8x^2 + 81x - 827 \\ r(x) = 8462x^2 - 1898x + 2484 \end{array} \right\}$$

$$\Rightarrow x^6 + 2x^5 - x^4 + 2x^3 + 6x^2 - x + 3 =$$

$$= (x^3 - 8x^2 + 81x - 827) \cdot (x^3 + 10x^2 - 2x + 3) + (8462x^2 - 1898x + 2484)$$

2.1-7 Factoriza según sus raíces reales los siguientes polinomios:

a) $p(x) = x^2 - 4x - 5$

b) $p(x) = x^3 - 5x^2 + 6x$

c) $p(x) = x^4 - 13x^2 + 36$

d) $p(x) = x^4 - 16$

Solución

a) $p(x) = x^2 - 4x - 5$

$$x^2 - 4x - 5 = 0 \Rightarrow x = \frac{4 \pm \sqrt{16 + 20}}{2} = \frac{4 \pm 6}{2} = 5, -1$$

$$p(x) = x^2 - 4x - 5 = (x - 5) \cdot (x + 1)$$

b) $p(x) = x^3 - 5x^2 + 6x$

$$x^3 - 5x^2 + 6x = x \cdot (x^2 - 5x + 6) = 0 \Rightarrow \begin{cases} x = 0 \\ x^2 - 5x + 6 = 0 \end{cases}$$

$$x^2 - 5x + 6 = 0 \Rightarrow x = \frac{5 \pm \sqrt{25 - 24}}{2} = \frac{5 \pm 1}{2} = 3, 2$$

$$p(x) = x^3 - 5x^2 + 6x = x \cdot (x - 3) \cdot (x - 2)$$

c) $p(x) = x^4 - 13x^2 + 36$

$$x^2 = t \Rightarrow x^4 - 13x^2 + 36 = 0 \Rightarrow t^2 - 13t + 36 = 0$$

$$t^2 - 13t + 36 = 0 \Rightarrow x^2 = t = \frac{13 \pm \sqrt{169 - 144}}{2} = \frac{13 \pm 5}{2} = 9, 4$$

$$x = \pm\sqrt{9}, \pm\sqrt{4} = 3, -3, 2, -2$$

$$p(x) = x^4 - 13x^2 + 36 = (x - 3) \cdot (x + 3) \cdot (x - 2) \cdot (x + 2)$$

d) $p(x) = x^4 - 16 = (x^2 - 4) \cdot (x^2 + 4)$

$$(x^2 - 4) \cdot (x^2 + 4) = 0 \Rightarrow \begin{cases} x^2 - 4 = 0 \\ x^2 + 4 = 0 \end{cases} \Rightarrow \begin{cases} x = 2, -2 \\ x^2 = \pm\sqrt{-4} \notin \mathbb{R} \end{cases}$$

$$p(x) = x^4 - 16 = (x^2 - 4) \cdot (x^2 + 4) = (x - 2) \cdot (x + 2) \cdot (x^2 + 4)$$

2.1-8 Factoriza según sus raíces reales los siguientes polinomios utilizando la regla de Ruffini:

$$a) p(x) = x^3 - 3x^2 - 6x + 8$$

$$b) p(x) = x^3 + 3x^2 - 13x - 15$$

$$c) p(x) = x^3 - 7x^2 + x - 7$$

$$d) p(x) = x^4 - 2x^3 - x + 2$$

$$e) p(x) = x^5 - x^4 - 10x^3 + 9x^2 + 9x$$

$$f) p(x) = x^4 - \frac{5}{6}x^3 - \frac{7}{3}x^2 - \frac{1}{6}x + \frac{1}{3}$$

Solución

$$a) p(x) = x^3 - 3x^2 - 6x + 8$$

Divisores enteros del término independiente: 1, -1, 2, -2, 4, -4, 8, -8

$$\begin{array}{r|rrrr} & 1 & -3 & -6 & 8 \\ 1 & & 1 & -2 & -8 \\ \hline & 1 & -2 & -8 & \boxed{0} \end{array} \quad \text{Resto cero}$$

$$p(x) = x^3 - 3x^2 - 6x + 8 = (x^2 - 2x - 8) \cdot (x - 1)$$

Ahora, el polinomio de grado 2 se puede factorizar obteniendo sus raíces:

$$x^2 - 2x - 8 = 0 \Rightarrow x = \frac{2 \pm \sqrt{4 + 32}}{2} = \frac{2 \pm 6}{2} = 4, -2$$

