

Deriva las siguientes expresiones:

a) $y = \text{sen}2x$; b) $y = \text{cos}(x-1)$; c) $y = \text{cos}3x^2$;

d) $y = \frac{x}{\sqrt[3]{x^2+4}}$; e) $y = \left(\frac{3x-1}{x^2+3}\right)^2$; f) $y = \sqrt{\text{sen}4x}$

Solución

a) $y = \text{sen}2x$. Derivada del seno por la derivada del ángulo:

$$y = \text{sen}2x \Rightarrow y' = \text{cos}2x \cdot (2x)' \Rightarrow y' = 2 \cdot \text{cos}2x$$

b) $y = \text{cos}(x-1)$. Derivada del coseno por la derivada del ángulo:

$$y = \text{cos}(x-1) \Rightarrow y' = -\text{sen}(x-1) \cdot (x-1)' \Rightarrow y' = -\text{sen}(x-1)$$

c) $y = \text{cos}3x^2$. Derivada del coseno por la derivada del ángulo:

$$y = \text{cos}3x^2 \Rightarrow y' = -\text{sen}3x^2 \cdot (3x^2)' \Rightarrow y' = -6x \cdot \text{sen}3x^2$$

d) $y = \frac{x}{\sqrt[3]{x^2+4}}$. Derivada de un cociente:

$$\begin{aligned} y &= \frac{x}{\sqrt[3]{x^2+4}} = \frac{x}{(x^2+4)^{1/3}} \\ y' &= \frac{(x^2+4)^{1/3} - \left((x^2+4)^{1/3}\right)' \cdot x}{\left((x^2+4)^{1/3}\right)^2} = \frac{(x^2+4)^{1/3} - \frac{1}{3}x \cdot (x^2+4)^{-2/3} \cdot 2x}{(x^2+4)^{2/3}} = \\ &= \frac{1}{3}(x^2+4)^{-2/3} \left[\frac{3(x^2+4) - 2x^2}{(x^2+4)^{2/3}} \right] = \frac{x^2+12}{3(x^2+4)^{4/3}} \end{aligned}$$

e) $y = \left(\frac{3x-1}{x^2+3}\right)^2$. Derivada de una potencia:

$$\begin{aligned} y' &= 2 \cdot \frac{3x-1}{x^2+3} \cdot \left(\frac{3x-1}{x^2+3}\right)' = \\ &= 2 \cdot \frac{3x-1}{x^2+3} \cdot \frac{3 \cdot (x^2+3) - 2x \cdot (3x-1)}{(x^2+3)^2} = \frac{2(3x-1)(-3x^2+2x+9)}{(x^2+3)^3} \end{aligned}$$

f) $y = \sqrt{\text{sen}4x}$. Derivada de una raíz cuadrada:

$$y = \sqrt{\text{sen}4x} \Rightarrow y = (\text{sen}4x)^{1/2}$$

$$y' = \frac{1}{2} \cdot (\text{sen}4x)^{-1/2} \cdot (\text{sen}4x)' = \frac{1}{2} (\text{sen}4x)^{-1/2} \cdot 4 \text{cos}4x = \frac{2 \text{cos}4x}{\sqrt{\text{sen}4x}}$$

Derivar:

$$y = \frac{2}{\sqrt{3}} \operatorname{arctag} \left(\frac{\operatorname{tag} \left(\frac{x}{2} \right)}{\sqrt{3}} \right) - \frac{1}{\sqrt{2}} \operatorname{arctag} \left(\frac{\operatorname{tag} \left(\frac{x}{2} \right)}{\sqrt{2}} \right)$$

Solución

$$\begin{aligned} y' &= \frac{2}{\sqrt{3}} \frac{1}{1 + \frac{\operatorname{tag}^2 \left(\frac{x}{2} \right)}{3}} \cdot \frac{1}{2\sqrt{3} \cos^2 \left(\frac{x}{2} \right)} - \frac{1}{\sqrt{2}} \frac{1}{1 + \frac{\operatorname{tag}^2 \left(\frac{x}{2} \right)}{2}} \cdot \frac{1}{2\sqrt{2} \cos^2 \left(\frac{x}{2} \right)} = \\ &= \frac{1}{\left(3 + \operatorname{tag}^2 \left(\frac{x}{2} \right) \right) \cos^2 \left(\frac{x}{2} \right)} - \frac{1}{\left(4 + 2\operatorname{tag}^2 \left(\frac{x}{2} \right) \right) \cos^2 \left(\frac{x}{2} \right)} = \\ &= \frac{1}{3 \cos^2 \left(\frac{x}{2} \right) + \operatorname{tag}^2 \left(\frac{x}{2} \right) \cdot \cos^2 \left(\frac{x}{2} \right)} - \frac{1}{4 \cos^2 \left(\frac{x}{2} \right) + 2\operatorname{tag}^2 \left(\frac{x}{2} \right) \cdot \cos^2 \left(\frac{x}{2} \right)} \end{aligned}$$

