

RESOLUCIÓN DE ECUACIONES IRRACIONALES

Resuelve las siguientes ecuaciones irracionales:

1) $\sqrt{x^2 - 1} = x - 1$

2) $\sqrt{x + 2} = x - 4$

3) $\sqrt{x + 3} + x = 3$

4) $3x - \sqrt{x + 2} = 4$

5) $\sqrt{x + 2} + x = 3x - 2$

6) $3 - 2\sqrt{x} = x$

7) $\sqrt{x^2 - 5} = x - 5$

8) $\sqrt{2x^2 - 7} - x = 3$

9) $x + \sqrt{x^2 + 3} = -1$

10) $\sqrt{x^2 + 5} = x^2 - 1$

11) $\sqrt{(2-x)(1-x)} = x - 1$

12) $x - \sqrt{169 - x^2} = 17$

13) $\sqrt[3]{x + 7} = 3$

14) $\sqrt[3]{2 - x} = -2$

15) $\sqrt[3]{x^2 - 2x} = -1$

16) $\sqrt[3]{x^2 + x + 2} = 2$

Soluciones:

$$1) \sqrt{x^2 - 1} = x - 1$$

$$x^2 - 1 = (x - 1)^2 \Rightarrow x^2 - 1 = x^2 + 1 - 2x \Rightarrow 2x = 2 \Rightarrow \boxed{x = 1}$$

Comprobación:

$$\text{Si } x = 1 \Rightarrow \sqrt{1-1} = 1-1$$

$$2) \sqrt{x+2} = x-4$$

$$x+2 = (x-4)^2 \Rightarrow x+2 = x^2 - 8x + 16 \Rightarrow x^2 - 9x + 14 = 0$$

$$x = \frac{9 \pm \sqrt{9^2 - 4 \cdot 14}}{2} = \frac{9 \pm 5}{2} \Rightarrow \begin{cases} x_1 = \frac{9+5}{2} = 7 \Rightarrow \boxed{x_1 = 7} \\ x_2 = \frac{9-5}{2} = 2 \Rightarrow \boxed{x_2 = 2} \end{cases}$$

Comprobación:

$$\text{Si } x = 7 \Rightarrow \sqrt{7+2} = 7-4 \Rightarrow 3=3$$

$$\text{Si } x = 2 \Rightarrow \sqrt{2+2} = 2-4 \Rightarrow 2 \neq -2$$

$$3) \sqrt{x+3} + x = 3$$

$$\sqrt{x+3} = 3-x \Rightarrow x+3 = (3-x)^2 \Rightarrow x+3 = 9 - 6x + x^2 \Rightarrow x^2 - 7x + 6 = 0$$

$$x = \frac{7 \pm \sqrt{7^2 - 4 \cdot 6}}{2} = \frac{7 \pm 5}{2} \Rightarrow \begin{cases} x_1 = \frac{7+5}{2} = 6 \Rightarrow \boxed{x_1 = 6} \\ x_2 = \frac{7-5}{2} = 1 \Rightarrow \boxed{x_2 = 1} \end{cases}$$

Comprobación:

$$\text{Si } x = 6 \Rightarrow \sqrt{6+3} + 6 = 3 \Rightarrow 9 \neq 3$$

$$\text{Si } x = 1 \Rightarrow \sqrt{1+3} + 1 = 3 \Rightarrow 2+1=3$$

$$4) 3x - \sqrt{x+2} = 4$$

$$3x - 4 = \sqrt{x+2} \Rightarrow (3x-4)^2 = x+2 \Rightarrow 9x^2 - 24x + 16 = x+2 \Rightarrow 9x^2 - 25x + 14 = 0$$

$$x = \frac{25 \pm \sqrt{25^2 - 4 \cdot 9 \cdot 14}}{2 \cdot 9} = \frac{25 \pm 11}{18} \Rightarrow \begin{cases} x_1 = \frac{25+11}{18} = 2 \Rightarrow \boxed{x_1 = 2} \\ x_2 = \frac{25-11}{18} = \frac{14}{18} \Rightarrow \boxed{x_2 = \frac{7}{9}} \end{cases}$$