Por tanto:

$$p(x) = x^3 - 3x^2 - 6x + 8 = (x - 4) \cdot (x + 2) \cdot (x - 1)$$

$$b) p(x) = x^3 + 3x^2 - 13x - 15$$

Divisores enteros del término independiente: 1, -1, 3, -3, 5, -5, 15, -15

$$\begin{array}{r|rrrr} & 1 & 3 & -13 & -15 \\ -1 & & -1 & -2 & 15 \\ \hline & 1 & 2 & -15 & \boxed{0} \end{array} \quad \text{Resto cero}$$

$$p(x) = x^3 + 3x^2 - 13x - 15 = (x^2 + 2x - 15) \cdot (x + 1)$$

Ahora, el polinomio de grado 2 se puede factorizar obteniendo sus raíces:

$$x^2 + 2x - 15 = 0 \Rightarrow x = \frac{-2 \pm \sqrt{4 + 60}}{2} = \frac{-2 \pm 8}{2} = 3, -5$$

Por tanto:

$$p(x) = x^3 + 3x^2 - 13x - 15 = (x - 3) \cdot (x + 5) \cdot (x + 1)$$

c) $p(x) = x^3 - 7x^2 + x - 7$

Divisores enteros del término independiente: 1, -1, 7, -7

$$\begin{array}{r|rrrr} & 1 & -7 & 1 & -7 \\ 7 & & 7 & 0 & 7 \\ \hline & 1 & 0 & 1 & 0 \end{array} \quad \text{Resto cero}$$

$$p(x) = x^3 - 7x^2 + x - 7 = (x^2 + 1) \cdot (x - 7)$$

Ahora, el polinomio de grado 2 se puede factorizar obteniendo sus raíces:

$$x^2 + 1 = 0 \Rightarrow x^2 = -1 \Rightarrow x = \pm\sqrt{-1}, \text{ que no son raíces reales.}$$

Por tanto:

$$p(x) = x^3 - 7x^2 + x - 7 = (x^2 + 1) \cdot (x - 7)$$

d) $p(x) = x^4 - 2x^3 - x + 2$

Divisores enteros del término independiente: 1, -1, 2, -2

$$\begin{array}{r|rrrrr} & 1 & -2 & 0 & -1 & 2 \\ 1 & & 1 & -1 & -1 & -2 \\ \hline & 1 & -1 & -1 & -2 & 0 \end{array} \quad \text{Resto cero}$$

$$p(x) = x^4 - 2x^3 - x + 2 = (x^3 - x^2 - x - 2) \cdot (x - 1)$$

Para el polinomio de grado 3 que resulta utilizamos otra vez Ruffini con los divisores enteros del término independiente: 1, -1, 2, -2

$$\begin{array}{r|rrrr} & 1 & -1 & -1 & -2 \\ 2 & & 2 & 2 & 2 \\ \hline & 1 & 1 & 1 & 0 \end{array} \quad \text{Resto cero}$$

Por tanto:

$$p(x) = x^4 - 2x^3 - x + 2 = (x^2 + x + 1) \cdot (x - 2) \cdot (x - 1)$$

Ahora, el polinomio de grado 2 se puede factorizar obteniendo sus raíces:

$$x^2 + x + 1 = 0 \Rightarrow x = \frac{-1 \pm \sqrt{1-4}}{2}, \text{ que no son raíces reales.}$$