Hemos de tener en cuenta que:

$$\operatorname{tag}^2 \left(\frac{x}{2} \right) = \frac{\operatorname{sen}^2 \left(\frac{x}{2} \right)}{\cos^2 \left(\frac{x}{2} \right)} \Rightarrow \operatorname{tag}^2 \frac{x}{2} \cdot \cos^2 \left(\frac{x}{2} \right) = \frac{\operatorname{sen}^2 \left(\frac{x}{2} \right)}{\cos^2 \left(\frac{x}{2} \right)} \cdot \cos^2 \left(\frac{x}{2} \right) = \operatorname{sen}^2 \left(\frac{x}{2} \right)$$

Por lo tanto:

$$\begin{aligned} y' &= \frac{1}{3 \cos^2 \left(\frac{x}{2} \right) + \operatorname{sen}^2 \left(\frac{x}{2} \right)} - \frac{1}{4 \cos^2 \left(\frac{x}{2} \right) + 2\operatorname{sen}^2 \left(\frac{x}{2} \right)} = \\ &= \left\{ \cos^2 \left(\frac{x}{2} \right) + \operatorname{sen}^2 \left(\frac{x}{2} \right) = 1 \right\} = \\ &= \frac{1}{1 + 2 \cos^2 \left(\frac{x}{2} \right)} - \frac{1}{2 + 2 \cos^2 \left(\frac{x}{2} \right)} = \left\{ \cos^2 \left(\frac{x}{2} \right) = \frac{1 + \cos x}{2} \right\} = \\ &= \frac{1}{1 + 1 + \cos x} - \frac{1}{2 + 1 + \cos x} = \frac{3 + \cos x - (2 + \cos x)}{(2 + \cos x)(3 + \cos x)} = \frac{1}{(2 + \cos x)(3 + \cos x)} \end{aligned}$$

Derivar:

$$y = \operatorname{arc}.\cos \left(\frac{1 - x^2}{1 + x^2} \right)$$

Solución

Derivada del arco coseno:

$$\begin{aligned}y' &= \frac{-1}{\sqrt{1-\left(\frac{1-x^2}{1+x^2}\right)^2}} \cdot \frac{-2x(1+x^2)-2x(1-x^2)}{(1+x^2)^2} = \frac{2x+2x^3+2x-2x^3}{(1+x^2)^2 \sqrt{1-\frac{1-2x^2+x^4}{1+2x^2+x^4}}} = \\ &= \frac{4x}{(1+x^2)^2 \sqrt{\frac{4x^2}{(1+x^2)^2}}} = \frac{4x}{2x(1+x^2)} = \frac{2}{1+x^2}\end{aligned}$$

Derivar:

$$y = (1 + \sqrt{1+x})^{3/2} - 3(1 + \sqrt{1+x})^{1/2}$$

Solución

Derivada de potencias:

$$\begin{aligned}y' &= \frac{3}{2}(1 + \sqrt{1+x})^{1/2} \cdot \frac{1}{2\sqrt{1+x}} - \frac{3}{2}(1 + \sqrt{1+x})^{-1/2} \cdot \frac{1}{2\sqrt{1+x}} = \\ &= \frac{3}{4\sqrt{1+x}} \left(\sqrt{1 + \sqrt{1+x}} - \frac{1}{\sqrt{1 + \sqrt{1+x}}} \right) = \frac{3}{4\sqrt{1+x}} \left(\frac{1 + \sqrt{1+x} - 1}{\sqrt{1 + \sqrt{1+x}}} \right) = \\ &= \frac{3}{4\sqrt{1 + \sqrt{1+x}}}\end{aligned}$$

Derivar:

$$y = \operatorname{arctag} x + \operatorname{arctag} \left(\frac{1}{x} \right) + \operatorname{arcsen} x + \operatorname{arccos} \sqrt{1-x^2}$$

Solución

$$\begin{aligned}y' &= \frac{1}{1+x^2} + \frac{1}{1+\left(\frac{1}{x^2}\right)} \cdot \frac{-1}{x^2} + \frac{1}{\sqrt{1-x^2}} + \frac{-1}{\sqrt{1-(1-x^2)}} \cdot \frac{-2x}{2\sqrt{1-x^2}} = \\ &= \frac{1}{1+x^2} - \frac{1}{1+x^2} + \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{x^2}} \cdot \frac{x}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}} = \\ &= \frac{2}{\sqrt{1-x^2}}\end{aligned}$$

Derivar:

$$y = \ln \sqrt[3]{1-x^4}$$

Solución

$$y = \ln \sqrt[3]{1-x^4} \Rightarrow y = \frac{1}{3} \ln(1-x^4)$$

Derivamos esta última expresión:

$$y' = \frac{1}{3} \cdot \frac{1}{(1-x^4)} \cdot (-4x^3) = -\frac{4x^3}{3(1-x^4)}$$

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