Comprobación:

$$\text{Si } x = 2 \Rightarrow 6 - \sqrt{2+2} = 4 \Rightarrow 6-2=4$$

$$\text{Si } x = \frac{7}{9} \Rightarrow \frac{21}{9} - \sqrt{\frac{7}{9}+2} = 4 \Rightarrow \frac{7}{3} - \frac{5}{3} \neq 4$$

5) $\sqrt{x+2} + x = 3x - 2$

$$\sqrt{x+2} = 2x - 2 \Rightarrow x + 2 = (2x - 2)^2 \Rightarrow x + 2 = 4x^2 - 8x + 4 \Rightarrow 4x^2 - 9x + 2 = 0$$

$$x = \frac{9 \pm \sqrt{9^2 - 4 \cdot 4 \cdot 2}}{2 \cdot 4} = \frac{9 \pm 7}{8} \Rightarrow \begin{cases} x_1 = \frac{9+7}{8} = 2 \Rightarrow \boxed{x_1 = 2} \\ x_2 = \frac{9-7}{8} = \frac{1}{4} \Rightarrow \boxed{x_2 = \frac{1}{4}} \end{cases}$$

Comprobación:

Si $x = 2 \Rightarrow 2 + \sqrt{2+2} = 6 - 2 \Rightarrow 2 + 2 = 4$

Si $x = \frac{7}{9} \Rightarrow \sqrt{\frac{1}{4} + 2} + \frac{1}{4} = \frac{3}{4} - 2 \Rightarrow \frac{3}{2} + \frac{1}{4} \neq -\frac{5}{4}$

6) $3 - 2\sqrt{x} = x$

$$3 - x = 2\sqrt{x} \Rightarrow (3 - x)^2 = 4x \Rightarrow 9 + x^2 - 6x = 4x \Rightarrow x^2 - 10x + 9 = 0$$

$$x = \frac{10 \pm \sqrt{10^2 - 4 \cdot 9}}{2} = \frac{10 \pm 8}{2} \Rightarrow \begin{cases} x_1 = \frac{10+8}{2} = 9 \Rightarrow \boxed{x_1 = 9} \\ x_2 = \frac{10-8}{2} = 1 \Rightarrow \boxed{x_2 = 1} \end{cases}$$

Comprobación:

Si $x = 9 \Rightarrow 3 - 2\sqrt{9} = 9 \Rightarrow 3 - 6 \neq 9$

Si $x = 1 \Rightarrow 3 - 2\sqrt{1} = 1 \Rightarrow 3 - 2 = 1$

7) $\sqrt{x^2 - 5} = x - 5$

$$x^2 - 5 = (x - 5)^2 \Rightarrow x^2 - 5 = x^2 - 10x + 25 \Rightarrow 10x = 30 \Rightarrow \boxed{x = 3} \Rightarrow \boxed{\text{No tiene solución}}$$

8) $\sqrt{2x^2 - 7} - x = 3$

$$\sqrt{2x^2 - 7} = 3 + x \Rightarrow 2x^2 - 7 = (x + 3)^2 \Rightarrow 2x^2 - 7 = x^2 + 6x + 9 \Rightarrow x^2 - 6x - 16 = 0$$

$$x = \frac{6 \pm \sqrt{6^2 - 4 \cdot (-16)}}{2} = \frac{6 \pm 10}{2} \Rightarrow \begin{cases} x_1 = \frac{6+10}{2} = 8 \Rightarrow \boxed{x_1 = 8} \\ x_2 = \frac{6-10}{2} = -2 \Rightarrow \boxed{x_2 = -2} \end{cases}$$

Comprobación:

Si $x = 8 \Rightarrow \sqrt{128 - 7} - 8 = 3 \Rightarrow 11 - 8 = 3$

Si $x = -2 \Rightarrow \sqrt{8 - 7} + 2 = 3 \Rightarrow 1 + 2 = 3$

$$9) x + \sqrt{x^2 + 3} = -1$$

$$\sqrt{x^2 + 3} = -1 - x \Rightarrow x^2 + 3 = (1 + x)^2 \Rightarrow x^2 + 3 = x^2 + 1 + 2x \Rightarrow 2x = 2 \Rightarrow \boxed{x=1} \Rightarrow \text{No tiene solución}$$