Por tanto:

$$p(x) = x^4 - 2x^3 - x + 2 = (x^2 + x + 1) \cdot (x - 2) \cdot (x - 1)$$

e) $p(x) = x^5 - x^4 - 10x^3 + 9x^2 + 9x$

Podemos factorizar fácilmente el polinomio sacando x factor común:

$$p(x) = x^5 - x^4 - 10x^3 + 9x^2 + 9x = (x^4 - x^3 - 10x^2 + 9x + 9) \cdot x$$

Para el polinomio de grado 4 que resulta utilizamos Ruffini con los divisores enteros del término independiente: 1, -1, 3, -3, 9, -9

$$\begin{array}{r|rrrrr} & 1 & -1 & -10 & 9 & 9 \\ 3 & & 3 & 6 & -12 & -9 \\ \hline & 1 & 2 & -4 & -3 & 0 \end{array} \quad \text{Resto cero}$$

$$p(x) = x^5 - x^4 - 10x^3 + 9x^2 + 9x = (x^3 + 2x^2 - 4x - 3) \cdot (x - 3) \cdot x$$

Para el polinomio de grado 3 que resulta utilizamos otra vez Ruffini con los divisores enteros del término independiente: 1, -1, 3, -3

$$\begin{array}{r|rrrr} & 1 & 2 & -4 & -3 \\ -3 & & -3 & 3 & 3 \\ \hline & 1 & -1 & -1 & 0 \end{array} \quad \text{Resto cero}$$

Por tanto:

$$p(x) = x^5 - x^4 - 10x^3 + 9x^2 + 9x = (x^2 - x - 1) \cdot (x + 3) \cdot (x - 3) \cdot x$$

Ahora, el polinomio de grado 2 se puede factorizar obteniendo sus raíces:

$$x^2 - x - 1 = 0 \Rightarrow x = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 + \sqrt{5}}{2}, \frac{1 - \sqrt{5}}{2}, \text{ que son raíces reales.}$$

Por tanto:

$$p(x) = \left(x - \frac{1+\sqrt{5}}{2}\right) \cdot \left(x - \frac{1-\sqrt{5}}{2}\right) \cdot (x+3) \cdot (x-3) \cdot x$$

$$f) p(x) = x^4 - \frac{5}{6}x^3 - \frac{7}{3}x^2 - \frac{1}{6}x + \frac{1}{3}$$

Como el término independiente no es entero y aparecen términos racionales, multiplicamos todo el polinomio por 6:

$$6p(x) = 6x^4 - 5x^3 - 14x^2 - x + 2$$

Para este polinomio de grado 4 que resulta utilizamos Ruffini con los divisores enteros del término independiente: 1, -1, 2, -2

$$\begin{array}{r|rrrrr} 2 & 6 & -5 & -14 & -1 & 2 \\ & & 12 & 14 & 0 & -2 \\ \hline & 6 & 7 & 0 & -1 & 0 \end{array} \quad \text{Resto cero}$$

$$6p(x) = 6x^4 - 5x^3 - 14x^2 - x + 2 = (6x^3 + 7x^2 - 1) \cdot (x - 2)$$

Para el polinomio de grado 3 que resulta utilizamos otra vez Ruffini con los divisores enteros del término independiente: 1, -1

$$\begin{array}{r|rrrr} -1 & 6 & 7 & 0 & -1 \\ & & -6 & -1 & 1 \\ \hline & 6 & 1 & -1 & 0 \end{array} \quad \text{Resto cero}$$

Por tanto:

$$6p(x) = 6x^4 - 5x^3 - 14x^2 - x + 2 = (6x^2 + x - 1) \cdot (x+1) \cdot (x-2)$$

Ahora, el polinomio de grado 2 se puede factorizar obteniendo sus raíces:

$$6x^2 + x - 1 = 0 \Rightarrow x = \frac{-1 \pm \sqrt{1+24}}{12} = \frac{-1 \pm 5}{12} = \frac{1}{3}, -\frac{1}{2}, \text{ son raíces reales.}$$

Por tanto:

$$6p(x) = 6x^4 - 5x^3 - 14x^2 - x + 2 = 6 \cdot \left(x - \frac{1}{3}\right) \cdot \left(x + \frac{1}{2}\right) \cdot (x+1) \cdot (x-2)$$

Finalmente:

$$p(x) = x^4 - \frac{5}{6}x^3 - \frac{7}{3}x^2 - \frac{1}{6}x + \frac{1}{3} = \left(x - \frac{1}{3}\right) \cdot \left(x + \frac{1}{2}\right) \cdot (x+1) \cdot (x-2)$$