Comprobación:

$$\text{Si } x = 1 \Rightarrow 1 + \sqrt{1+3} = -1 \Rightarrow 1+2 \neq -1$$

$$10) \sqrt{x^2 + 5} = x^2 - 1$$

$$x^2 + 5 = (x^2 - 1)^2 \Rightarrow x^2 + 5 = x^4 - 2x^2 + 1 \Rightarrow x^4 - 3x^2 - 4 = 0 \Rightarrow y^2 - 3y - 4 = 0$$

$$y = \frac{3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot (-4)}}{2 \cdot 1} = \frac{3 \pm 5}{2} \Rightarrow \begin{cases} y_1 = \frac{3+5}{2} = 4 \Rightarrow \boxed{x_1 = \pm 2} \\ y_2 = \frac{3-5}{2} = -1 \end{cases}$$

Comprobación:

$$\text{Si } x = 2 \Rightarrow \sqrt{4+5} = 4-1 \Rightarrow 3=3$$

$$\text{Si } x = -2 \Rightarrow \sqrt{4+5} = 4-1 \Rightarrow 3=3$$

$$11) \sqrt{(2-x)(1-x)} = x-1$$

$$(2-x)(1-x) = (x-1)^2 \Rightarrow 2-2x-x+x^2 = x^2-2x+1 \Rightarrow \boxed{x=1}$$

$$12) x - \sqrt{169 - x^2} = 17$$

$$x - 17 = \sqrt{169 - x^2} \Rightarrow (x - 17)^2 = 169 - x^2 \Rightarrow x^2 + 289 - 34x = 169 - x^2 \Rightarrow 2x^2 + 120 - 34x = 0$$

$$x^2 - 17x + 60 = 0 \Rightarrow x = \frac{17 \pm \sqrt{17^2 - 4 \cdot 60}}{2} = \frac{17 \pm 7}{2} \Rightarrow \begin{cases} x_1 = \frac{17+7}{2} = 12 \Rightarrow \boxed{x_1=12} \\ x_2 = \frac{17-7}{2} = 5 \Rightarrow \boxed{x_2=5} \end{cases}$$

Comprobación:

$$\text{Si } x = 12 \Rightarrow 12 - \sqrt{169-144} = 17 \Rightarrow 12-5 \neq 17$$

$$\text{Si } x = 5 \Rightarrow 5 - \sqrt{169-25} = 17 \Rightarrow 5-12 \neq 17$$

$$13) \sqrt[3]{x+7} = 3$$

$$x + 7 = 27 \Rightarrow \boxed{x=20}$$

Comprobación:

$$\text{Si } x = 20 \Rightarrow \sqrt[3]{20+7} = \sqrt[3]{27} = 3$$

$$14) \sqrt[3]{2-x} = -2$$

$$2 - x = -8 \Rightarrow 10 = x$$

Comprobación:

$$\text{Si } x = 10 \Rightarrow \sqrt[3]{2-10} = \sqrt[3]{-8} = -2$$

$$15) \sqrt[3]{x^2 - 2x} = -1$$

$$x^2 - 2x = -1 \Rightarrow x^2 - 2x + 1 = 0 \Rightarrow (x - 1)^2 = 0 \Rightarrow \boxed{x=1}$$

Comprobación:

$$\text{Si } x = 1 \Rightarrow \sqrt[3]{1-2} = \sqrt[3]{-1} = -1$$

$$16) \sqrt[3]{x^2 + x + 2} = 2$$

$$x^2 + x + 2 = 8 \Rightarrow x^2 + x - 6 = 0$$

$$x = \frac{-1 \pm \sqrt{(-1)^2 - 4 \cdot (-6)}}{2} = \frac{-1 \pm 5}{2} \Rightarrow \begin{cases} x_1 = \frac{-1+5}{2} = 2 \Rightarrow \boxed{x=2} \\ x_2 = \frac{-1-5}{2} = -3 \Rightarrow \boxed{x=-3} \end{cases}$$

Comprobación:

$$\text{Si } x = 2 \Rightarrow \sqrt[3]{4+2+2} = \sqrt[3]{8} = 2$$

$$\text{Si } x = -3 \Rightarrow \sqrt[3]{9-3+2} = \sqrt[3]{8} = 2$$